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Abstract

We study steady state and business cycle properties of a model with heterogeneous firms and on-the-job search in the spirit of Moscarini and Postel-Vinay (2012). We extend the setup by including capital in the production function and show how this change influences model properties. The model is solved using a novel numerical method, projection within perturbation, which uses Chebyshev polynomial approximation and Clenshaw-Curtis quadrature for dealing with heterogeneity. We analyze worker flows between firms, distribution of firm size and wages, and study how productivity and other shocks affect them. When we introduce a working capital channel into the model we find that costly borrowing that finances firms' wage and vacancy bill shifts the distribution of firms to the right.

Keywords: job search, business cycle, unemployment, computation method, heterogeneous firms

JEL Codes: J64, E32, J31

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1 Introduction

Financial markets and financial regulation (or lack thereof) affect the macroeconomy and the Great Recession is a stark reminder of this fact. The recent recession has illustrated the particular importance of financial markets for the performance of the labor market: the salient feature of the 2008-2009 financial crisis has been high and very persistent unemployment that remains elevated in many countries.[1]

There exists a fast growing literature that studies the nexus of financial and labor markets by incorporating financial frictions into macroeconomic models with labor-search frictions. However, the question of how the state of and developments in financial markets affects the workers' movements across different employers and the distribution of wages remains largely unanswered.

In this paper we do the first step toward this issue and study the business cycle on the reallocation of workers across employers (on-the-job flows). We use a version of a stochastic Burdett and Mortensen (1998) on-the-job-search model based on recent work by Moscarini and Poster-Vinay (2012, 2013, 2016),henceforth referred to as MPV. Importantly we consider the case with exogenously heterogeneous firms that differs in the individual productivity. Solving the model with heterogenous firms and aggregate shocks is non-trivial. To tackle the issue we propose a two-step approach. First, we solve for the stochastic distribution of firms using projection method and Chebyshev polynomials. Next, we approximate the dynamics in the model using perturbation method (in our case we linearize the model around the steady state) and track the evolution of Chebyshev nodes under this dynamics. Using the approach we can efficiently solve for the stochastic equilibrium and study the effects of aggregate shocks and on the distribution of firms, wages, employment, etc.

To study the effects of financial markets we introduce in the second part of the paper working capital channel. We extend the standard on-the-job-search model and introduce cost of borrowing as a production cost: firms must borrow to finance their labor input.

Our paper relates to two strands of literature. It draws on recent developments in on-the-job search models.[2]. It connects also to a recent literature that incorporates financial frictions into macroeconomic models with search frictions in the labor market.[3]

2 Model

We consider a dynamic version of Burdett and Mortensen (1998) using the Moscarini and Postel-Vinay (2012) specification. One of the key features of our model setup is that firms invest in and hold capital, allowing for a richer production function. We later extend the standard framework and allow for a basic form of financial friction. In this model it takes the form of working capital: firms have to finance their wage and investment bill with the help of costly lending.

[1]See Elsby, Hobijn, and Sahin (2010) for the analysis of U.S. labor market developments in the Great Recession and Elsby, Hobijn, and Sahin (2013) for analysis for OECD countries.

[2]See, for example, Coles and Mortensen (2011), Manning (2012), Moscarini and Poster-Vinay (2009, 2012, 2013, 2016), Wolthoff (2014)

[3]See Wasmer and Weil (2004), Monacelli et al. (2011), Petrosky-Nadeau (2014), Kaas and Kircher (*forthcoming*).

2.1 Household

We assume that the economy is populated by a unit mass of workers and that in each period an individual worker can be either employed or unemployed. We assume that unemployed U_t receive utility b and search for jobs with a constant intensity λ_u, whereas employed N_t search on the job with constant intensity equal to λ_e. All jobs are destroyed with an exogenous probability of δ. The representative household aims to maximize the stream of discounted utility from consumption:

$$\max \sum_{t=0}^{\infty} \beta^t \log(C_t). \tag{1}$$

The budget constraint of the household is the following:

$$C_t = N_t W_t + \Pi_t, \tag{2}$$

where W_t is the average wage, which described later on, and Π_t is the aggregate profit of all firms, which transferred to the household.

2.2 Firm

We assume that there is a mass of firms distributed according to pdf given by Γ with support on the interval $[\underline{p}, \overline{p}]$. Moreover, we assume that the probability density function given by $\gamma = \Gamma'$ is strictly positive and continuous on the support. A type p firm[4] aims to maximize the sum of discounted future profits:

$$\max \sum_{t=0}^{\infty} \Lambda^t \log(Pi_t(p)), \tag{3}$$

where $Lambda_t$ is the stochastic discount factor due to the Lagrange multiplier linked with the household budget constraint, and $\Pi_t(p)$ is profit of type-p firms. The firm operates a standard Cobb-Douglas production function:

$$Y_t(p) = A_t L_t(p)^{r_s(1-\alpha)} K_{t-1}^{r_s\alpha}(p), \tag{4}$$

where $Y_t(p)$ is product, $L_t(p)$ is employment and $K_t(p)$ is capital holding of type-p firms. The parameter α sets the elasticity of the production function with respect to capital, and parameter r_s allows for non-constant returns to scale. Aggregate productivity $A_t p$ in period t which follows a standard AR(1) process. Capital is accumulated by means of a standard capital accumulation equation augmented with investment frictions:

$$K_t(p) = (1-\rho)K_{t-1}(p) + \left(\frac{I_t(p)}{K_{t-1}}\right)_I^{\sigma} K_{t-1}. \tag{5}$$

In the above, ρ is the depreciation rate of capital, and the extent of investment frictions is set by parameter σ_I. In order to hire workers firms must post vacancies $V_t(p)$, for which they incur an increasing convex cost $c(V)$. The amount of vacancies posted by a type-p firm is set by means of free entry condition which is discussed later on. Finally, firm profit can be set as:

$$\Pi_t(s) = Y_t(p) - W_t(p)L_t(p) - I_t(p) - C(V(p)). \tag{6}$$

[4] In the following exposition, we will use the phrase type p firm and a firm of productivity p interchangeably, despite the fact that when A_t is not equal to one the two do not mean exactly the same.

2.3 Labour Market

Let $N_t(p) : [\underline{p}, \overline{p}] \rightarrow [0, 1]$ be the cumulative distribution function of employed workers at firms of different productivity in period t. Using this notation we can define total employment, N_t, as $N_t = N_t(\overline{p})$ and unemployment rate, U_t, as $U_t = 1 - N_t$. We define the average size of a firm of productivity p, $L_t(p)$, as:

$$L_t(p) = \frac{dN_t(p)/dp}{\gamma(p)}. \tag{7}$$

Workers that search for job, O_t, both unemployed and employed, are matched with vacancies posted by firms, V_t, according to aggregate matching function:

$$M_t = \upsilon V_t^{1-\mu} O_t^{\mu}. \tag{8}$$

The number of job applications sent by unemployed O_t^U and employed O_t^N members of the household are given by:

$$O_t^U = U_t \lambda_u \qquad\qquad O_t^N = N_t \lambda_e, \tag{9}$$

giving the total number of job offers as:

$$O_t = O_t^U + O_t^N. \tag{10}$$

Using the number of job offers sent by unemployed and employed job seekers and the assumption of random search, we can determine how many of the job matches are of persons who were previously unemployed M_t^U and employed M_t^N:

$$M_t^U = \frac{O_t^U}{O_t} M_t \qquad\qquad M_t^N = \frac{O_t^N}{O_t} M_t \tag{11}$$

Aggregate employment evolves according to:

$$N_t = (1 - \delta) N_{t-1} + M_t^U, \tag{12}$$

where δ is the exogenous job separation rate.

We assume that all job matches of previously unemployed persons are productive and that such job seekers always take up employment.[5] The same is not true for on-the-job matches - only those workers who find a higher wage offer will switch jobs. The Nash wage bargaining procedure that is used for setting wages ensures that firms of higher productivity post higher wages. We can express the probability for an unemployed person to find a job, Φ_t^U, and the probability that an on-the-job seeker meet a new employer, Φ_t^U, as

$$\Phi_t^U = \frac{M_t^U}{U_t} \qquad\qquad \Phi_t^N = \frac{M_t^N}{N_t} \tag{13}$$

The probability for an employer to meet any job-seeker (per vacancy) is given by

$$\Psi_t = \frac{M_t}{V_t}, \tag{14}$$

[5]In order for this to be the case, the parameters of the model must result in respective value functions (which are described later on) being positive for all firm productivity.

probabilities for a firm to meet a previously unemployed and employed job seeker equal

$$\Psi_t^U = \frac{M_t^U}{V_t} \qquad\qquad \Psi_t^N = \frac{M_t^N}{V_t}, \tag{15}$$

respectively.

The total number of posted vacancies V_t is given by the following equation:

$$V_t = \int_{\underline{p}}^{\overline{p}} V_t(p)\gamma(p)dp, \tag{16}$$

where $V_t(p)$ denotes the number of vacancies posted by a a firm of productivity p at time t.

The share of total vacancies posted by firms of productivity of more than p, $\overline{V}_t(p)$, and the share of vacancies posted by firms of productivity no greater than p, $\underline{V}_t(p)$, can be computed as

$$\overline{V}_t(p) = \frac{\int_p^{\overline{p}} V_t(p)\gamma(p)dp}{V_t} \qquad\qquad \underline{V}_t(p) = \frac{\int_{\underline{p}}^{p} V_t(p)\gamma(p)dp}{V_t} \tag{17}$$

Using the above notation we can now discuss the dynamics of the distribution of workers among firms. In the following we assume that employed job seekers will switch jobs only if they have are offered a higher than currently receiving wage. In equilibrium, given the assumption of Nash wage bargaining with constant bargaining power for wage setting, which is discussed in greater detail later on, firms of higher productivity offer higher wages. The average size of a type-p firm evolves as follows:

$$L_{t+1}(p) = (1-\delta)\left(1 - \Phi_t^N \frac{\overline{V}_t(p)}{V_t}\right) L_t(p) + U_t \Phi_t^U \frac{V_t(p)}{V_t} + (1-\delta) N_t \Psi_t^N \frac{V_t(p)}{V_t} \frac{N_t(p)}{N_t} \tag{18}$$

The first term of equation (18) describes workers that do not change their job due to either exogenous separation or better job offer. Recall that an exogenous fraction δ of workers are exogenously separated from their job and a fraction $(1-\delta)$ does not become unemployed. The probability of a worker to meet another employer is given by Φ_t^N but this worker to quit his current job only if the job offer comes from a firm of higher productivity. Hence the probability that a worker receive a better-paying offer is $\Phi_t^N \frac{\overline{V}_t(p)}{V_t}$. In the second term, The probability of employing a previously unemployed job-seeker is given by $U_t \Phi_t^U$, and this probability must be corrected to account for the intensity of vacancy posting by the type-p firm. The last term relates to the poaching behavior of a firm. The probability for a type-p firm of meeting a random previously employed job-seeker is $N_t \Phi_t^U \frac{V_t(p)}{V_t}$. This value must be multiplied by the probability that this worker was previously employed at a firm of lower productivity, which is $\frac{N_t(p)}{N_t}$. Finally, under the assumption that only existing workers can be poached, the on-the-job search inflow is multiplied by the $(1-\delta)$.

Using the definition for the distribution of employment across firms:

$$N_t(p) = \int_{\underline{p}}^{p} L_t(s)\gamma(s)ds \tag{19}$$

we can also specify the law of motion for the distribution of employment as:

$$N_{t+1}(p) = (1-\delta)\left(1 - \Phi_t^N \frac{\overline{V}_t(p)}{V_t}\right) N_t(p) + U_t \Phi_t^U \frac{\underline{V}_t(p)}{V_t} \tag{20}$$

We assume that wages are determined through Nash wage bargaining, whose aim is to maximize the weighted surplus of the worker and firm. The wage rate for a type-p firm, $W_t(p)$, is the solution to the following optimization problem:

$$W_t(p) = \arg\max_{W_t(p)} \left(V_t^N(p) - V_t^U\right)^{\xi} \left(V_t^J(p) - V_t^V(p)\right)^{1-\xi} \tag{21}$$

where ξ is the bargaining power of the worker, $V_t^N(p)$ is the value of being employed at a type-p firm for a worker, V_t^U is the value of being unemployed (which does not depend on p), $V_t^J(p)$ is the value of employing a worker for a type-p firm, and finally $V_t^V(p)$ is the value of posting a vacancy for a type-p firm. The solution to the Nash wage bargaining problem is the following implicit equation for wages:

$$\xi\left(V_t^J(p) - V_t^V(p)\right) = (1-\xi)\left(V_t^N(p) - V_t^U\right) \tag{22}$$

We now turn to the discussion of the value functions. The value for a worker of being employed at a type-p firm can be written as follows:

$$V_t^N(p) = W_t(p) + \beta E_t\left((1-\delta)\left(\Phi_{t,p}^N \int_p^{\overline{p}} \frac{V_t(s)}{\overline{V}_t(p)} V_{t+1}^N(s)\gamma(s)ds + (1-\Phi_{t,p}^N)V_{t+1}^N(p)\right) + \delta V_{t+1}^U\right) \tag{23}$$

where $\Phi_{t,p}^N = \Phi_t^N \frac{\overline{V}_t(p)}{V_t}$ is the probability that a worker employed at a type-p firm finds a job in a firm of higher productivity. The explanation for the value function is similar to the discussion of the equation describing the dynamics of average firm size. A worker employed at a type-p firm receives a wage $W_t(p)$ in period t. In period $t+1$ with probability δ they become unemployed and receive continuation value equal to the value of being unemployed, V_{t+1}^U, or with probability $(1-\delta)$ they will remain employed. If they remain employed at the same firm (which occurs with probability $(1-\Phi_{t,p}^N)$, they will receive continuation value $V_{t+1}^N(p)$. If they find a better job, through on-the-job search, the expected value of their new employment is calculated as the vacancy posting intensity weighted value of employment of firms of productivity greater than p.

The value of being unemployed is given by

$$V_t^U = b + \beta E_t\left(\Phi_t^U \int_{\underline{p}}^{\overline{p}} \frac{V_t(s)}{V_t} V_{t+1}^N(s)\gamma(s)ds + (1-\Phi_t^U)V_{t+1}^U\right) \tag{24}$$

where b is the exogenous utility flow from being unemployed. The value of being unemployed consists of current unemployment benefits and the discounted expected value of either being employed in the next period, $V_{t+1}^N(p)$, or the future value of continued unemployment. The latter occurs with the probability $(1-\Phi_t^U)$.

The value for a type-p firm of employing a worker is equal to:

$$V_t^J(p) = (A_t P(p) - W_t(p)) + \beta E_t\left((1-\delta)\left(1 - \Phi_t^N \frac{\overline{V}_t(p)}{V_t}\right) V_{t+1}^J(p) + \delta V_{t+1}^V\right) \tag{25}$$

The value of employing a worker is equal to the current revenue generated by the worker, $A_t p$, minus the wage paid, $W_t(p)$, and the next period value of employment multiplied by the probability of the job relationship not being severed. The exogenous probability of the job relationship being severed is given by δ, whereas the probability of the worker switching jobs to a better firm is given by Φ_t^N, corrected by the probability of the job offer coming from a better firm $\frac{\overline{V}_t(p)}{V_t}$.

Finally, the value of posting a vacancy is given by:

$$V_t^V(p) = -\Xi(V_t(p)) + \beta E_t\left((\Psi_t^U + \Psi_t^N \frac{N_t(p)}{N_t})V_{t+1}^J(p)\right) \tag{26}$$

where $\Xi(V_t(p))$ is the marginal cost of posting a vacancy. The value of posting a vacancy is equal to the discounted expected future value of employing a worker multiplied by the probability of employing a worker net of cost of posting the vacancy. The probability of employing a previously unemployed job seeker is given by Ψ_t^U, whereas the probability of employing an employed job seeker is given by Ψ_t^N, corrected for the probability that the job seeker was employed by a worse firm (which is again given by the distribution of employment across firms). Finally, the number of vacancies posted by any firm is given by the free entry condition, namely $V_t^V(p) = 0$.

2.4 Aggregation

Aggregate values of firm profit, product, capital, investment, employment, wage bill are given by the following equations:

$$\Pi_t = \int_{\underline{p}}^{\overline{p}} \Pi_t(p)\gamma(p)dp \qquad Y_t = \int_{\underline{p}}^{\overline{p}} Y_t(p)\gamma(p)dp \tag{27}$$

$$K_t = \int_{\underline{p}}^{\overline{p}} K_t(p)\gamma(p)dp \qquad I_t = \int_{\underline{p}}^{\overline{p}} I_t(p)\gamma(p)dp \tag{28}$$

$$N_t = \int_{\underline{p}}^{\overline{p}} L_t(p)\gamma(p)dp \qquad W_t = \int_{\underline{p}}^{\overline{p}} L_t(p)W_t(p)\gamma(p)dp \tag{29}$$

3 Solution method

To solve the model we introduce the following two step numerical procedure. In the first step we approximate all functions of firm productivity using Chebyshev polynomial approximation. The functions that need to be approximated are in particular the firm and worker value functions $V_t^X(p)$, wage function $W_t(p)$, vacancy intensity posting $V_t(p)$, average firm size $L_t(p)$ and distribution of employment $N_t(p)$. Following Boyd (2001), the basic N-point approximation P_N^f for a function $f(p)$ is the following:

$$P_N^f(p) \approx \sum_{n=0}^{N} b_n^f \times T_n(h(p)) \tag{30}$$

where b_n^f are Chebyshev weights, T_n is the n-th Chebyshev polynomial and $h(p)$ is a linear transformation between the interval $[\underline{p}, \overline{p}]$ to the canonical interval $[-1, 1]$. The approximation weights are set by the collocation method with Gauss-Lobatto nodes. The nodes y_k are set as follows:

$$x_k = -cos\left(\frac{k\pi}{N}\right) \qquad y_k = h^{-1}(x_k), \tag{31}$$

whereas the weights are defined as follows;

$$b_n^f = \sum_{k=0}^{N} f(y_k)T_{k,n} \qquad T_{k,n} = \frac{2\delta_n\delta_k T_n(x_k)}{N}, \tag{32}$$

where $\delta_k = \frac{1}{2}$ for $k = 0, N$ or $\delta_k = 1$ otherwise. Therefore, in order to calculate the value of function $f(p)$ at any point it is enough to know the values at the nodes. This property of Chebyshev polynomial approximation is central to our solution method - every function of productivity can be represented by its value at N nodes.

Solving the model also requires calculating a number of integrals of these functions, such as those that appear in equations (16), (17) or (23). The limits of this integration can be either the endpoints of the interval $[\underline{p}, \overline{p}]$ or any point inside the interval. However, a closer inspection of the equations that contain an integral taken together with the fact that we are only seeking to calculate the values of functions at the nodes y_k reveals that in the solution method the limits of the integration will only be the nodes y_k. Therefore, in order to calculate an integral of the function $f(p)$, we use the following scheme:[6]

$$\int_{-1}^{a} f_t(x)dx \approx \int_{-1}^{a} P_N^{f_t}(x)dx = \sum_{n=0}^{N} b_n^{f_t} \int_{-1}^{a} T_n(x)dx = \tag{33}$$

$$= \sum_{n=0}^{N} \frac{2\delta_n}{N} \sum_{k=0}^{N} f(x_k)T_n(x_k) \int_{-1}^{a} T_n(x)dx \tag{34}$$

Using a matrix multiplication view, the integral can be written as follows:

$$\int_{-1}^{a} f_t(x)dx \approx F_t^f \times W \times T = F_t^f \times P \tag{35}$$

where

- F_t^f is a row vector of length $N+1$ which contains the values of the function f,
- W is an $N+1 \times N+1$ matrix which contains the values of consequent Chebyshev polynomials in consequent nodes,
- T is a column vector of length $N+1$ which contains the integrals over the interval $-1, a$ of consequent Chebyshev polynomials,
- P is a column vector of length $N+1$ which is the result of multiplication of W and T.

Notice that the matrices W and T do not depend either on time or on the function f, and hence they can be calculated analytically outside of the model. The key result that we obtain from using Chebyshev approximations is that all the integrals that appear in the model equations can be replaced by a scalar product of the values of the integrand function in the nodes and exogenously calculated parameters. Finally, we transform the equations of the model into a system of equations in which integrals are replaced by their Chebyshev approximations.

In the second step we take this system of equations and solve (i) for the deterministic steady state and (ii) for the dynamics using a first order log-linearization around the steady state. The results shown in the next section are based on a model solved using a 20-point approximation ($N = 19$), which requires the external calculation of approx $(N+1)^2$ parameters for the integrals embedded in the equations of the model. The key advantage of using this solution method is that it allows for easy calculation of model dynamics and facilitates the implementation of various elements (frictions, capital etc.) which are of interest for the researcher.

[6] In order to make exposition more clear we assume canonical interval, without need to use linear transformation of intervals.

Table 1: Model parameters

Parameter	Description	Baseline value
γ	Pareto exponent	3
β	discount factor	0.99
α	prod function elasticity	0.3
r_s	degree of returns to scale	1
ρ	capital depreciation rate	0.1
σ_I	degree of investment frictions	1
δ	job destruction rate	0.05
b	utility of unemployed	0
υ	matching function efficiency	1.2
μ	match elasticity wrt job offers	0.5
ξ	bargaining power	0.5
ν_1	linear vacancy cost	0.01
ν_2	quadratic vacancy cost	500
λ^e	search intensity of employed	0.9
λ^u	search intensity of unemployed	1

4 Results

In this section we first discuss the baseline parametrization of the model and present results. We proceed as follows: we first document basic steady state properties model of the model, including the distribution of firm size and wage and job flows. Next we show the dynamic properties of the model. We document moments of selected variables show the dynamic response of the distribution of workers across firms in response to different shocks. We also introduce a working capital constraint and study its effects on the distribution of employment. Finally, we conduct a sensitivity analysis with respect to main model parameters.

4.1 Baseline parametrisation

Since the main goal of this paper is to present the solution methodology and general quantitative properties of a dynamic Burdett-Mortensen on-the-job search model with capital, we opt for a rough parametrization for the USA economy and use some parameters from the literature. For the baseline parametrization we assume that firms are distributed according to a truncated Pareto distribution with exponent equal to 3 on the interval $[\underline{p}\overline{p}] = [1, 10]$:

$$\Gamma(p) = \frac{\gamma}{p^{\gamma+1}}\Gamma^*, \tag{36}$$

where Γ^* is the normalization coefficient due to the truncation of the distribution to a finite interval. Solving the model rests crucially on the functional form and parametrization of vacancy posting. It turns out that one must impose a strong degree of convexity on this function. A linear vacancy posting cost would result in high-p firms posting a large amount of vacancies and consequently they would poach all the workers from low-p firms driving them out of the market. In our specification we assume a quadratic cost of posting vacancies:

$$c(V) = \nu_1 V + \nu_2 V^2, \tag{37}$$

which results in the following form for $\Xi(V_t(p))$:

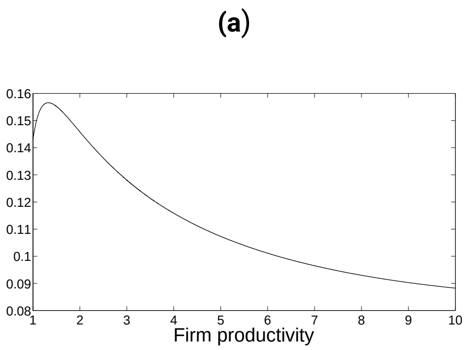
$$\Xi(V_t(p)) = \nu_1 + 2\nu_2 V_t(p). \tag{38}$$

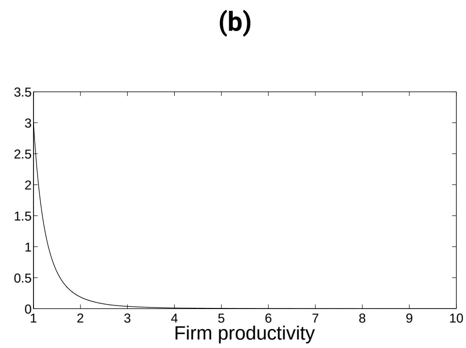
In our parametrization the linear term is set at a small value of 0.01, whereas the quadratic term is set at a large value of 500, which results in a large difference in the vacancy posting cost between firms of low and high productivity. While this may seem like a large degree of convexity, it is still much smaller than the value assumed in the MPV model.

The parameters of the model are summarized in Table 1. Most values are taken as standard in the literature. The model is calibrated to quarterly frequency with the discount factor set to 0.99. The parameters governing the firm are set as follows: share of capital is set to 0.3 and capital depreciation rate is set to 0.1. For the baseline parametrization we assume constant returns to scale and no investment rigidities. The steady state job destruction rate is set to 0.05, implying a roughly 20% annual job destruction rate. We allow the job destruction rate to fluctuate exogenously by modeling it as and AR(1) shock, similarly to the productivity shock. The elasticity of matches with respect to vacancies and the wage bargaining power parameters are set at 0.5, which are values commonly used in these models. The search intensity of unemployed is normalized to unity, whereas the value of the search intensity of employed is determined in order to set the on-the-job search flows to approximately 3.5% quarterly. The matching function scale parameter is used to calibrate employment in the economy to approximately 0.7, which means that the remaining population of the household can be thought of as the unemployed and inactive, instead of strictly unemployed. Finally, the utility of unemployed is set to zero. While this parameter is normally set to a value much closer to the wage, we are forced to use a low value for the same reason as with the convexity of vacancy posting. A high value of this parameter would increase the value of the unemployment, which would increase the wage relative to productivity primarily for small-p firms, thus driving them out of the market.

4.2 Steady state properties of the model

Figure 1: Steady state distribution of (a) workers across firms and (b) distribution of firms.





In this subsection we document steady state and properties of the model for the baseline parametrization. Figure 1 shows the exogenous assumed Pareto distribution of productivity across firms (panel b) and the

Figure 2: Average firm size by employment (a) and wage (b).

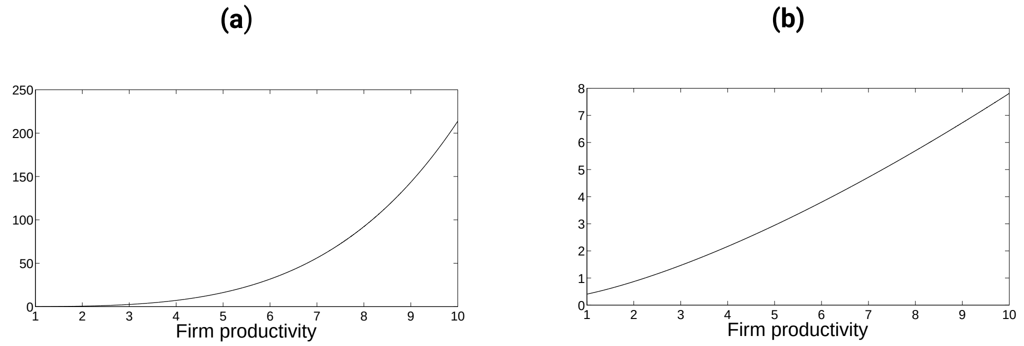
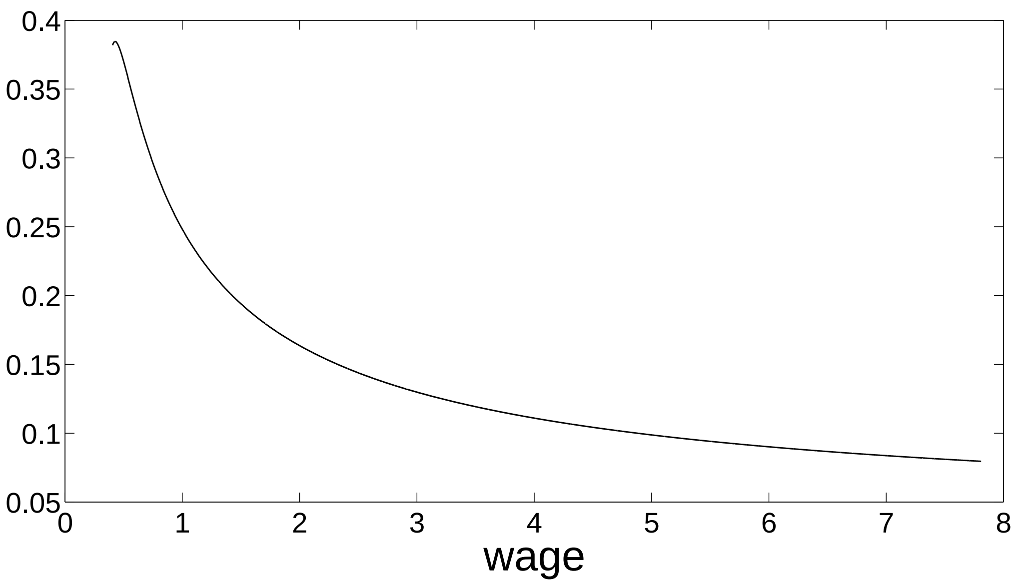


Figure 3: Distribution of wages



steady state distribution of employment across firms (panel a), which is the result of the on-the-job search mechanism of the mode. The distribution of employment differs from the distribution of firms in two respects. First of all, it is slightly hump-shaped. The firm size of the lowest-p firms is very small, and despite the fact that the number of such firms is the largest, their overall share in total employment is rather small. As average firm size increases with productivity, so does the contribution of total employment of firms. However, after exceeding a certain threshold, the decreasing Pareto density of firms of higher productivity overweighs the increasing average firm size which results in an overall decrease of share of total employment for firms of high productivity. The second interesting feature of the employment distribution is the heavy right tail, which is due to large average firm size of firms of high productivity (which is shown and discussed later in Figure 2. This second feature of the employment is most surprising as it is at least qualitatively in line with data about employment distribution for the United States. According to data from the Business Dynamics Statistics Database, the share of total employment for groups of firm size revolves at around 0.07 %, only to reach approximately 26% of employment for firms employing over 10000 employees. While a direct comparison with our model is not possible, the share of employment for analogically defined firm group sizes is strikingly similar.

Figure 4: Ratio of on-the-job search arrivals and arrivals from unemployment

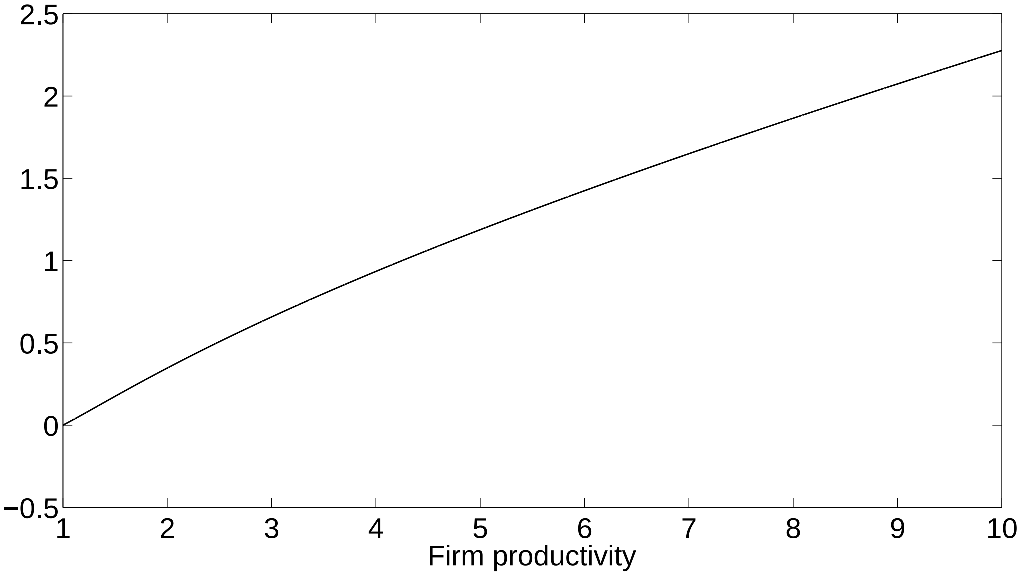
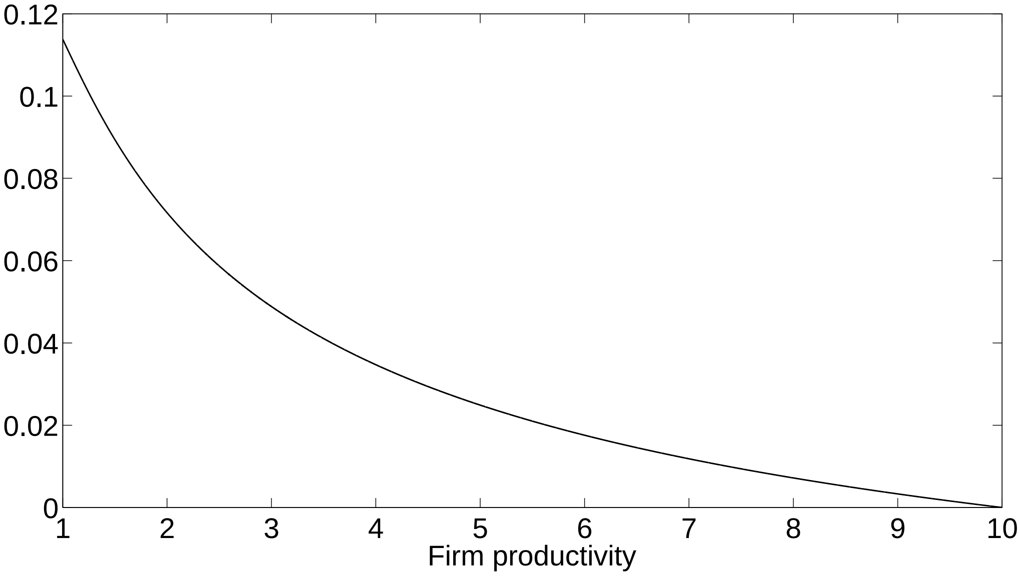


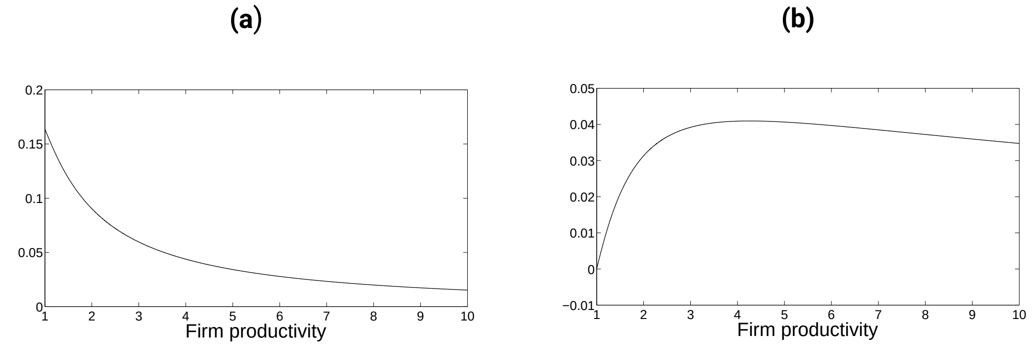
Figure 5: Rate of on the job search separations from firms



The main properties of the Burdett and Mortensen (1998) model hold - that is the wage and average firm size increase along with productivity, as shown in Figure 2. Employment at the smallest firm is equal to 0.0346, while at the largest it is equal to 213, resulting in a ratio of firm size of approximately 6150. This is a value which is much larger than reported in MPV2016 for both their baseline parametrization (the value is equal to 4.24), which uses much more highly convex hiring cost function, and for their parametrization with hiring cost convexity of similar magnitude (for which the ratio is equal to 137.4). The wage function, which is shown on the right panel of Figure 2, follows marginal labour productivity and displays some convexity, which is a result of the Nash wage bargaining assumption and the inclusion of capital in the model. In a model without capital, where production is a linear function of employment, the wage function would be exactly linear. The distribution of wages is shown on Figure 3. The difference between smallest and largest wage is equal to approximately 20, which is of course much less than in the data. The convexity of the wage function increases wage inequality, which for this parametrization is equal to approx 0.38, as measured by the Gini coefficient, a value slightly lower than the data for the USA.

We now turn to the discussion of the steady state employment flows between firms. We first analyze the inflow of workers to firms and look at how many workers are poached from other firms relative to hires from the pool of the unemployed. The results for this ratio is shown in Figure 4. The plot starts at the value zero, since firms of the lowest possible productivity do not have the possibility to poach any workers, and rises to up to 2.2 for the most productive firm, meaning that a little over two thirds of new hires are poached for these firms. Figure 5 shows the rate at which firms are losing workers to other firms through on the job search. The smallest firms are losing a little over 11% of their workforce quarterly to poaches from other firms, a value which is probably higher than in the data. The OJS separation rate reaches zero for firms of highest productivity.

Figure 6: New hires arrival rates for (a) unemployed (b) poached



The panels of Figure 6 show the arrival rate for hires from the pool of the unemployed and OJS hires. The picture here is a bit more complicated than in the case of on the job search separations, since in order for their to be an equilibrium, these rates have to implicitly take into account the job losses due to poaching. The hiring rate is a monotonically decreasing function of productivity. Since low-p firms can only (or mainly) rely on hires from unemployed and lose a lot of workers due to poaching, their hiring rate has to be high. On the other hand, the ojs arrival rate is not monotonic, for firms of high productivity it switches to being a decreasing function. This is due to the fact that for higher-p the decreasing probability of having a worker poached has a relatively stronger effect than the need for new hires (in equilibrium).

4.3 Dynamic properties of the model

We document the business cycle properties of the model by showing impulse response functions for the distribution of employment to the productivity and job destruction rate shocks and basic standard and relative-to-GDP standard deviations of model variables. We assume that both shocks follow an AR(1) process with autocorrelation coefficient equal to 0.95, and the standard deviation of the innovation equal to 0.01 and 0.05 for the shocks respectively.

We now turn to analyze the Figures 7 and 8 show the dynamic response of the distribution of firms in response to an aggregate productivity shock A_t after 4, and 12 quarters after the shock hits plotted against

Figure 7: Change in distribution of employment in response to technology shock in levels

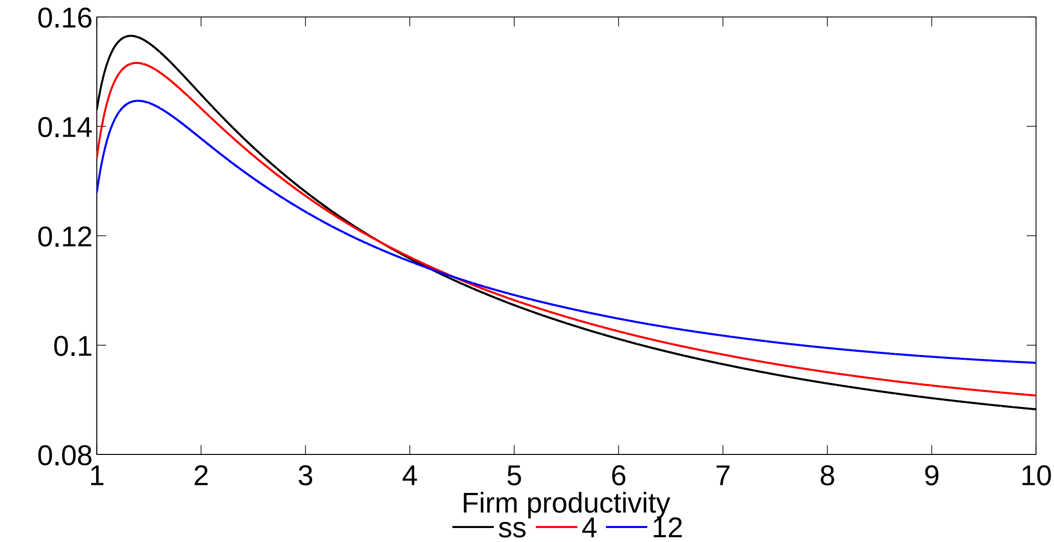
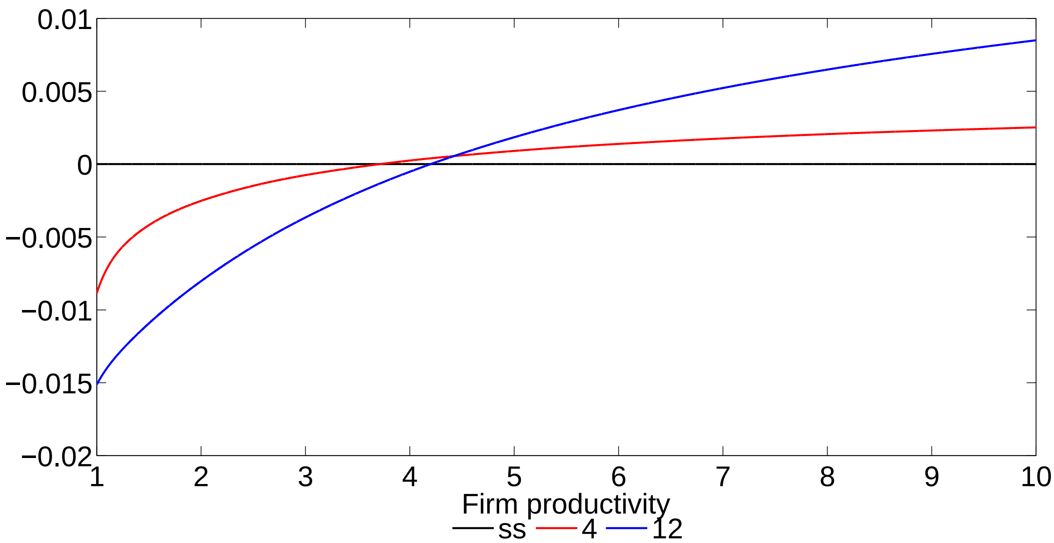
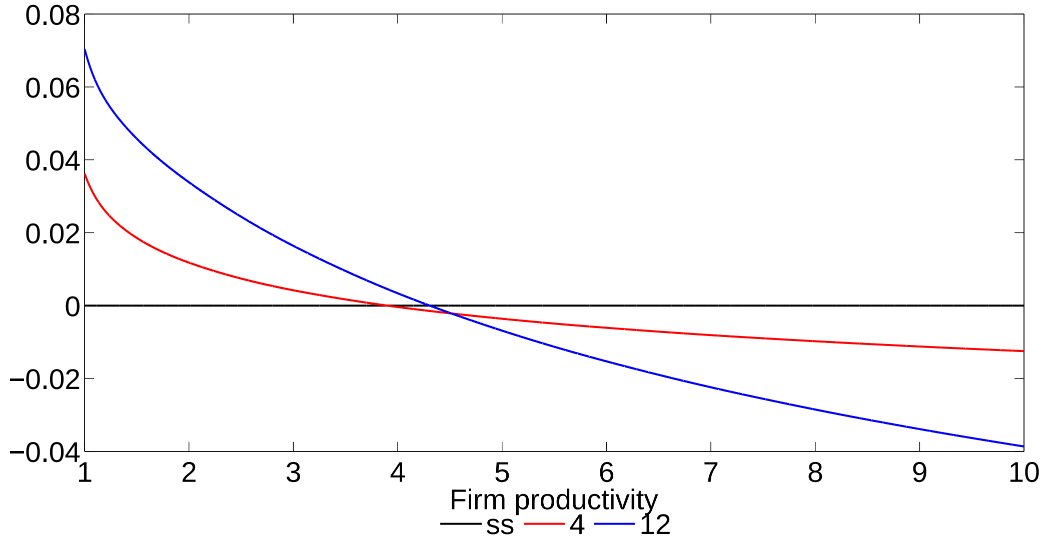


Figure 8: Change in distribution of employment in response to technology shock shown as difference from steady state distribution



the steady state distribution. Overall, the model predicts a shift in the distribution of workers to the right. In absolute terms, the 1% increase in productivity is more pronounced for high-p firms, since is a stronger incentive for them to post vacancies. This model property is consistent with basic empirical facts documented in MPV. What is interesting is the fact that for this baseline parametrization there is a positive correlation between the initial size of firm and the change in employment of a firm. It is not the case that the impulse response function starts decreasing in the highest p range.

Figure 9: Change in distribution of employment in response to job destruction rate shock shown as difference from steady state distribution



The second shock for which we show the response of the distribution of employment is the job destruction rate shock which is shown on Figure 9. Since low-p firms incur smaller costs of posting vacancies, an increase in the job destruction rate hurts them less and shifts the distribution of workers to the left. This dynamic model property is also in line with empirical facts from MPV, since, as one would expect, the job destruction rate is negatively correlated with GDP (and also with the productivity shock).

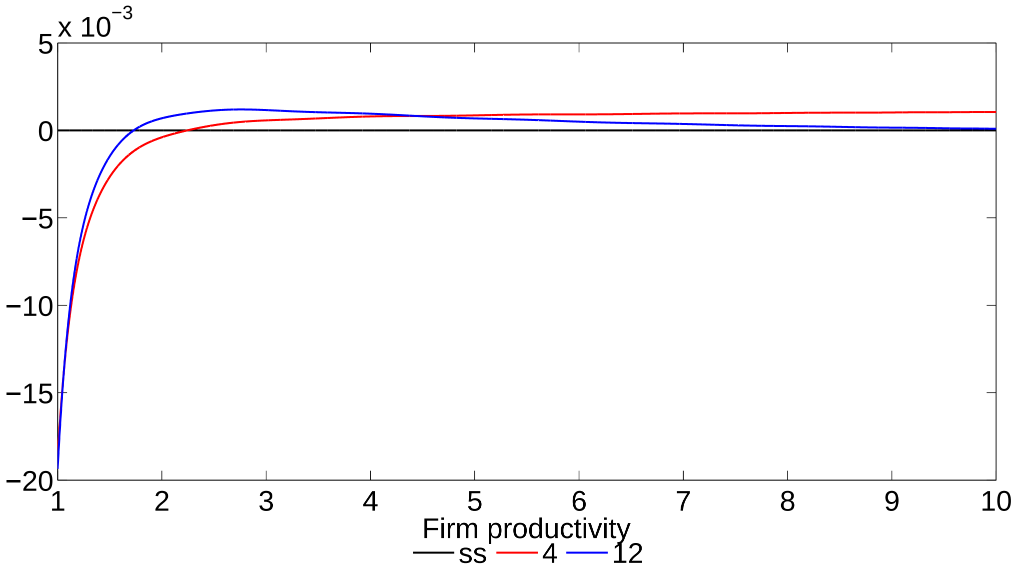
Table 2 shows the standard deviations and relative-to-GDP standard deviations of basic model variables. The values are shown for the data for the USA and separately for two shocks - the productivity shock and shock to job destruction rate. The model does a fairly good job in replicating the volatility of main macroeconomic aggregates with the technology shock. However, as can be seen from the results, the aggregate productivity shock alone is not able to replicate the volatility of labour market variables such as unemployment, vacancies and labour market flows, therefore we also consider an exogenously varying job destruction rate shock. The driver behind the dynamics of the MPV2106 model was also the two shocks, with the difference that in their model job destruction rate was perfectly correlated with (a function of) aggregate productivity level. What is more, in our setup we calibrate employment to approx 0.7, leaving the remaining mass of workers as unemployed job seekers, which by itself lowers the cyclical volatility of unemployment. One crucial property of our model, is that thanks to the inclusion of capital, we are able to replicate the volatility of product and do not face the tradeoff between its volatility and the ratio between the largest and smallest firm, as is reported in MPV2016 model. The inclusion of the job destruction shock helps in replicating some of the volatility of labour market variables, most notably unemployment.

Faced with the average performance in replicating the volatility of labour market variables using the productivity and job destruction shocks, we investigate the impact on dynamic model properties if we assume that the job search intensity parameters λ_u and λ_e were also shocks following an AR(1) process. These results are reported in table 3. Overall, these shocks have a significant impact on the functioning of the labour market, with the relative-to-GDP standard deviation of vacancies, job finding rate, labour market tightness

Table 2: Standard deviations and relative-to-GDP standard deviations of model variables

Variable	data USA		techn shock		destr rate shock		both shocks	
	σ_X	$\sigma_{X/Y}$	σ_X	$\sigma_{X/Y}$	σ_X	$\sigma_{X/Y}$	σ_X	$\sigma_{X/Y}$
Product	0.013	1.00	0.013	1	0.009	1	0.015	1
Consumption	0.0075	0.58	0.009	0.696	0.007	0.781	0.011	0.724
Investment	0.044	3.46	0.022	1.782	0.02	2.339	0.03	1.978
Capital	–	–	0.008	0.657	0.009	0.99	0.012	0.78
Average Wage	0.007	0.70	0.013	1.05	0.007	0.827	0.015	0.984
Variance of Wages	–	–	0.027	2.156	0.022	2.534	0.035	2.284
Unemployment	0.121	9.56	0.001	0.097	0.022	2.508	0.022	1.426
Vacancies	0.111	8.84	0.006	0.49	0.008	0.924	0.01	0.662
Vac / Unemployment	0.366	28.15	0.007	0.558	0.016	1.873	0.018	1.158
Vac / Search effort	–	–	0.006	0.492	0.007	0.867	0.01	0.638
Average Labour Prod.	0.007	0.54	0.012	0.973	0.005	0.633	0.013	0.878
OJS rate	–	–	0.003	0.279	0.015	1.748	0.015	1.019
UE rate	0.068	5.23	0.003	0.246	0.004	0.433	0.005	0.319

Figure 10: Effect of wage markup on the distribution of workers across firms



and the job-to-job transition rate being quite high. These results suggest that endogenizing search intensity could go a long way in improving model quality. Regarding the Beveridge curve, the correlation between vacancies and unemployment is negative and is equal to -0.65 for the productivity shock.

4.4 Working capital constraint

In this subsection we introduce a working capital constraint in which we assume that a firm must borrow in order to pay the wage bill. In order to capture this phenomenon we modify the equation for the value of

Table 3: Standard deviations and relative-to-GDP standard deviations of model variables

Variable	λ_u **shock** σ_X	λ_u **shock** $\sigma_{X/Y}$	λ_e **shock** σ_X	λ_e **shock** $\sigma_{X/Y}$
Product	0.001	1	0.001	1
Consumption	0.001	0.781	0.001	1.003
Investment	0.004	2.442	0.001	2.216
Capital	0.002	1.024	0	0.975
Average Wage	0.002	1.621	0.002	4.444
Variance of Wages	0.003	2.294	0.002	4.911
Unemployment	0.002	1.026	0.001	1.343
Vacancies	0.011	7.774	0.011	22.425
Vac / Unemployment	0.01	6.956	0.012	23.22
Vac / Search effort	0.015	10.003	0.003	6.678
Average Labour Prod.	0.001	0.781	0.001	1.464
OJS rate	0.004	2.883	0.005	10.042
UE rate	0.004	2.754	0.002	3.339

employment for a firm:

$$V_t^J(p) = \left(Y_t^L(p) - (1+r_k^w)W_t(p)\right) + \beta(1-\delta)E_t\left(\left(1-\Phi_t^N\frac{\overline{V}_t(p)}{V_t}\right)V_{t+1}^J(p)\right) \tag{39}$$

where r_k^w is a shock that sets the interest that the firm must pay on that loan. We analyze the dynamic impact on the distribution for a unit shock to this interest rate. As can be seen from Figure 10, the introduction of a markup on the wage bill shifts the distribution of workers to the right - there are more firms of higher productivity and higher wages and the expense of low type-p firms. The reason for this is that the low type-p firms are much more vulnerable to the wage bill cost - the profit they make on each worker is reduced by a much larger share then for more productive firms - which in turn causes a much higher reduction in their vacancy posting and employment.

We also examine what happens when the working capital constraint is introduced in the same way for the linear term of the vacancy posting cost. The result, as can be seen on Figure 11 is also a shift of the distribution to the right, since the overall vacancy posting cost is relatively smaller for low-p firms.

$$V_t^V(p) = -(\nu_1(1+r_k^v) + 2\nu_2 V_t(p)) + \beta E_t\left((\Psi_t^U + \Psi_t^N\frac{N_t(p)}{N_t})V_{t+1}^J(p)\right) \tag{40}$$

4.5 Sensitivity Analysis

This subsection shows the sensitivity analysis of the steady state distribution of the distribution of employment under different values of selected parameters. Figures 12 - 16 show the steady state distribution of employment across firms for for various values of the following parameters: returns to scale, Pareto exponent of the exogenous productivity distribution, search intensity of unemployed and employed job seekers, and finally the elasticity of matches with respect to vacancies. As can be expected and is shown on Figure

Figure 11: Effect of vacancy cost markup on the distribution of workers across firms

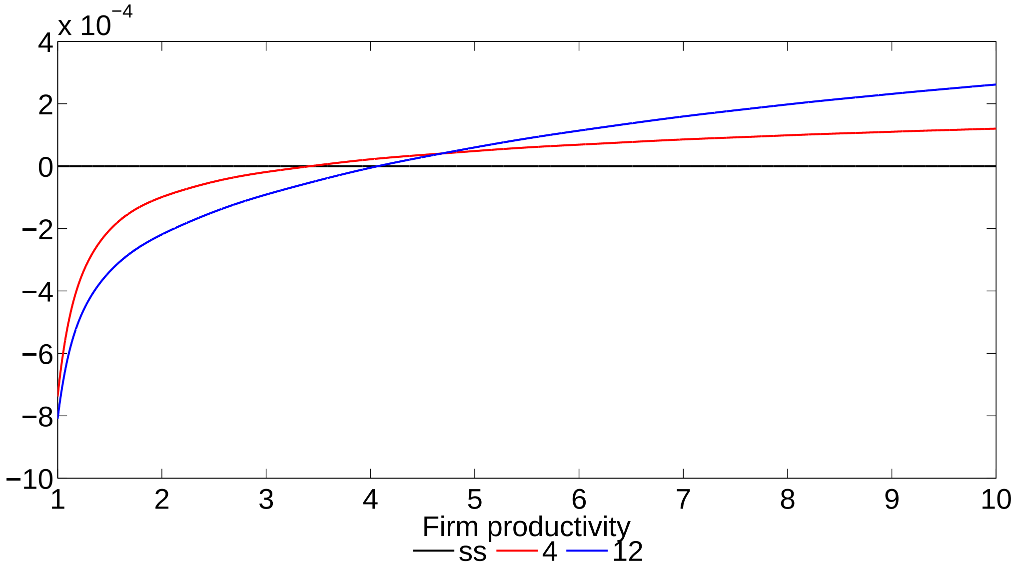
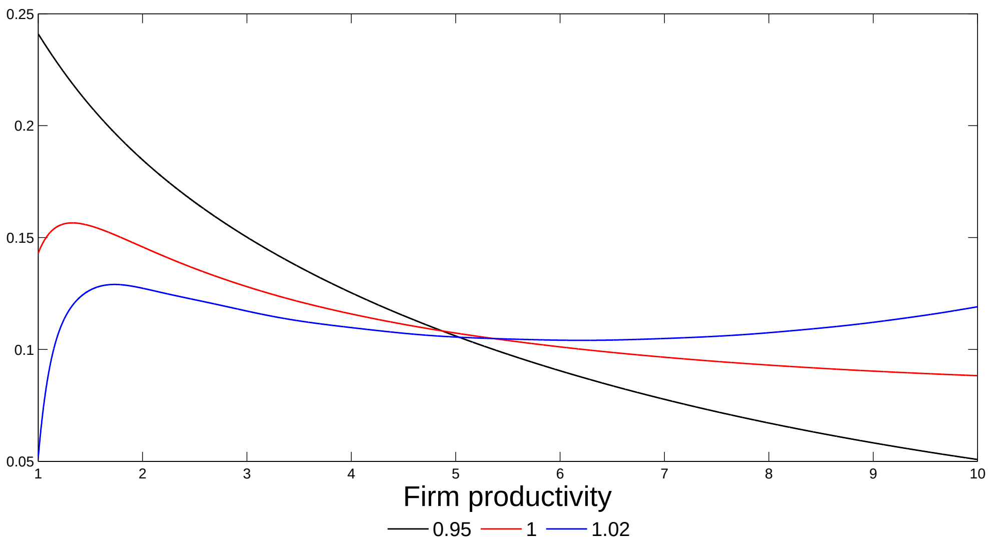


Figure 12: Distribution of employment across firms for different values of returns to scale parameter.



12, decreasing returns to scale affects negatively the most productive firms, which employ more factors of production. What is more, this parameter also has a huge impact on the ratio between employment in the largest and smallest firm - for returns to scale equal to 1.02 the value is approximately 23000, whereas for 0.95 it is an order of magnitude smaller at approximately 2000.

The effect of the Pareto exponent parameter can be seen on Figure 13. The overall effect is similar to the effects of the returns to scale, a higher density of more productive firms translates into higher overall employment at larger firms, however some significant differences emerge. First of all, their is no change in

Figure 13: Distribution of employment across firms for different values of Pareto exponent.

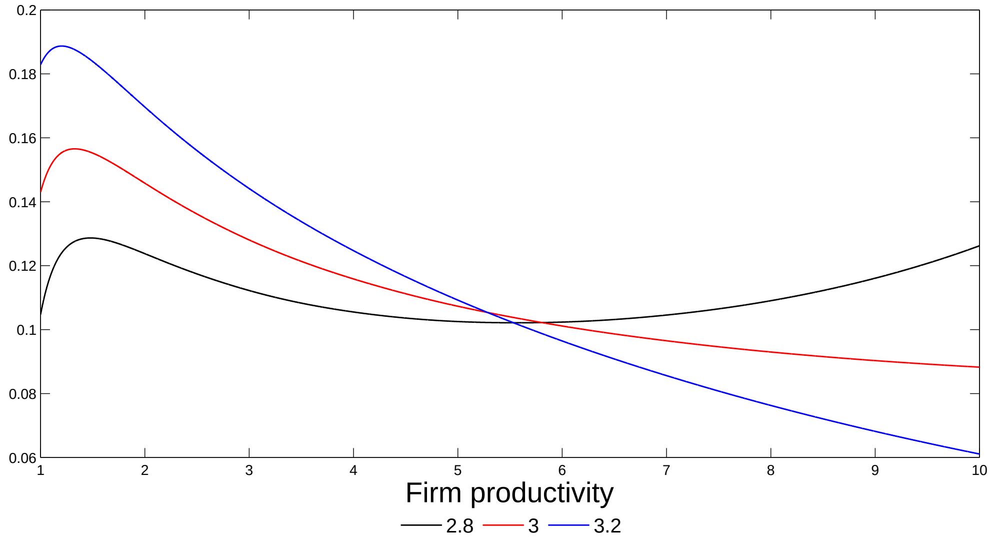
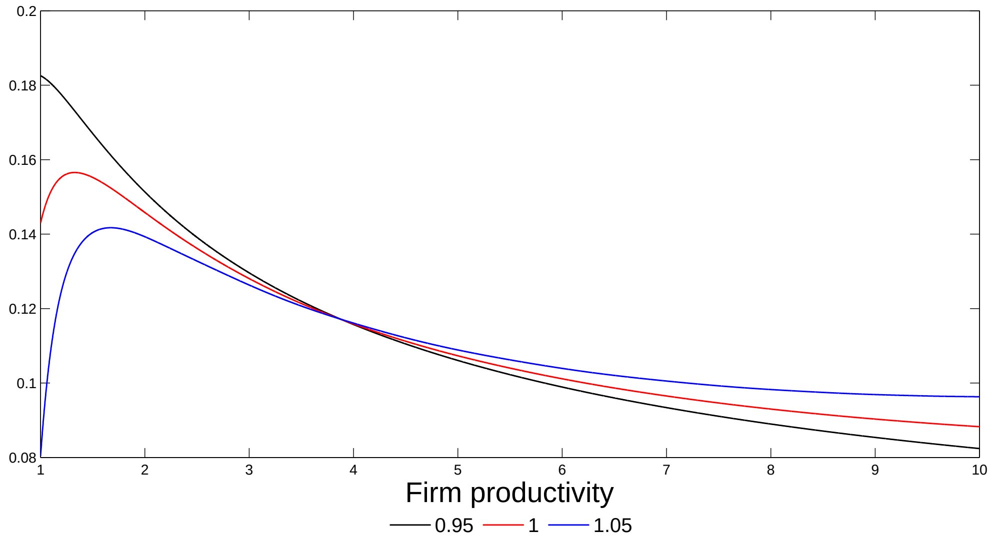


Figure 14: Distribution of employment across firms for different values search intensity of unemployed.



the shape of the distribution for low-p firms - for all values of the Pareto exponent they are hump-shaped. Also, there is practically no changes in the ratio of largest to smallest firm size - the differences in total employment are the result of the changes in the exogenous productivity distribution and not the model properties.

The effect of the search intensity of unemployed is shown in Figure 14. Higher search intensity of unemployed workers results in increased vacancy posting, especially by larger firms, and hence the distribution is moved to the right. This results suggests that incorporating endogenous search intensity by unemployed

Figure 15: Distribution of employment across firms for different values of search intensity of employed.

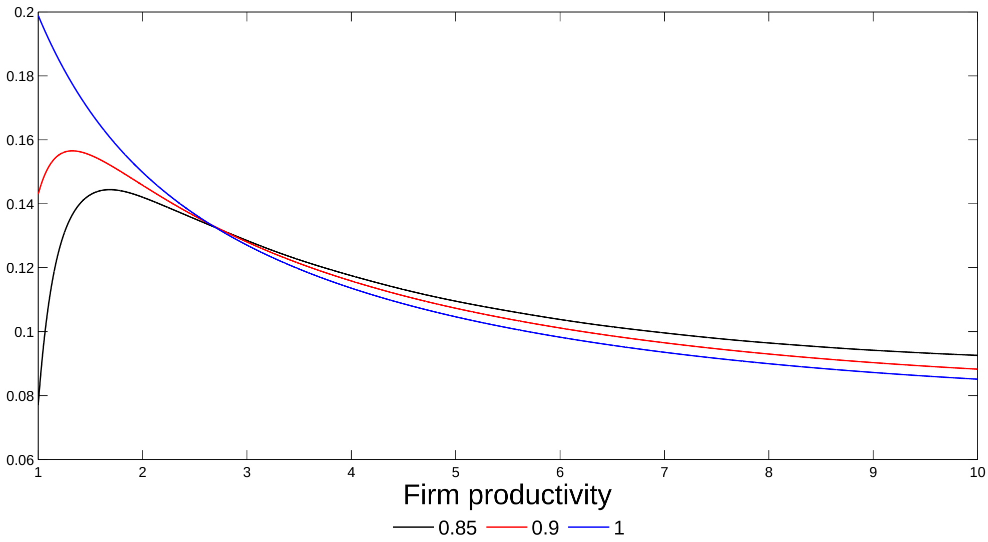
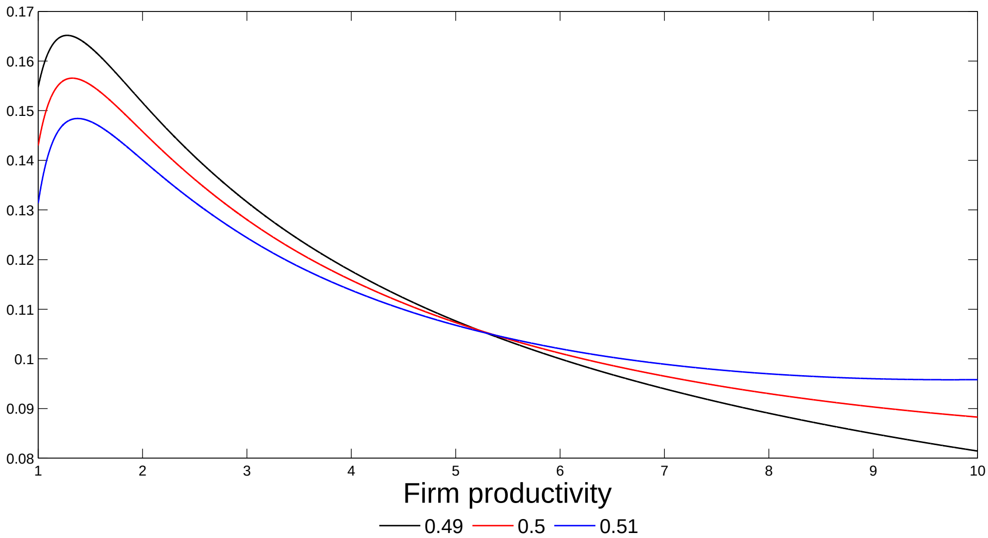


Figure 16: Distribution of employment across firms for different values of elasticity of matching function wrt vacancies.



into the model could help the model amplify the response of employment and unemployment to productivity shocks.

The effect of the search intensity of employed is shown in Figure 15. Surprisingly, higher search intensity of employed workers results in smaller employment at more productive firms. The reason behind this surprising result is the following: firms in the middle of the distribution take into account that more of their workers will be poached by firms of higher productivity and hence post a smaller amount of vacancies, and this mechanism propagates itself all the way to the most productive firms.

The effect of the elasticity of matches with respect to vacancy posting is shown in Figure 16. Overall, this parameter has a very strong impact on the distribution of employment. Even a small increase from 0.5 to 0.51 of the elasticity results in a relatively large shift of the distribution of workers to the right.

5 Conclusions

In this paper we consider a stochastic version of on-the-job search model a la Burdett-Mortensen based on Moscarini and Postel-Vinay (2012). We propose a novel technique to solve and then simulate the model that uses both projection and perturbation methods. We first characterize the resulting steady state distribution of employment, wages and flows between firms and from unemployment. We then show how the aggregate productivity shock affect the flows of workers to and from unemployment as well as across firms. Model implications are in line with empirical data - employment at larger firms varies more over the business cycle than at small firms. We then extend the model to allow for the presence of the financial markets in the form of working capital. We show how higher costs of borrowing affect the distribution of wages offered by firms. We also find that including capital in the model helps in resolving the tradeoff between model dynamic properties and steady state distribution of firm size - we are able to generate a model where the ratio of largest to smallest firm is much closer than in the data. What is more, the resulting distribution of employment is close to the data. While a direct comparison with Business Dynamics Statistics data is difficult, total employment in subsequent firm employment groups in the model remains at a constant level.

References

Andolfatto, D. (1996). Business cycles and labor-market search. *The american economic review*, pages 112–132.

Barth III, M. J. and Ramey, V. A. (2002). The cost channel of monetary transmission. In *NBER Macroeconomics Annual 2001, Volume 16*, pages 199–256. MIT Press.

Boyd, J. P. (2001). *Chebyshev and Fourier spectral methods*. Courier Corporation.

Burdett, K. and Mortensen, D. T. (1998). Wage differentials, employer size, and unemployment. *International Economic Review*, pages 257–273.

Cahuc, P., Postel-Vinay, F., and Robin, J.-M. (2006). Wage bargaining with on-the-job search: Theory and evidence. *Econometrica*, 74(2):323–364.

Cheron, A. and Langot, F. (2004). Labor market search and real business cycles: reconciling nash bargaining with the real wage dynamics. *Review of Economic Dynamics*, 7(2):476–493.

Christensen, B. J., Lentz, R., Mortensen, D. T., Neumann, G. R., and Werwatz, A. (2005). On-the-job search and the wage distribution. *Journal of Labor Economics*, 23(1):31–58.

Elsby, M. W., Hobijn, B., and Sahin, A. (2010). The labor market in the great recession. Technical report, National Bureau of Economic Research.

Elsby, M. W., Hobijn, B., and Şahin, A. (2013). Unemployment dynamics in the oecd. *Review of Economics and Statistics*, 95(2):530–548.

Fuerst, T. S. (1992). Liquidity, loanable funds, and real activity. *Journal of monetary economics*, 29(1):3–24.

Haltiwanger, J., Hyatt, H., and McEntarfer, E. (2015). Cyclical reallocation of workers across employers by firm size and firm wage. Technical report, National Bureau of Economic Research.

Hosios, A. J. (1990). On the efficiency of matching and related models of search and unemployment. *The Review of Economic Studies*, 57(2):279–298.

Kaas, L. and Kircher, P. (2015). Efficient firm dynamics in a frictional labor market. *American Economic Review*, 105(10):3030–60.

Manning, A. et al. (2012). Steady-state equilibrium in a model of short-term wage-posting. Technical report, Centre for Economic Performance, LSE.

Merz, M. (1995). Search in the labor market and the real business cycle. *Journal of Monetary Economics*, 36(2):269–300.

Mortensen, D. (2005). *Wage dispersion: why are similar workers paid differently?* MIT press.

Mortensen, D. T. et al. (2010). Dynamic monopsonistic competition and labor market equilibrium. In *Society for Economic Dynamics, 2010 Meeting Paper*, volume 164. Citeseer.

Moscarini, G. and Postel-Vinay, F. (2009). The timing of labor market expansions: New facts and a new hypothesis. In *NBER Macroeconomics Annual 2008, Volume 23*, pages 1–51. University of Chicago Press.

Moscarini, G. and Postel-Vinay, F. (2012). The contribution of large and small employers to job creation in times of high and low unemployment. *The American Economic Review*, 102(6):2509–2539.

Moscarini, G. and Postel-Vinay, F. (2013). Stochastic search equilibrium. *The Review of Economic Studies*, pages 1545–1581.

Moscarini, G. and Postel-Vinay, F. (2016). Wage posting and business cycles: A quantitative exploration. *Review of Economic Dynamics*.

Nagypal, E. (2005). On the extent of job-to-job transitions. *Unpublished Manuscript, Northwestern University*.

Petrosky-Nadeau, N. (2014). Credit, vacancies and unemployment fluctuations. *Review of Economic Dynamics*, 17(2):191–205.

Wasmer, E. and Weil, P. (2000). The macroeconomics of labor and credit market imperfections.

Wolthoff, R. (2014). It's about time: Implications of the period length in an equilibrium search model. *International Economic Review*, 55(3):839–867.

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