

# Firm Dynamics with Frictional Product and Labor Markets

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**Work in progress**

# Motivation

- ▶ Firm dynamics and heterogeneity is central for the labor market and for aggregate outcomes (hires, separations, wages, productivity, ...).
- ▶ Much of the theoretical and quantitative literature considers shocks to *revenue* productivity to account for firm dynamics

(e.g. Hopenhayn & Rogerson 1993, Cooper, Haltiwanger & Willis 2007, Veracierto 2007, Elsby & Michaels 2013,...)

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- ▶ But supply and demand shocks affect firms differently.
- ▶ Foster, Haltiwanger and Syverson (2008, 2012):
  - ▶ Demand is important for firm growth (more than productivity)
  - ▶ Price dispersion: younger firms are more demand constrained and charge lower prices.

# Research Question

What are the respective roles of demand and productivity for the firm-level dynamics of prices, output, employment and wages?

# This paper

- ▶ Document the joint dynamics of prices, output, employment, working hours and wages for German manufacturing firms.
- ▶ Document patterns of price, labor productivity (and wage) dispersion.

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- ▶ Document the joint dynamics of prices, output, employment, working hours and wages for German manufacturing firms.
- ▶ Document patterns of price, labor productivity (and wage) dispersion.
- ▶ Develop an equilibrium model of firm dynamics with
  - ▶ product and labor market frictions
  - ▶ costly recruitment and sales, wage and price dispersion
  - ▶ separate roles for demand and supply shocks
- ▶ Quantitative evaluation, counterfactual experiments (in progress)

# Literature

## Price and productivity dispersion

Abbott 1992, Foster, Haltiwanger & Syverson 2008, 2012, Smeets & Warzynski 2013, Kugler & Verhoogen 2012

## Firm-level price and employment dynamics

Carlson & Skans 2012, Carlson, Messina & Skans 2014

## Firm dynamics with labor market frictions

Smith 1999, Veracierto 2007, Elsby & Michaels 2013, Acemoglu & Hawkins 2014, Kaas & Kircher 2015

## Firm dynamics with product market frictions

Gourio & Rudanko 2014

# Data (I)

- ▶ Administrative Firm Data (AFiD) of the German Federal Statistical Office.
- ▶ All establishments in manufacturing (& mining, quarrying) with  $\geq 20$  employees
- ▶ 1995–2014 (annual). (So far, we work with 2005–2007)
- ▶ Sales value and quantity for nine-digit product categories
- ▶ Employment, working hours, wages
- ▶ Detailed worker information (matched employer-employee) for subsample of establishments in 1996, 2001, 2006, 2010, 2014.



## Data (II)

- ▶ Consider one-establishment firms.
- ▶ Two samples of goods: [▶ Examples](#)
  - ▶ **Full**: All goods with quantity info  $\Rightarrow$  **Firm dynamics**
  - ▶ **Homogeneous**: Those measured in length, area, volume, or weight; drop goods produced by less than 6 firms  $\Rightarrow$  **Price & productivity dispersion**

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  - ▶ **Homogeneous**: Those measured in length, area, volume, or weight; drop goods produced by less than 6 firms ⇒ **Price & productivity dispersion**
- ▶ Drop firm observations where sample sales value is less than 50 percent of total sales:
  - ▶ Full sample: 61,034 firm-years, 13,177 product-years
  - ▶ Homogeneous sample: 38,651 firm-years, 3,730 product-years

## Price and productivity dispersion

- ▶  $\bar{P}_{jt}$  quantity-weighted mean price of good  $j$  in year  $t$ .
- ▶ Firm  $i$ 's relative price:

$$\tilde{P}_{it} = \frac{\sum_j P_{jit} Q_{jit}}{\sum_j \bar{P}_{jt} Q_{jit}}$$

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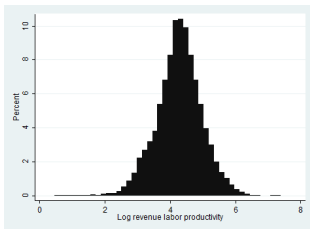
- ▶ Revenue and quantity labor productivity (per hour):

$$RLP_{it} = \frac{\sum_j Q_{jit} P_{jit}}{H_{it}} \quad , \quad QLP_{it} = \frac{\sum_j Q_{jit} \bar{P}_{jt}}{H_{it}} .$$

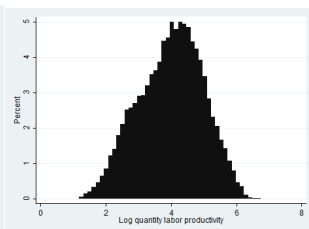
- ▶

$$RLP_{it} = \tilde{P}_{it} \cdot QLP_{it} .$$

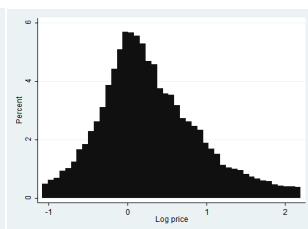
# Revenue and quantity productivity, and prices



RLP



QLP



$\tilde{P}$

## Correlations and standard deviations

Correlations	RLP	QLP	$\tilde{P}$	Empl.	wage/hour
RLP	1				
QLP	0.775	1			
$\tilde{P}$	-0.108	-0.712	1		
Empl	0.293	0.229	-0.035	1	
wage/h.	0.558	0.383	0.017	0.308	1
Std.dev.	0.697	0.986	0.629	0.871	0.374

All variables in logs.

▶ Weighted

▶ Residuals

# Firm dynamics

- ▶ Measure firm  $i$ 's output growth:

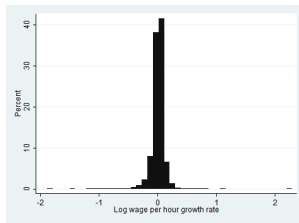
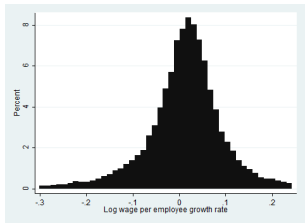
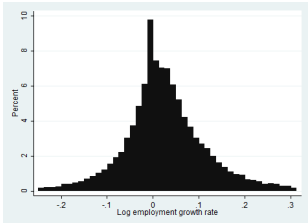
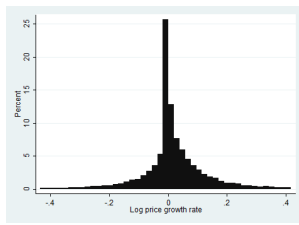
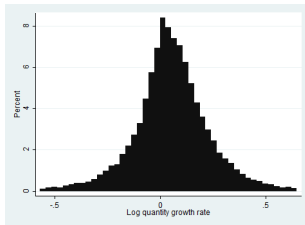
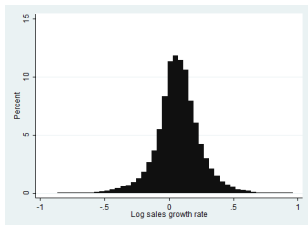
$$\frac{Q_{i,t+1}}{Q_{i,t}} = \frac{\sum_j P_{jit} Q_{ji,t+1}}{\sum_j P_{jit} Q_{jit}} .$$

- ▶ Log revenue growth is split into log output growth and log growth of the firm's Paasche price index:

$$\widehat{R}_{i,t} = \widehat{Q}_{i,t} + \widehat{P}_{i,t} .$$

- ▶ Further consider log growth rates of employment, hours, wages, revenue and quantity productivity.

# Firm growth rates





## Correlations and standard deviations

Correlations	$\widehat{PQ}$	$\widehat{Q}$	$\widehat{P}$	$\widehat{E}$	$\widehat{w/h}$
$\widehat{PQ}$	1				
$\widehat{Q}$	0.795	1			
$\widehat{P}$	0.284	-0.356	1		
$\widehat{E}$	0.307	0.276	0.035	1	
$\widehat{w/h}$	-0.009	-0.014	0.009	-0.013	1
Std.dev.	0.172	0.176	0.109	0.087	0.100

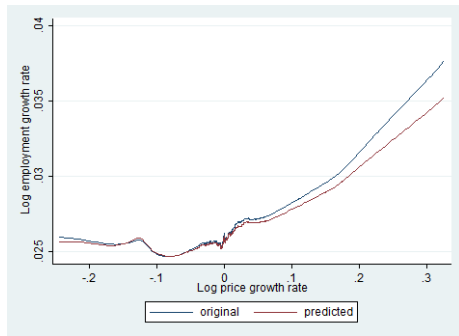
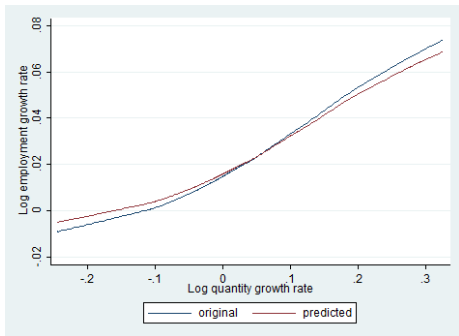
All variables are log growth rates.

Variance decomposition:  $\widehat{P}$  accounts for 18% of revenue growth and 16% of the growth of hourly labor productivity.

▶ Weighted

▶ Residuals

# Nonlinear relations between $\hat{P}$ , $\hat{Q}$ and $\hat{E}$



# The model

- ▶ Canonical model of firm dynamics with trading frictions in product and labor markets.
- ▶ Risk-neutral representative household with  $\bar{L}$  worker members and  $\bar{B}$  shopper members.
- ▶ A worker member supplies one unit of labor per period.
- ▶ A shopper member can buy one unit of a good per period.
- ▶ Household's preferences are

$$\sum_{t \geq 0} \beta^t \left[ e_t + \int y_t(f) c_t(f) d\mu_t(f) \right].$$

$e_t$  consumption of a numeraire good,

$y_t(f)$  firm-specific demand state (e.g. product quality),

$c_t(f)$  consumption of firm  $f$ 's output.

# Firms

- ▶ Consider a firm with  $L$  workers and  $B$  customers.
- ▶ Output  $xF(L)$  with  $F' > 0$ ,  $F'' \leq 0$ .  $x$  is firm-specific productivity.
- ▶ The firm sells  $B \leq xF(L)$  units of output. (Waste if inequality is strict).
- ▶  $z = (x, y)$  follows a Markov process.
- ▶ Any firm's policy depends on the shock history  $z^a$  where  $a$  is firm age (stationary equilibrium).

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- ▶ Any firm's policy depends on the shock history  $z^a$  where  $a$  is firm age (stationary equilibrium).
- ▶ Recruitment and sales activities are costly. With recruitment effort  $R$  and sales effort  $S$ , costs are  $r(R, L)$  and  $s(S, L)$ .
- ▶ Costs are increasing & convex in effort and possibly declining in size (scale effects).

## Search and matching

- ▶ Firms offer long-term wage contracts to new hires and price discounts (initial period) to new customers.
- ▶ Search is directed: Unemployed workers and unmatched shoppers search in submarkets that differ by their matching rates and match values.

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- ▶ Firm hires  $m(\lambda)R$  where  $\lambda$  are unemployed workers per unit of recruitment effort in the submarket ( $m' > 0$ ,  $m'' < 0$ ).
- ▶ Firm attracts  $q(\varphi)S$  new customers where  $\varphi$  are unmatched shoppers per unit of sales effort in the submarket ( $q' > 0$ ,  $q'' < 0$ ).

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- ▶ Matching rate for workers:  $m(\lambda)/\lambda$ .
- ▶ Matching rate for shoppers:  $q(\varphi)/\varphi$ .



## Separations, entry and exit

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- ▶ Firms exit with probability  $\delta$ .
- ▶ Firms choose customer separation rates  $\delta_b$ . Worker separation rates  $\delta_w$  are pre-committed in long-term contracts.
- ▶ Separation rates are bounded below by exogenous quit rates  $\bar{\delta}_w$  and  $\bar{\delta}_b$ .

## Stationary competitive search equilibrium

Value functions for workers  $U$ ,  $W$ , shoppers  $V$ ,  $Q$ , firms  $J$ , firm policies  $\lambda$ ,  $R$ ,  $\varphi$ ,  $S$ ,  $\delta_b$ ,  $C^a = (w^a(\cdot), \delta_w^a(\cdot))$ ,  $(L^\tau)_{\tau=0}^a$ ,  $L$ ,  $B$ ,  $p$ ,  $p^R$ , entrant firms  $N_0$ , and search values  $c^*$  and  $\rho^*$  such that

- (a) Workers search optimally.
- (b) Shoppers search optimally.
- (c) Firms' value functions  $J$  and policy functions solve the recursive firm problem. [▶ more](#)
- (d) Entry is optimal:

$$K = \sum_{z^0} \pi^0(z^0) J(0, z^0)$$

- (e) Aggregate resource feasibility:

$$\bar{L} = \sum_{z^a} N(z^a) \left\{ L(z^a) + [\lambda(z^a) - m(\lambda(z^a))] R(z^a) \right\},$$

$$\bar{B} = \sum_{z^a} N(z^a) \left\{ B(z^a) + [\varphi(z^a) - q(\varphi(z^a))] S(z^a) \right\}.$$

## Social optimality

The competitive search equilibrium is socially optimal.

Recursive planning problem: Maximize the social firm value

$$G(L_-, B_-, z) = \max \left\{ yB - bL - r(R, L_-(1 - \delta_w)) - s(S, L_-(1 - \delta_w)) \right. \\ \left. - \rho[L + (\lambda - m(\lambda))R] - c[B + (\varphi - q(\varphi))S] + \beta(1 - \delta)\mathbb{E}_z G(L, B, z_+) \right\},$$

subject to

$$\begin{aligned} L &= L_-(1 - \delta_w) + m(\lambda)R, \\ B &= B_-(1 - \delta_b) + q(\varphi)S, \\ B &\leq xF(L), \quad \delta_w \geq \bar{\delta}_w, \quad \delta_b \geq \bar{\delta}_b. \end{aligned}$$

$c$  and  $\rho$  are the social costs of shoppers and workers (multipliers on aggregate resource constraints).

## Firm policies

- ▶ Recruitment expenditures and job matching rates are positively related. If  $R > 0$ ,

$$r'_1(.) = \rho \left[ \frac{m(\lambda)}{m'(\lambda)} - \lambda \right]$$

- ▶ Sales expenditures and customer matching rates are positively related. If  $S > 0$ ,

$$s'_1(.) = c \left[ \frac{q(\varphi)}{q'(\varphi)} - \varphi \right]$$

- ▶ Faster growing firms offer higher salaries to workers and greater discounts to customers.

## Prices and revenue

- ▶ Discount price  $p = y - \frac{c\varphi}{q(\varphi)}$  falls in  $\varphi$  (and  $S$ ).
- ▶ Reservation price  $p^R = y - c$ .
- ▶ Younger firms charge lower prices to build a customer base.

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- ▶ Reservation price  $p^R = y - c$ .
- ▶ Younger firms charge lower prices to build a customer base.
- ▶ Revenue

$$p^R B_-(1 - \delta_b) + pq(\varphi)S$$



## Calibrated example

- ▶ Functional forms:

$$F(L) = L^\alpha, \quad r(R, L_0) = \frac{r_0}{1+\nu} \left(\frac{R}{L_0}\right)^\nu R, \quad s(S, L_0) = \frac{s_0}{1+\sigma} \left(\frac{S}{L_0}\right)^\sigma S,$$

$$m(\lambda) = m_0 \lambda^{0.5}, \quad q(\varphi) = q_0 \varphi^{0.5}.$$

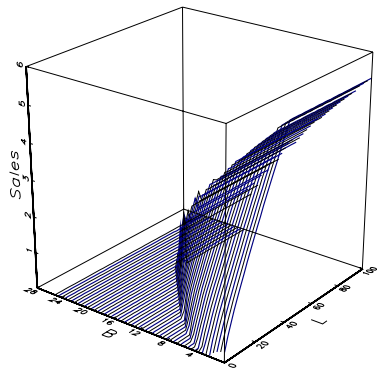
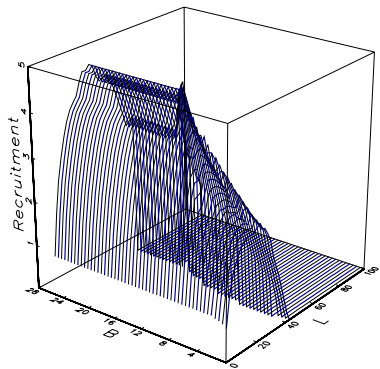
- ▶ Parameters

$$\alpha = 0.7, \quad \nu = \sigma = 2,$$

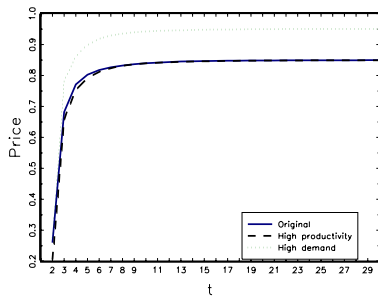
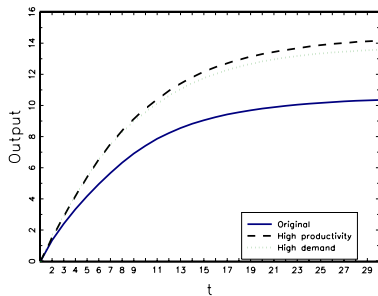
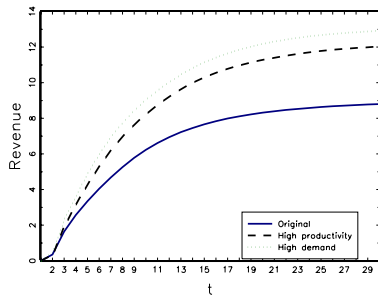
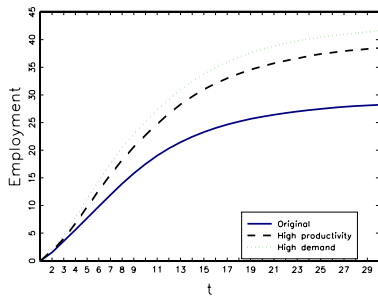
$$\bar{\delta}_w = 0.05, \quad \bar{\delta}_b = 0.15, \quad \delta = 0.05, \quad \beta = 0.96.$$

- ▶ Matching rates for workers (shoppers) are 0.49 (0.70).
- ▶  $x = y = 1$  (no heterogeneity).
- ▶ Expenditures for recruitment (sales) are 1%–2% of output.

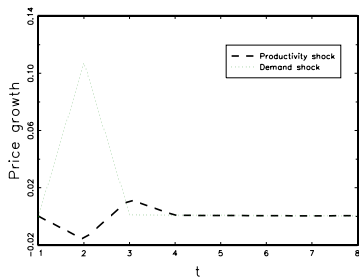
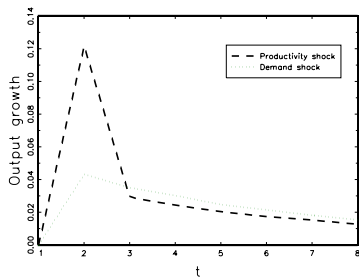
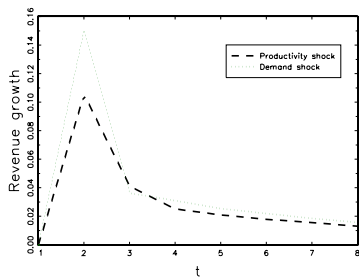
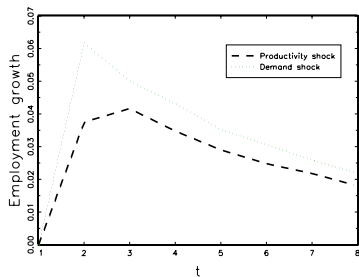
# Firm policies



# Firm growth



# Response to demand shock (dashed) and productivity shock (solid)



# Conclusions and outlook

- ▶ Firm dynamics with product and labor market frictions: separate role of demand shocks.
- ▶ Quantitative application: calibrate productivity and demand shocks to capture price and output dynamics.
- ▶ Implications for wage and price dispersion?
- ▶ Experiments:
  - ▶ Impact of product market regulation on the labor market?
  - ▶ Implications of aggregate demand versus aggregate productivity shocks?

## Examples of nine-digit products

- ▶ “Homogeneous” goods:
  - ▶ 1720 32 144 Fabric of synthetic fibers (with more than 85% synthetic) for curtains (measured in  $m^2$ ).
  - ▶ 2112 30 200 Cigarette paper, not in the form of booklets, husks, or rolls less than 5 cm broad (measured in  $t$ ).
  - ▶ 2125 14 130 Cigarette paper, in the form of booklets or husks (measured in  $kg$ ).
- ▶ Other goods
  - ▶ 1720 32 144 Sleeping bags (measured in “items”).
  - ▶ 2513 60 550 Gloves made of vulcanized rubber for housework usage (measured in “pairs”).
  - ▶ 2971 21 130 Vacuum cleaner with voltage 110 V or more (measured in “items”).

## Descriptive statistics

Correlations	RLP	QLP	$\tilde{P}$	Empl.	wage/hour
RLP	1				
QLP	0.790	1			
$\tilde{P}$	-0.142	-0.719	1		
Empl	0.387	0.315	-0.070	1	
wage/h.	0.543	0.381	0.000	0.439	1
Std.dev.	0.670	0.954	0.594	1.132	0.362

Statistics weighted by employment size. All variables in logs.

[▶ Back](#)

## Descriptive statistics

Correlations	RLP	QLP	$\tilde{P}$	Empl.	wage/hour
RLP	1				
QLP	0.623	1			
$\tilde{P}$	0.083	-0.686	1		
Empl	-0.092	-0.053	-0.010	1	
wage/h.	0.330	0.205	0.027	-0.049	1
Std.dev.	0.115	0.166	0.139	0.089	0.063

All variables in logs. Residuals after controlling for year, 2-digit industry and German region.

[▶ back](#)



## Correlations and standard deviations

Correlations	$\widehat{PQ}$	$\widehat{Q}$	$\widehat{P}$	$\widehat{E}$	$\widehat{w/h}$
$\widehat{PQ}$	1				
$\widehat{Q}$	0.782	1			
$\widehat{P}$	0.321	-0.339	1		
$\widehat{E}$	0.339	0.301	0.047	1	
$\widehat{w/h}$	-0.016	-0.024	0.012	-0.019	1
Std.dev.	0.159	0.160	0.105	0.076	0.090

Statistics weighted by employment size. All variables are log growth rates.

Variance decomposition:  $\widehat{P}$  accounts for 21% of revenue growth and 19% of the growth of hourly labor productivity.

▶ Back

## Correlations and standard deviations

Correlations	$\widehat{PQ}$	$\widehat{Q}$	$\widehat{P}$	$\widehat{E}$	$\widehat{w/h}$
$\widehat{PQ}$	1				
$\widehat{Q}$	0.792	1			
$\widehat{P}$	0.281	-0.364	1		
$\widehat{E}$	0.247	0.227	0.021	1	
$\widehat{w/h}$	-0.016	-0.016	0.001	-0.033	1
Std.dev.	0.154	0.160	0.099	0.062	0.088

Residuals after controlling for year, 2-digit industry and German region. All variables are log growth rates.

▶ Back

## Firms' problem

Firm with shock history  $z^a$  has state vector  $\sigma = [(L^\tau, C^\tau)_{\tau=0}^{a-1}, B_-, z^a]$ .

Recursive problem

$$J_a(\sigma) = \max_{(\lambda, R, C^a), (\delta_b, \varphi, S, p, p^R)} \left\{ p^R B_- (1 - \delta_b) + pq(\varphi)S - W - r(R, L_0) - s(S, L_0) + \beta(1 - \delta) \mathbb{E} J_{a+1}(\sigma_+) \right\} \quad \text{s.t.}$$

$$L^{\tau+} = (1 - \delta_w^\tau(z^a))L^\tau, \quad \tau = 0, \dots, a-1, \quad L^{a+} = m(\lambda)R, \quad L_0 = \sum_{\tau=0}^{a-1} L^{\tau+},$$

$$W = \sum_{\tau=0}^a w^\tau(z^a) L^{\tau+},$$

$$B = B_- (1 - \delta_b) + q(\varphi)S \leq xF(L), \quad L = \sum_{\tau=0}^a L^{\tau+},$$

$$\rho^* = \frac{m(\lambda)}{\lambda} [W(C^a, z^a) - b - \beta U] \quad \text{if } \lambda > 0,$$

$$p = y_a - \frac{c^* \varphi}{q(\varphi)} \quad \text{if } \varphi > 0, \quad p^R = y_a - c^*.$$