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Abstract

The recent contributions in directed technological change theory show that technological progress in clean industries (i.e. industries which do no produce CO2 emissions) lead to emission decline only if clean and dirty goods are sufficiently substitutable. The result raises a question whether a government could design a policy which increases this substitutability. In this paper I show that the elasticity of substitution between clean and dirty goods increases with the number of varieties of the clean good. This is shown in the theoretical model, which combines the insights from the directed technological change literature and discrete choice literature. The policy implications of the finding is that a policy that promotes development of clean industries should be supplemented with a policy that ensures the diversity of clean goods. For example a subsidy for R&D in clean transport should support a wide range of alternative technologies rather than selected few.

Keywords: Green growth, directed technological change, clean and dirty goods substitutability, optimal variety.

JEL Codes: O44, D11, Q55

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1 Introduction

In a 1950 paper, Edward Chamberlin observed that allowing for some monopoly power is a necessary condition for sustaining product diversity. Further, he argued that variety might benefit the society because it gives a higher chance that the individual needs of consumers are satisfied. Nearly 30 years later, Dixit and Stiglitz attempted to formalize Chamberlin's argument in an economic model. Their model possessed two features that turned out to be useful for other macroeconomic models: first, it endogenously generates product variety, and second, it provided a simple way to derive monopoly power and avoid an infinite elasticity of demand for firms. In fact, the elasticity of demand became a simple function of the elasticity of substitution between goods. These two features made the framework a convinient setup for numerous macroeconomic models.

Although the original motivation of Dixit and Stiglitz was to formalize Chamberlin's logic, the model is not an exact illustriation of his argument. Specifically, although the model shows how monopoly power could be derived from the consumers' love for variety, the source of the love for variety in the Dixit-Stiglitz model and in Chamberlin's argument differ substantially. In the Dixit-Stiglitz model, it originates from the convexity of indifference curves of a representative consumer. Chamberlin in turn argued that variety is desired because consumers' tastes differ, and a higher number of products implies a higher chance that the taste of an individual consumer will be matched. To give an illustrative example, according to Chamberlin, the reason why Mercedes and Lexus could both enjoy monopoly power is that there are consumers who value the former much more than the latter brand, and there are other consumers who value latter more than the former. Hence, an increase in price of one of the brands will have little impact on consumer choices. According to the Dixit and Stiglitz model, the reason why Mercedes and Lexus could exercise monopoly power is that a representative consumer does not consider the two brands to be perfect substitutes.

Intuitively, the Dixit-Stiglitz model and Chamberlin's arguments are related: a higher heterogeneity of taste in population of consumers would correspond to a smaller elasticity of substitution of the representative consumer. This paper intends to establish a formal link between these two variables. By taking the individual consumer optimization as a starting point, it sets the microfoundation for the Dixit-Stiglitz model. It derives how the convexity of indifference curves of the representative consumer (i.e. at the aggregate level) depends on the elasticity of substitution of *individual* consumers and the variance of valuation of goods by different consumers (heterogeneity of taste). It turns out that the elasticity of substitution of the representative agent (and the elasticity of demand for each good) can be written as a simple function of the two.

Relating the elasticity of substitution to taste heterogeneity has consequences for any macroeconomic model that uses the Dixit-Stiglitz setup. An example, which I analyze in detail in section 4, is the model on Directed Technological Change by Acemoglu, Aghion, Bursztyn and Hemous (2012), henceforth AABH. The model assumes that the economy is composed of two industries: a dirty industry generating greenhouse gases emission which is proportional to its production. and a clean industry, which does not generate any emission. The outputs of the dirty and clean industries is labeled as dirty and clean good. The model predicts that a subsidy for R&D in clean industry could redirect a research effort towards this industry starting the era of green growth. Technological progress in clean industry incentivises consumers to substitute dirty goods with clean goods driving. If the substitution effect is strong enough,green technological progress leads to phasing out of the dirty industry and zero-emission economy in the long run. However, if the substitution effect is smaller than the income effect, green technological progress increases production of dirty industry leading to accelerating accumulation of CO2 and eventually to environmental diaser.

The central role of elasticity of substitution highlighted in the AABH paper raises the interest on what determines the value of the parameter and is there a policy which could increase the substitutability between clean and dirty goods. The theoretical result of this paper allows to shed some light on this question. In section 4, I merge the AABH framework with the insight of section 3. I show that heterogeneity of taste delays phasing out of dirty industry. If the dispersion of consumers' tastes is large, technological progress in clean industry leads to environmental disaster (exponential growth of emissions).

To understand the intuition behind this result, one could consider an extreme case when consumers tastes are polarized: half of the population derives no utility from the consumption of clean good (e.g. they do not have access to public transportation), while the other half value the clean good highly. More efficient production of the clean good increases income of all consumers, including the income of those who do not value clean good. These consumers will consume higher amount of dirty good, which will produce a growth in emissions.

Interestingly, the result highlights also the role of the variety of clean goods. The model presented in section 4.3 shows that low elasticity of substitution can result from the fact that some consumers cannot find an attractive alternative to a dirty good among existing varieties of clean good. Consumers in rural areas cannot benefit from more comfortable trams. Additional varieties - e.g. cars fueled with biofuels - increase the chances that those consumers find a variety which could compete with the dirty good. This suggests that environmental policy which promotes development of clean industries should be supplemented with a policy which ensures the diversity of clean goods. For example a subsidy for R&D in clean transport should support a wide range of alternative technologies rather than selected few.

2 Related Literature

Chamberlin's argument deriving monopoly power from the differentiation in consumers' tastes has been formally described with economic models by a number of authors. One may distinguish two branches in this literature. The first consists of the models of spatial comptetion with the most prominent examples by Hotelling (1929), Salop (1979) and d'Aspremont, Gabszewicz and Thisse (1979). The second branch has been inspired by the econometric discrete choice theory (Manski and McFadden (1981) and Berry, Pakes and Levinsohn (1991)). This approach postulates using stochastic utility function to model taste heterogeneity. It was first proposed by Perloff and Salop (1985) and then adopted by Caplin and Nalebuff (1991) and Anderson, de Palma and Nesterov (1995). The paper by de Palma, Ginsburgh, Papageorgiou and Thisse (1985) combines the approaches from the two branches.

Contrary to all studies listed above, this paper is not an alternative to the Dixit and Stiglitz model. Instead, it aims to extend the Dixit and Stiglitz framework by allowing for heterogeneity of taste. As demonstrated in section 4, the compatability with the Dixit and Stiglitz framework allows merging the heterogeneity model with other macroeconomic models using Dixit and Stiglitz setup.

This paper is also closely related to the work by Anderson, de Palma and Thisse (1988, 1989), who show the equivalence between demand generated by the CES utility function and the logit model of the discrete choice theory. However, their model assumes that an individual consumer treats different goods as perfect substitutes. In contrast, the model presented in this paper allows the individual consumer to have a finite elasticity of substitution between goods.

The section of the paper that evaluates the model by Acemoglu et al. (2012) aims to contribute to the recent literature on directed technological change in the context of climate change. The principles and the origins of directed technological change theory are outlined in Acemoglu (2014). Aghion et al. (2014) show that the intensity of clean R&D in automobile industry depends on fuel prices. Andre and Smulders (2014) and Hassler et al. (2014) use the directed technological change setup to determine long run energy efficiency. None of these papers examines how their results are affected if consumers are heterogenous.

3 Consumer's Heterogeneity and Elasticity of Substitution

The number of goods in the economy is given by N_t. Consumers' utility function takes the CES form:

$$U_{it} = \left(\sum_{j=1}^{N_t} (\theta_{ij} \lambda_{jt} x_{ijt})^\rho \right)^{\frac{1}{\rho}} \tag{1}$$

where x_{ijt} is the quantity of product j consumed by individual i at time t, λ_{jt} is the quality of product j at time t and θ_{ij} is the idiosyncratic taste parameter[1].

In order to explore different types of competition for customers, I will consider two demand systems. In the first case, goods are imperfect substitutes for the individual consumer ($\rho < 1$). As a result, everyone consumes all products but possibly in different proportions. This partly addresses concerns raised by Pettengill (1979) that the Dixit-Stiglitz framework predicts that all consumers consume exactly equal (and very small) amounts of all products. One example of an application is the model with different types of labour (e.g. high skill, medium skill and low skill) as goods and heterogeneity of taste representing heterogeneity of production functions across sectors.

In the second scenario, goods are perfect substitutes ($\rho = 1$) and consumers choose only one product from the set of products available on the market. Again, each consumer might have his/her own valuation of each brand. This specification goes in line with discrete choice theory and corresponds to sectors like automobiles or personal computers.

3.1 Imperfect Substitutes

In case of imperfect substitutes, corner solutions are ruled out. The demand of consumer i for product j is given by:

$$x_{ijt} = \frac{\left(\frac{\theta_{ij}\lambda_j}{p_j}\right)^{\frac{\rho}{1-\rho}}}{\sum_k \left(\frac{\theta_{ik}\lambda_k}{p_k}\right)^{\frac{\rho}{1-\rho}}} p_j^{-1} y_i \tag{2}$$

Notice that the value of $\phi = \frac{\left(\frac{\theta_{ij}\lambda_j}{p_j}\right)^{\frac{\rho}{1-\rho}}}{\sum_k \left(\frac{\theta_{ik}\lambda_k}{p_k}\right)^{\frac{\rho}{1-\rho}}}$ can be interpreted as a fraction of total real expenditure which consumer i is willing to spend on the purchase of product j if prices and qualities of all goods are the same.

Integrating over consumers with different tastes provides the total demand for good j:

[1] time indices are added to ease the incorporation of this framework into Young's endogenous growth model in section 4.

$Q_j = \int \ldots \int \frac{\left(\frac{\theta_{ij}\lambda_j}{p_j}\right)^{\frac{\rho}{1-\rho}}}{\sum_k \left(\frac{\theta_{ik}\lambda_k}{p_k}\right)^{\frac{\rho}{1-\rho}}} p_j^{-1} y_i g\left(\underline{\theta}\right) d\underline{\theta}$

When choosing the optimal level of quality and price, firm takes qualities and prices of others as given. The marginal change of quantity due to change of prices will be given by:

$$\frac{dQ_j}{dp_j} = \frac{-1}{1-\rho} \int \int \ldots \int \left\{ \frac{\left(\frac{\theta_{ij}\lambda_j}{p_j}\right)^{\frac{\rho}{1-\rho}}}{\sum_k \left(\frac{\theta_{ik}\lambda_k}{p_k}\right)^{\frac{\rho}{1-\rho}}} p_j^{-2} y_i \right.$$

$$\left. -\rho \left(\frac{\left(\frac{\theta_{ij}\lambda_j}{p_j}\right)^{\frac{\rho}{1-\rho}}}{\sum_k \left(\frac{\theta_{ik}\lambda_k}{p_k}\right)^{\frac{\rho}{1-\rho}}} \right)^2 p_j^{-2} y_i \right\} g\left(\underline{\theta}\right) d\underline{\theta}$$

The elasticity of demand with respect to price will be given by

$$\frac{dQ_{jt}}{dp_{jt}} \frac{p_{jt}}{Q_{jt}} = -\frac{1}{1-\rho} \left(1 - \rho \frac{\int \int \ldots \int \phi_{ij}^2 y_i g\left(\underline{\theta}\right) d\underline{\theta}}{\int \int \ldots \int \phi_{ij} y_i g\left(\underline{\theta}\right) d\underline{\theta}} \right) =$$

$$= -\frac{1}{1-\rho} \left(1 - \rho \frac{E\left(\phi_j^2 y\right)}{E\left(\phi_j y\right)} \right)$$

If consumers' income is uncorrelated with their tastes, the expression further simplifies to:

$$\frac{dQ_{jt}}{dp_{jt}} \frac{p_{jt}}{Q_{jt}} = -\frac{1}{1-\rho} \left(1 - \rho \frac{E\left(\phi_j^2\right)}{E\left(\phi_j\right)} \right) \tag{3}$$

Thus elasticity of demand is fully characterized by the substitutability parameter rho and the first two moments of the distribution of taste. In fact, in statistics,$D\left(\phi_j\right) = \frac{E\left(\phi_j^2\right)}{E\left(\phi_j\right)} - E\left(\phi_j\right)$ is the coefficient of dispersion of ϕ's distribution. The formula indicates that if these first two moments are the same for all goods, the demand curve will be the same for all goods. If all goods also have the same (upward sloping or flat) supply curves, the symmetric equilibrium exists.

For future reference, I also derive the cross-price elasticity of demand. For $kneqj$

$$\frac{dQ_{jt}}{dp_{kt}} \frac{p_{kt}}{Q_{jt}} = \frac{\rho}{1-\rho} \frac{E\left(\phi_j \phi_k\right)}{E\left(\phi_j\right)}$$

Now define $\psi_{ij} = \frac{\theta_{ij}^{\frac{\rho}{1-\rho}}}{\sum_k \theta_{ik}^{\frac{\rho}{1-\rho}}}$. If the distribution of tastes is symmetric in the sense that $E\left(\psi_j\right) = E\left(\psi_k\right)$, $E\left(\psi_j^2\right) = E\left(\psi_k^2\right)$ and$Cov\left(\psi_j, \psi_k\right) = Cov\left(\psi_j, \psi_h\right)$ for any tripling of goods j, k and h and if all goods have the same supply curve, then symetric equilibrium exists, $p_j = p_k$, $\phi_{ij} = \psi_{ij}$, $E\left(\psi_j\right) = \frac{1}{N}$ and $Cov\left(\psi_j, \psi_k\right) = \frac{D(\psi_j)}{N(N-1)}$. Thus, under symmetry, we find that

$$\frac{dQ_{jt}}{dp_{jt}} \frac{p_{jt}}{Q_{jt}} = -\frac{1}{1-\rho} \left(1 - \rho D\left(\psi_j\right) - \frac{\rho}{N} \right) \equiv -\epsilon \tag{4}$$

and

$$\frac{dQ_j}{dp_k}\frac{p_k}{Q_j} = \frac{\rho}{1-\rho}\left(-\frac{D(\psi_j)}{N-1} + \frac{1}{N}\right) \tag{5}$$

The elasticity, $|\epsilon|$, is a decreasing function of taste dispersion if $\rho > 0$ (goods would be gross substitutes in the absence of heterogeneity) and increasing function of taste dispersion if $\rho > 0$ (goods would be gross compliments).

By analogous derivations, the elasticity of demand with respect to quality is can be found:[2]

$$\frac{dQ_j}{d\lambda_j}\frac{\lambda_j}{Q_j} = \frac{\rho}{1-\rho}\left(1 - \mathsf{D}\left(\psi\right) - \frac{1}{N}\right) \equiv \epsilon - 1 \tag{6}$$

3.1.1 Corollary: Representative Consumer and Heterogeneity of Taste

Consider an economy in symmetric equilibrium (as defined above). Equations (4) and (5) predict that the Walrasian demand for good j is

$$log\left(Q_j\right) = -\frac{1}{1-\rho}\left(1 - \rho D\left(\psi\right) - \frac{\rho}{N}\right) * log\left(p_j\right)$$

$$+\frac{\rho}{1-\rho}\left(\frac{-D\left(\psi\right)}{N-1} + \frac{1}{N}\right)\sum_{k\neq j}\log\left(p_k\right) + \log\left(\frac{y}{N}\right)$$

Now consider a consumer with utility function

$$U = \left((0.5 * x_j)^{\eta} + (0.5 * x_k)^{\eta}\right)^{\frac{1}{\eta}} \tag{7}$$

where $\eta = \frac{\epsilon - 1}{\epsilon - \frac{1}{1}} = \rho\frac{1-\frac{1}{N}-D}{1-\frac{1}{N}-\rho D}$. The Walrasian demand for good j in the economy populated only by this consumer is exactly the same as the one stated in equation (7). The consumer is therefore a representative consumer for an economy with heterogenous agents with utility function from equation (1).

There are two points that follows this observation. First, for the economy in symmetric equilibrium, the elasticity of substitution in the CES utility function (or production function) is a function of taste heterogeneity. If $\rho > 0$, lower values of η correspond to greater taste heterogeneity. If a model assumes very high values of η implicitly, it assumes low heterogeneity of tastes.

Second, the corollary suggests that heterogeneity does not have to be explicitly modelled. Any model with CES utility function (or production function) implicitly allows for heterogeneity of taste between consumers. Whenever an analysis should account for taste heterogeneity of consumers, it is enough to perform comparative statics for parameter η. There is no need to build a sophisticated model with heterogenous tastes of agents.

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[2] The two elasticities can be also expressed in terms of variance of ψ: $\frac{dQ_{jt}}{dp_{jt}}\frac{p_{jt}}{Q_{jt}} = -\frac{1}{1-\rho}\left(1 - \rho N \mathsf{Var}\left(\psi_{ij}\right) - \frac{\rho}{N}\right) \equiv -\epsilon$ and $\frac{dQ_j}{d\lambda_j}\frac{\lambda_j}{Q_j} = \frac{\rho}{1-\rho}\left(1 - N\mathsf{Var}\left(\psi\right) + \frac{1}{N}\right) \equiv \epsilon - 1$

3.1.2 The Ideal Measure of Heterogeneity

The choice of measure for taste heterogeneity is not trivial. First, two candidates are the variance and the dispersion of ϕ_j, an income share devoted for good j. If income share devoted to good j differs between consumers, it must be due to taste heterogeneity. The spread of income shares devoted to good j across the population may therefore serve as a measure of heterogeneity in preferences. The major advantage of the two measures is that they are easily empirically observable. Their major problem is that the distribution of ϕ is defined on the simplex[3] and therefore depends on the number of goods: if a new good becomes available, the distribution of ϕ will change and thus its variance and dispersion.

In models where N is fixed, $Var(\psi_j)$ and $D(\psi_j)$ can be treated as determined purely by factors outside the model. However, in models in which N is endogenous, $Var(\psi_j)$ and $D(\psi_j)$ can no longer be taken as exogenous. How could we find the alternative? What remains exogenous is the distribution of taste in the utility function, θ. Recall that θ_{ij} is the idiosyncratic taste parameter, a weight each customer i puts on consumption of good j; since θ_{ij} can be any positive number chosen by the consumer and the sum $\sum \theta_{ij}$ does not have to be unity, the distribution of θ_j can be completely independent of N.

A handy alternative meassure of heterogeneity of taste between consumers is a coefficient of dispersion of $\theta_j^{\frac{\rho}{1-\rho}}$, $D\left(\theta_j^{\frac{\rho}{1-\rho}}\right)$[4].

Consider the case of symmetric tastes, i.e. the same distribution of ψ for all goods. If the goal is to express the right hand side of equation (4) in terms of $D\left(\theta^{\frac{\rho}{1-\rho}}\right)$, the relation between $\frac{E(\psi^2)}{E(\psi)}$ and $D\left(\theta^{\frac{\rho}{1-\rho}}\right)$ shall be found. The relation will depend on the particular distribution of $\theta^{\frac{\rho}{1-\rho}}$. An example of a distribution which allows for an elegant closed form solution is the gamma distribution.

If $\theta^{\frac{\rho}{1-\rho}} \sim Gamma\left(\alpha, \beta\right)$ then the dispersion of $\theta^{\frac{\rho}{1-\rho}}$ is $D = \frac{1}{\beta}$ and its expected value is $\mu = \alpha D$. Under the mean preserving spread - i.e. if upon increase in dispersion, parameter α adjusts to keep the mean unchanged - the distribution of income shares is Dirichlet, $\psi = \frac{\theta^{\frac{\rho}{1-\rho}}}{\sum_k \theta^{\frac{\rho}{1-\rho}}} \sim Dirichlet\left(\frac{\mu}{D}, \frac{\mu}{D}, ..., \frac{\mu}{D}\right)$ and

$$\frac{E\left(\psi^2\right)}{E\left(\psi\right)} = \frac{\mu + D}{N\mu + D} \tag{8}$$

which is an increasing function of D for $n \geq 2$.

If the assumption on the symmetry of distributions is dropped and the taste for each product is allowed to follow its own distribution, $\theta_j^{\frac{\rho}{1-\rho}} \sim Gamma\left(\alpha_j, \beta_j\right)$, then $D_j = \frac{1}{\beta_j}$. The income share for product j is distributed according to $\phi_j = \left(\frac{\theta_j \lambda_j}{p_j}\right)^{\frac{\rho}{1-\rho}} \sim Gamma\left(\alpha_j, \beta_j \left(\frac{\lambda_j}{p_j}\right)^{-\frac{\rho}{1-\rho}}\right)$ and its expected value is $\mu_j = \frac{\alpha_j}{\beta_j}\left(\frac{\lambda_j}{p_j}\right)^{\frac{\rho}{1-\rho}} = \alpha_j D_j \left(\frac{\lambda_j}{p_j}\right)^{\frac{\rho}{1-\rho}}$. Again, under the mean preserving spread, the distribution of income

[3] Since every consumer chooses an arrow $(\phi_1, \phi_2 ... \phi_N)$ such that $\sum_{j=1}^N \phi_j = 1$, the distribution of ϕ is a multivariate distribution with simplex Δ^{N-1} as a support.

[4] Perhaps a more natural meassure of heterogeneity would be $Var(\theta_j)$ or $D(\theta_j)$. It turns out, however, that such meassure involves much more problematic and less tractable derivations. In the appendix I show that, if $\theta_j^{\frac{\rho}{1-\rho}}$ is distributed with the gamma distribution, the ralation between $D(\theta_j^{\frac{\rho}{1-\rho}})$ and$D(\theta_j)$ is positive. I also show numerically that the mean preserving spread of $\theta^{\frac{1-\rho}{\rho}}$ involves an increase in dispersion of θ

shares is Dirichlet, $\psi = \frac{\theta^{\frac{\rho}{1-\rho}}}{\sum_k \theta^{\frac{\rho}{1-\rho}}} \sim Dirichlet\left(\frac{\mu_1}{D_1}\left(\frac{\lambda_1}{p_{1j}}\right)^{-\frac{\rho}{1-\rho}}, ..., \frac{\mu_n}{D_n}\left(\frac{\lambda_n}{p_n}\right)^{-\frac{\rho}{1-\rho}}\right)$. It follows that

$$\frac{E\left(\phi_j^2\right)}{E\left(\phi_j\right)} = \frac{\frac{\mu_j}{D_j}\left(\frac{\lambda_j}{p_j}\right)^{-\frac{\rho}{1-\rho}}+1}{\sum_k \frac{\mu_k}{D_k}\left(\frac{\lambda_k}{p_k}\right)^{-\frac{\rho}{1-\rho}}+1}$$

The expression is decreasing in D_j and increasing in D_k, implying that elasticity of demand for good j increases when the dispersion of $\theta_j^{\frac{\rho}{1-\rho}}$ across population increases and decreases when the dispersion of $\theta_k^{\frac{\rho}{1-\rho}}$ with $k \neq j$ increases. If the dispersion of taste increases for all products by the same factor, elasticity of demand for each product decreases.

3.2 Perfect Substitutes

It turns out that mathematical analysis of this case is complex, and some simplifying assumptions on the distribution of idiosyncratic taste are necessary to proceed. In particular, in this subsection we assume that θ_{ij} is independently and identically distributed across products (the assumption which is typical in the discrete choice theory). Later, I will also assume that the log of θ_{ij} follows a logistic distribution. These assumptions reduce the generality of the problem but help with picturing the basic mechanism which this paper intends to describe.

Under the perfect substitution case, consumer maximization can be written as

$$U_i = \max \sum_{j=1}^{N} \left(\theta_{ij}\lambda_j x_{ij}\right)$$

subject to the budget constraint

$$y = \sum_j x_{ij}p_j$$

In fact, this problem reduces to the simple choice of the product which gives the highest value to the consumer:

$$U_i = \max_j \left\{\theta_{ij}\lambda_j \frac{y_i}{p_j}\right\} = \max_j \left\{\ln\theta_{ij} + \ln\lambda_j + \ln y_i - \ln p_j\right\}$$

To find aggregate demand for each product, I follow the same strategy as in Perloff and Salop (1985). The probability that consumer i prefers product j to product k:

$$\Pr\left(\ln\theta_{ij} + \ln\lambda_j + \ln y - \ln p_j > \ln\theta_{ik} + \ln\lambda_k + \ln y - \ln p_k\right) =$$

$$= \Pr\left(\ln\theta_{ik} < \ln\theta_{ij} + \ln\lambda_j - \ln\lambda_k - \ln p_j + \ln p_k\right) =$$

$$= G\left(\ln\theta_{ij} + \ln\lambda_j - \ln\lambda_k - \ln p_j + \ln p_k\right)$$

where G denotes the cumulative distribution function of θ_{ij}.

Since θ_{ik} is independent and identically distributed across products, I find that the probability of choosing product j given θ_{ij} is given by

$$\Pr\left(j \succ 1 \cap ... \cap j \succ j-1 \cap j \succ j+1 \cap ... \cap j \succ N\right) =$$

$$= \prod_{k \neq j} G\left(\ln\theta_{ij} + \ln\lambda_j - \ln\lambda_k - \ln p_j + \ln p_k\right)$$

To find the aggregate demand for product j, I integrate over population:

$$Q_j = \frac{y}{p_j} L \int \prod_{k \neq j} G\left(\ln\theta_{ij} + \ln\lambda_j - \ln\lambda_k - \ln p_j + \ln p_k\right) g\left(\ln\theta_{ij}\right) d\ln\theta_{ij} \quad (9)$$

I consider only the symmetric equilibria. This allows further simplification of (9):

$$\frac{y}{p_j} L \int G\left(\ln\theta_{ij} + \ln\lambda_j - \ln\lambda - \ln p_j + \ln p\right)^{N-1} g\left(\ln\theta_{ij}\right) d\ln\theta_{ij} \quad (10)$$

Now I am able to derive the elasticities of demand with respect to price and quality. I start with the latter. Using (10) I obtain:

$$\frac{\partial Q_j}{\partial p_j}\frac{p_j}{Q_j} = -1 - \frac{p_{jt}}{Q_j}\frac{y}{p_j^2} L \int (N-1)\, G\left(\ln\theta_{ij} + \ln\lambda_j - \ln\lambda - \ln p_j + \ln p\right)^{N-2} *$$

$$* g\left(\ln\theta_{ij} + \ln\lambda_j - \ln\lambda - \ln p_j + \ln p\right) g\left(\ln\theta_{ij}\right) d\ln\theta_{ij}$$

and since in symmetric equilibrium $\lambda_j = \lambda$ and $p_j = p$, this simplifies to

$$\frac{\partial Q_j}{\partial p_j}\frac{p_j}{Q_j} = -\left(1 + \frac{p_{jt}}{Q_j}\frac{y}{p_j^2} L \int (N-1)\, G\left(\ln\theta_{ij}\right)^{N-2} g\left(\ln\theta_{ij}\right)^2 d\ln\theta_{ij}\right)$$

Analogous derivations gives the expression of elasticity of demand with respect to quality

$$\frac{\partial Q_j}{\partial \lambda_j}\frac{\lambda_j}{Q_j} = \frac{\lambda_j}{Q_j}\frac{y}{p_j\lambda_j} L \int (N-1)\, G\left(\ln\theta_{ij}\right)^{N-2} g\left(\ln\theta_{ij}\right)^2 d\ln\theta_{ij}$$

The assumption that $\ln\theta_{ij}$ follows the exponential distribution with $E\left[\ln\theta\right] = 0$ and $Var\left[\ln\theta\right] = \sigma^2$ allows to find a simple closed form solution (see appendix, section 3)

$$\frac{\partial Q_{jt}}{\partial p_{jt}}\frac{p_{jt}}{Q_{jt}} = -\epsilon = -\left(1 + \frac{1}{\sigma}\right) \quad (11)$$

$$\frac{\partial Q_{jt}}{\partial \lambda_{jt}}\frac{\lambda_{jt}}{Q_{jt}} = \epsilon - 1 = \frac{1}{\sigma}$$

4 Aplication to the Directed Technological Change model

4.1 Directed Technological Change framework

In this subsection I present a framework, based on the model by Acemoglu et al. (2012) to under what conditions technological progress in clean industries could mitigate climate change and environmentla disasters.

Consumer i utility function is given by

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} u_i \left(C_{it,}, S_t\right)$$

where C_{it} stands for consumption, S_t, quality of the environment and $r > 0$ is a discount rate. I assume that the instantenous utility is strictly indreasing in consumption and environmental quality. I also assume it is concave in C and $\lim_{C_{it} \to \infty} \frac{\partial u_i(C_{it,}, S_t)}{\partial C_{it}}$. Finally, I assume that that $\lim_{S \to 0} u_i \left(C_{it}, S\right) = -\infty$, i.e. the state of $S = 0$ is an equivalent of environmental disaster. The consumption is defined for each consumer as:

$$C_{it} = \left(\theta_{id} x_{id}^{\rho} + \Sigma_j^{n_c} \theta_{ij} x_{ijc}^{\rho}\right)^{\frac{1}{\rho}}$$

We also assume that the valuation of the dirty good, θ_d, is fixed i.e. does not vary across consumers.

This setting extends the Acemoglu et al. (2012) framework in two ways: First, I allow the valuation of the clean good to vary across consumers. Second, I allow for more than one clean good. This extention will be used in section 3.4 do demonstrate how elasticity of substitution depends on the variety of clean goods accessible for consumers.

Although, the consumers have different tastes, all households supply the same amount of labour.

The clean and dirty goods are produced with labour and the composite of capital good (machines) dedicated to the particular good

$$Q_{jk} = l_{jk}^{1-\alpha} \int_0^1 A_{vjk}^{1-\alpha} z_{vjk}^{\alpha} dv$$

the capital good (machine), z_{vjk}, is produced by a monopolist with constant returns to scale and unit cost of production, expressed in terms of final good, ψ.

I assume that the production of dirty good is associated with pollution (or CO2 emission), $P = \vartheta Q_d$. Pollution leads to deterioration of environmental quality with $S_t = S_{t-1} - P_t$.

I consider the scenario when there is no technological progress in the dirty sector (A_d is fixes) and there is a constant technological progress in all clean sectors:

$$A_{vjct} = (1+\gamma) A_{vjct-1}$$

This scenario corresponds to the case in which all research effort is directed towards clean research. The conditions under which such redirection of effort could be achieved are examined in the Acemoglu et al. model. In this section I leave this examination aside and istead focus on the consequences of clean technological progress for the economy and the environment.

If in the long run, pollution grows at the constatn rate, environmental quality hits zero leading to environemental distater.

To find the growth rate of pollution, I first note that

$$P = \vartheta Q_d = \vartheta Y \frac{s_d}{p_d}$$

Given the production function for dirty and clean goods,

$$p_{jk} = \frac{(\mu_j \psi)^{\alpha} w^{1-\alpha}}{A_j^{1-\alpha}} \tag{12}$$

where μ_{vj} is the mark-up of the monopolitic producers of machines.

Since the number of machines is infinite, the demand curve faced by monopolistic producers is given by

$$z_{vjk} = \left(\frac{\alpha p_{vjk}}{p_{jk}}\right)^{\frac{1}{\alpha-1}} (A_{jk} l_{jk})$$

which implies a constant elasticity of demand:

$$\frac{dz_{vjk}}{dp_{vjk}} \frac{p_{vjk}}{z_{vjk}} = \frac{1}{\alpha - 1}$$

Hence, the monopolist markup is given by

$$\mu_{jk} = \frac{1}{\alpha}$$

Next, I derive wage from clean and dirty producers' First Order Condition with respect to the labour input:

$$(1-\alpha) p_{jk} Q_{jk} = w l_{jk}$$

Summing over all goods

$$(1-\alpha) \Sigma_{jk} p_{jk} Q_{jk} = \Sigma_{jk} w l_{jk}$$

and rearranging

$$(1-\alpha) = \frac{wL}{Y}$$

and normalizing $L = 1$

$$w = (1-\alpha) Y$$

Putting this result back in the equation (12):

$$p_{ij} = \frac{\left(\frac{\psi}{\alpha}\right)^{\alpha} (1-\alpha)^{1-\alpha} Y^{1-\alpha}}{A_{jk}^{1-\alpha}} \tag{13}$$

Thus the emissions are given by

$$P = \vartheta \left(\frac{\psi}{\alpha}\right)^{\alpha} (1-\alpha)^{1-\alpha} \left(\frac{Y}{A_d}\right)^{\alpha-1} Y s_d$$

Taking logs and differentiating with respect to time brings:

$$\frac{d\log(P)}{dt} = \alpha\frac{d\log(Y)}{dt} + \frac{d\log(s_d)}{d\log(A_c)}\frac{d\log(A_c)}{dt}$$

4.2 Imperfect substitutes

In this subsection we assume that there is only one clean good with valuation θ_{ic}. The optimization of the consumer brings:

$$s_d = \int \frac{1}{1+\left(\frac{\theta_{ic}}{\theta_d}\frac{p_d}{p_c}\right)^{\frac{\rho}{1-\rho}}} f(\theta_{ic})\, d\theta_{ic}$$

Using (13)

$$s_d = \int \frac{1}{1+\left(\frac{\theta_{ic}}{\theta_d}\frac{A_c^{1-\alpha}}{A_d^{1-\alpha}}\right)^{\frac{\rho}{1-\rho}}} f(\theta_{ic})\, d\theta_{ic}$$

Then,

$$\frac{d\log(s_d)}{d\log(A_c)} = -\frac{\rho}{1-\rho}(1-\alpha)\frac{\int \phi_{id}(1-\phi_{id}) f(\theta_{ic})\, d\theta_{ic}}{\int \phi_{id} f(\theta_{ic})\, d\theta_{ic}}$$

which reduces to

$$\frac{d\log(s_d)}{d\log(A_c)} = -\frac{\rho}{1-\rho}(1-\alpha)\left(1-\frac{E\left[\phi_{id}^2\right]}{E\left[\phi_{id}\right]}\right) =$$

$$= -\frac{\rho}{1-\rho}(1-\alpha)(1-D[\phi_{id}])$$

In this case, the growth of pollution is given by:

$$\frac{d\log(P)}{dt} = \alpha\frac{d\log(Y)}{dt} - \frac{\rho(1-\alpha)}{1-\rho}(1-D[\phi_{id}])\frac{d\log(A_c)}{dt}$$

If a group of consumers derives no utility from the clean good, while other consumers have no valuation of the dirty good, then $E(\phi_{id}) = 1 * f(0) + 0 * (1-f(0))$ and $E\left(\phi_{id}^2\right) = 1^2 * f(0) + 0^2 * (1-f(0)) = E(\phi_{id})$. Consequently, in this case, $D[\phi_{id}] = 1$ and $\frac{d\log(P)}{dt} = \alpha\frac{d\log(Y)}{dt}$

Proposition 1. If consumer heterogeneity (meassured as the dispersion of consumers' spending on dirty good) is large, environmental disaster cannot be avoided. In the extreme case, if a group of consumers derives no utility from the clean good, while other consumers have no valuation of the dirty good, growth of pollution is given by $g_P = \alpha g_y$, where g_y is a GDP growth rate.

To understand the intuition behind this result, one could consider an extreme case when consumers' tastes are polarized: half of the population derives no utility from the consumption of clean good (e.g. they do not have access to public transportation), while the other half value the clean good highly. More efficient production of the clean good increases income of all consumers, including the income of those who do not value clean good. These consumers will consume higher amount of dirty good, which will produce a growth in emissions.

4.3 Perfect substitutes

Next, I consider the case when, in the utility function, of individual consumer, dirty and clean good are perfect substitutes ($\rho = 1$). In this subsection, we assume that there is only one dirty good and the variety of clean goods. In contrast to the previous subsection, we allow a consumer to have differentiated valuation between different clean varieties[5]. In particular, we let each θ_{ijd} to be drawn from the same probability distribution, $G(\theta)$. The drawas of θ are independent.

The aggregate expenditure on good d can be derived as

Let $s_d = \Pr\left(A_d \frac{\theta_d}{p_d} > \max_j \left\{\frac{\theta_{ijc}}{p_{jc}}\right\} * A_c\right)$denote the share of expenditure spent on dirty good. Suppose that there is only one dirty good and θ_{id} is fixed, i.e. they are the same for all consumers (then need to show that the demand curve is still downdward sloping). Suppose that θ_{ijc} can be only in the range of $(0,1)$ has distribution with cdf $G(\theta) = \theta^{\sigma}$ for all j's in clean. Then the probability that the dirty goods is chosen is

$$s_d = \Pr\left(\frac{\theta_d}{p_d} > \max_j \left\{\frac{\theta_{ijc}}{p_{jc}}\right\}\right) =$$

$$= \prod_j G\left(\frac{p_{jc}\theta_d}{p_d}\right)$$

in symmetric equailibrium with $p_{jc} = p_c$

$$s_d = G\left(\frac{p_c\theta_d}{p_d}\right)^{n_c}$$

and using 13

$$s_d = G\left(\frac{A_d^{1-\alpha}}{A_c^{1-\alpha}}\theta_d\right)^{n_c}$$

Taking logs and differentiating with respect of $\ln(A_d)$ gives:

$$\frac{d\ln(s_d)}{d\ln(A_c)} = -(1-\alpha)\, n_c \frac{g\left(\frac{A_d^{1-\alpha}}{A_c^{1-\alpha}}\theta_d\right)\frac{A_d^{1-\alpha}}{A_c^{1-\alpha}}\theta_d}{G\left(\frac{A_d^{1-\alpha}}{A_c^{1-\alpha}}\theta_d\right)}$$

In this case, the growth of pollution is given by:

[5] In section 4.2 we restricted $\theta_{ijc} = \theta_{ic}$ for all j, since otherwise the case becomes not tractable

$$\frac{d\log(P)}{dt} = \alpha\frac{d\log(Y)}{dt} - (1-\alpha)\,n_c\frac{g\left(\frac{A_d^{1-\alpha}}{A_c^{1-\alpha}}\theta_d\right)\frac{A_d^{1-\alpha}}{A_c^{1-\alpha}}\theta_d}{G\left(\frac{A_d^{1-\alpha}}{A_c^{1-\alpha}}\theta_d\right)}\frac{d\log(A_c)}{dt}$$

In particular, if $\ln(\theta) \sim exponential\left(\frac{1}{\sigma}\right)$ (thus σ is the standard deviation of $\ln(\theta)$) then

$$\frac{d\log(P)}{dt} = \alpha\frac{d\log(Y)}{dt} - (1-\alpha)\frac{n_c}{\sigma}\frac{d\log(A_c)}{dt}$$

The proposition follows:

Proposition 2 Suppose that for any consumer clean and dirty goods are perfect substitutes. Furthermore, consider the case in which there is only technological progress in the industry producing clean goods. If number of clean varieties is sufficiently large, the environmental disaster will be avoided for any growth of GDP.

Proof in the text

The result highlights also the role of the variety of clean goods. In the setup presented above, if $\ln(\theta) \sim exponential\left(\frac{1}{\sigma}\right)$, high σ implies that high portion of consumers cannot find an attractive alternative to a dirty good among existing varieties of clean good. Additional varieties - e.g. larger variety of clean transport technologies - increase the chances that those consumers find a variety which could compete with the dirty good. This suggests that environmental policy which promotes development of clean industries should be supplemented with a policy which ensures the diversity of clean goods. For example a subsidy for R&D in clean transport should support a wide range of alternative technologies rather than selected few.

5 Conclusion

Although Chamberlin's proposition that differences in taste between consumers can be a source of desire for products variety has been extensively discussed in numerous microeconomic models (e.g. Salop (1979), Hoteling (1929), Perloff and Salop (1985), de Palma et al. (1985)), this paper is the first study that formalizes Chamberlin's proposition in direct reference to the setup of the Dixit and Stiglitz model. In particular, it demonstrates that in symmetric equilibrium - whenever it exists - elasticity of substituion, which governs optimal product differentiation in the Dixit and Stiglitz model, can be shown to be a decreasing function of consumer taste dispersion. More generally, for any kind of equilibrium, elasticity of demand for a good can be expressed as a dereasing function of dispersion (across consumers) of income shares devoted for this good.

The result has important consequences for the model on Directed Technological Change by Acemoglu, Aghion, Bursztyn and Hemous (2012), which predicts that technological progress in clean industries (i.e. industries which do no produce CO2 emissions) leads to decline in emissions only if clean and dirty goods are sufficiently substitutable at the aggregate level. I show that heterogeneity of taste delays phasing out of dirty industry. If the dispersion of consumers' tastes is large, technological progress in clean industry leads to environmental disaster (exponential growth of emissions).

The model presented in section 4.3 highlights also the role of the variety of clean goods. The model presented in section 4.3 shows that low elasticity of substitution can result from the fact that some consumers cannot find an attractive alternative to a dirty good among existing varieties of clean good. Additional varieties - e.g. cars fueled with biofuels - increase the chances that those consumers find a variety which could compete with the dirty good. This suggests that environmental policy which promotes development of clean industries should be supplemented with a policy which ensures the diversity of clean goods. For example a subsidy for R&D in clean transport should support a wide range of alternative technologies rather than selected few.

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