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Abstract

In this paper we describe large scale, multisector dynamic stochastic general equilibrium model that was constructed for the purpose of CO2 reduction policy assessment. Despite its large size (compared to most of the other policy oriented models of the class) our model is directly calibrated and estimated.

Keywords: DSGE

JEL Codes: D50

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Introduction

The main objective is to construct a DSGE model that can be calibrated based on data, which can be used to simulate mitigation scenarios. This document presents the model structure and solution procedure, as well as calibration approach.

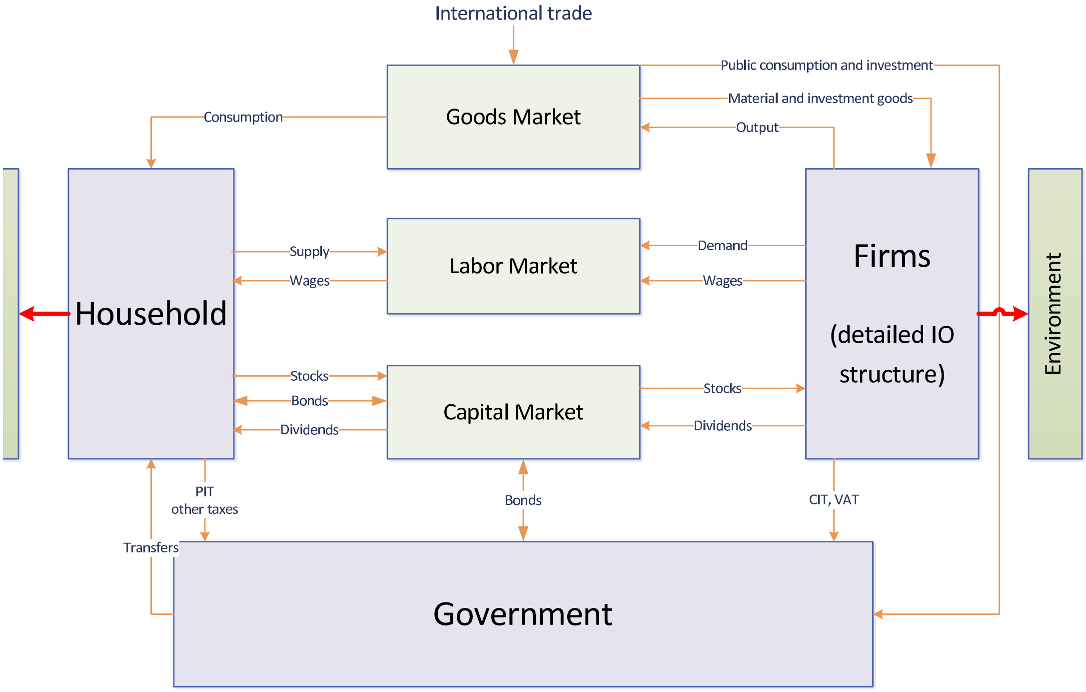
The model considers several different economic sectors, including the energy sector, and the structure of interconnections between sectors directly relates to empirical input-output tables. This allows realistic representation of interrelations among all sectors, and accounting for various sources of GHG emissions (direct sectoral emissions and emissions resulting from consumption of fuels). The model accounts for labor market imperfections, investment frictions, endogenous technological progress and government. It allows simulating the impact of policies on, among others: gross domestic product (GDP) level and growth and its components (investment, consumption, net exports, public consumption); total and sectoral value added, energy demand and GHG emissions; unemployment, employment and wages; real exchange rate, exports and imports; fiscal balance and welfare of society. The structure of the document is as follows. In section 1 we present the model structure and solution procedure. In section 2 the calibration procedure are shown.

1 Model structure

1.1 Main model segments

The model structure is divided into three main blocks: (1) households, (2) firms, and (3) government. These blocks are interconnected on three separate markets: (1) labor (2) capital, and (3) goods market (see Figure 1). Households supply labor, decide on the level of their demand for consumption goods as well as for government bonds and firm stocks. Households interact with producers on the labor market where wages are negotiated and vacancies filled in a search and matching process. This market is operated by a special intermediatory firm that buys labor from households and sells it to firms in basic production sectors described later. In exchange for their work and savings they receive dividends and wages from firms, interest payments from the government, paying at the same time taxes directly imposed on them by the government. Firms produce final goods that are later consumed by households, re-invested by producers or utilized by the government. Both production and consumption evoke CO2 emission, that is modeled on sectoral and household level. In the production process that we describe in detail in section 1.3, firms employ labor, capital, intermediate goods and energy. As they are owners of capital and have some monopolistic power, their profits are positive, which allows them to pay dividends for their shareholders. Additionally, they pay income and excise taxes to the government. On the other hand, the government divides its tax income into public investment, public consumption and social transfers to households for unemployed and retired.

Figure 1: Main blocks of the model and their interrelations



1.2 Households

In time $t \geq 0$ our model economy is populated with POP_t consumers that form a representative household which maximizes the following expected discounted utility from the stream of consumption:

$$\max_{\{\tilde{C}_t\}_{t=0}^{\infty}} U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \times u\left(\frac{\tilde{C}_t - h\tilde{C}_t^{ext}}{POP_t}\right) \tag{1}$$

$$u(C_t) = \frac{C_t^{1-\sigma} - 1}{1 - \sigma} \tag{2}$$

where β denotes the subjective discount factor, σ determines the inter-temporal elasticity of substitution between current and future consumption, $\tilde{C}_t$ is the effective consumption per capita of the representative household. The utility function also exhibits habit formation modelled by the the "keeping up" with the Joneses effect. The level of consumption by neighbours is proxied by $h\tilde{C}_t^{ext}$, where h is a parameter and $\tilde{C}_t^{ext}$ is a variable that is equal to aggregate consumption $\tilde{C}_t$. Please note that such a setup ensures that the habit variable is not taken into account in the optimization process by the household.

Effective consumption $\tilde{C}_t$ depends on the consumption of market goods, C_t and home produced goods, H_t and is given by the following equation:

$$\tilde{C}_t = [C_t^{\epsilon_{CH}} + H_t^{\epsilon_{CH}}]^{\frac{1}{\epsilon_{CH}}} \tag{3}$$

where ϵ_{CH} determines the elasticity of substitution between the consumption of market and home produced goods. Home produced consumption goods are produced by non-employed NE_t persons according to the following linear technology function:

$$H_t = b \times NE_t \tag{4}$$

The income of the representative household consists of (i) labor income W_tN_t, (ii) dividends transferred from firms in the economy, Π_t, (iii) pension payouts from the pension system PEN_t as well as (iv) income from saving goods B_t^C created by the saving good producer described in section 1.5, yielding interest rate R_t.

The expenditure side of the household's budget consists of consumption expenditures, $P_t^C C_t$, PIT, VAT and lump sum taxes, T_t, pension contributions as well as labor market search costs, Ξ_t. The household also invests in assets A_t, which yield one period interest $r_t - 1$.

The budget constraint of the household takes the form:

$$P_t^C C_t + T_t + VAT_t^H + \Xi_t + A_t \tag{5}$$
$$= A_{t-1}(r_{t-1}) + \Delta_t^B + \Pi_t + (1-\tau_t^S)(1-\tau_t^W)W_tN_t + PEN_t \tag{6}$$

where τ_t^S is the pension fund contribution rate, τ_t^W is income tax rate and the changes in bond holdings are given by:

$$\Delta_t^B = \left(B_{t-1}^C - \frac{B_t^C}{R_t} \right). \tag{7}$$

The amount of VAT paid by the household is given by:

$$VAT_t^H = \sum_{s \in \mathcal{S}} \tau_s^V C_{t,s}. \tag{8}$$

In the above equation $\mathcal{S}$ is the set of sectors in the model, described in detail in the next subsection. Please note that this formula for VAT allows for the differentiation of VAT rates among different sectors of the economy. Finally, labor search cost is given by:

$$\Xi_t = \left(\bar{c}_U \times (e_t - \bar{e}) + \psi_u \times (e_t - \bar{e})^2 \right) \times NE_t P_t^C \tag{9}$$

where e_t is the search intensity of nonemployed person with steady state value set to $\bar{e}$. We assume that this cost is zero in steady state. Parameters $\bar{c}_U$, ψ_u give the second order approximation of the true search cost function. Labor market search cost is also expressed in terms of the consumption good. The entire household population POP_t is divided as follows. Firstly, we can distinguish persons that are out of the workforce, who we denote by OUT_t (this group can be identified primarily with retired persons and children). The remaining population can be split into those employed N_t, and nonemployed NE_t, who in turn can be further divided into unemployed U_t and inactive IN_t. The number of persons that are out of the workforce is taken as exogenous, while the equations governing the remaining variables are described in the section describing the labor market (1.4). The above can be summarized by the following equation:

$$POP_t = N_t + U_t + IN_t + OUT_t \tag{10}$$

1.3 Firms

1.3.1 Production structure

The model allows for considering up to ten sectors producing basic sectoral goods. There is a lot of freedom in defining a list of sectors with the exception, than energy sector and raw material sector must be present. In the baseline version we consider the following sector: (1) agriculture (AGR), (2) raw materials production (RMP), (3) industry and manufacturing (IND), (4) energy production (ENG), (5) construction services (CST), (6) retail and whole trade services (TRD), (7) market services (SRV), (8) transport services (TRN), (9) financial services (FIN) and (10) public services (PUB). Sectors are defined in a way that enables calibration of the model directly on the data, in particular input-output tables (activity by activity) with a split by domestic and foreign inputs and outputs (more on that in section 2). It is possible to implement different disaggregation of the economy into sectors, however for the model to have applied potential any disaggregation has to be consistent with (statistical) input output tables, fuel consumption and emission data (so the calibration is possible). Antosiewicz (2014) discusses further the modeling of economic sectors in Memo III and shows under which conditions redefining of sectors, for example isolating mining sector from raw materials production, can be done.

Production is divided into three stages (see Fig. 2). In the first stage a basic sectoral good is produced by monopolistically competitive firms that employ capital, labor, materials and energy as production factors. This good is thereafter sold to trading firms operating on both domestic and foreign sectoral market. Finally, trading firms' product is purchased by: (i) (as intermediate demand) producers of basic goods (in each sector); (ii) (sectoral) export firms, which distribute domestic production in foreign markets; and (iii) and three types of domestic final goods producers, yielding (1) investment, (2) government, and (3) private consumption goods. Final production is traded on the goods market with households, basic producers and government in accordance with the flows established from the input/output matrix.

1.3.2 Production firms

Inter-temporal optimization problem. In each sector $s \in \mathcal{S}$ there exist infinitely many identical monopolistically competitive firms producing basic good Y_t^s and selling it for a price P_t^s, taking a demand function for their product as given. The firms use capital, K_t^s, labor, N_t^s, materials, M_t^s and electricity ENG_t^s, as input factors. Firms also pay CIT, excise and CO_2 taxes. A firm's decision process is based on the maximization of expected discounted cash-flows from production:

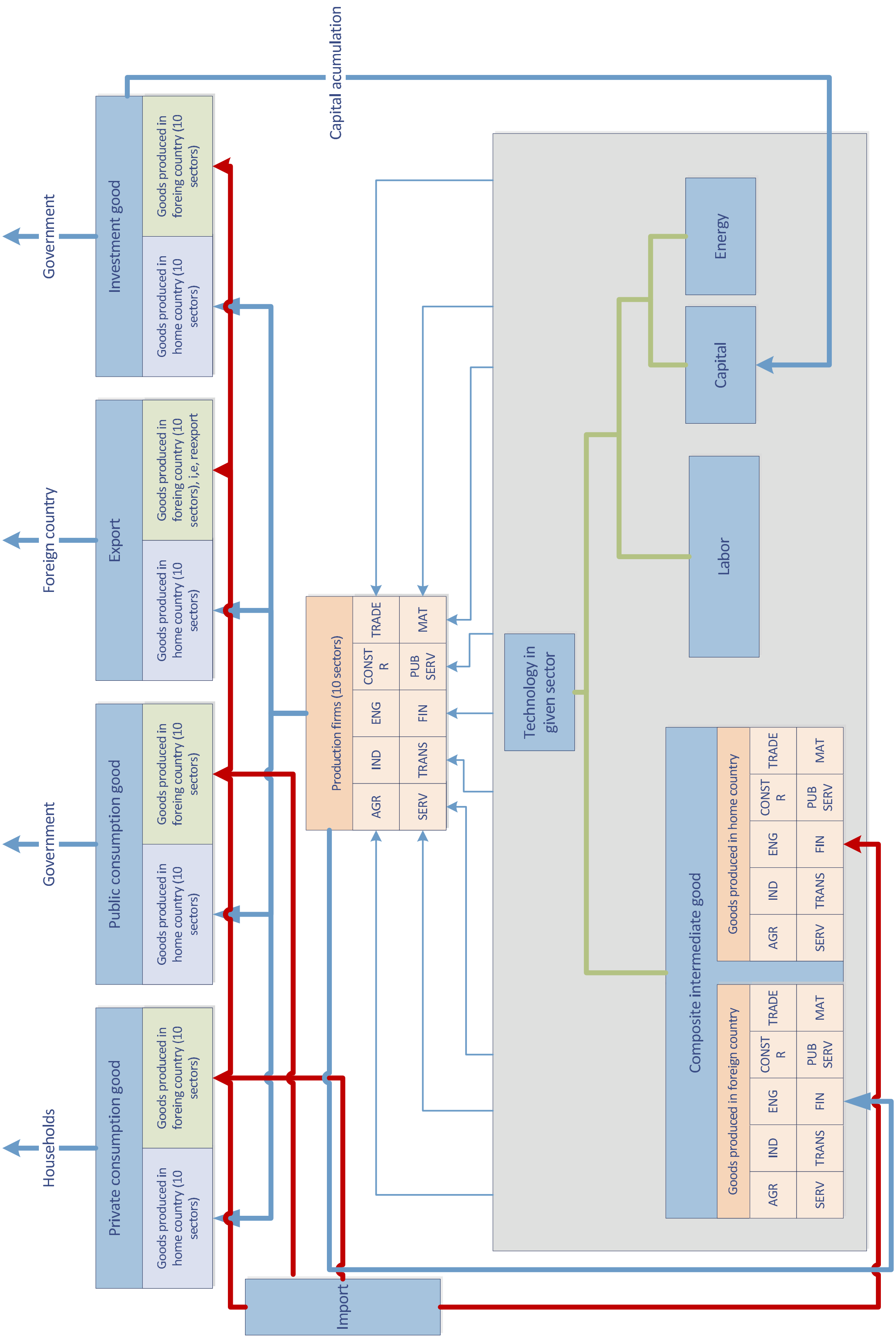
$$\max \widetilde{\Pi_0}^s, \qquad \widetilde{\Pi}_t^s = \Pi_t^s + E_t\{\Lambda_{t+1}\widetilde{\Pi}_{t+1}^s\}. \tag{11}$$

where Π_t^s denotes the temporary cash-flow obtained at time t and Λ_t represents the stochastic discount factor mirroring the preferences of the household, which is the owner of the firm.

Technology. Capital, intermediate goods, electricity and labor are involved in a three-stage production process of the basic good Y_t^s (see bottom of Figure 2). In the first stage, effective capital, $u_t^s K_t^s$, and electricity, ENG_t^s, are used to produce a composite good KE_t^s according to the following CES production technology:

$$KE_t^s = \left[(1-\theta_{ENG,t}^s)^{\frac{1}{\epsilon_E^s}}(u_t^s K_t^s)^{\frac{\epsilon_E^s-1}{\epsilon_E^s}} + (\theta_{ENG,t}^s)^{\frac{1}{\epsilon_E^s}}(ENG_t^s)^{\frac{\epsilon_E^s-1}{\epsilon_E^s}}\right]^{\frac{\epsilon_E^s}{\epsilon_E^s-1}} \tag{12}$$

Figure 2: Production process



where u_t^s denotes capital utilization rate, $\theta_{ENG,t}^s$ is intermediate consumption of electricity in sector s and ϵ_E^s is the elasticity of substitution between capital and electricity.

In the second stage the composite good KE_t^s is combined with labor N_t^s according to CES technology to produce another composite good KLE_t^s:

$$KLE_t^s = \left[(\theta_{KE,t}^s)^{\frac{1}{\epsilon_{KE}^s}} (KE_t^s)^{\frac{\epsilon_{KE}^s - 1}{\epsilon_{KE}^s}} + (1 - \theta_{KE,t}^s)^{\frac{1}{\epsilon_{KE}^s}} (N_t^s)^{\frac{\epsilon_{KE}^s - 1}{\epsilon_{KE}^s}} \right]^{\frac{\epsilon_{KE}^s}{\epsilon_{KE}^s - 1}} \tag{13}$$

where $(1 - \theta_{KE}^s)$ sets the share of labor in the production technology, while ϵ_{KE}^s is the elasticity of substitution between the capital-energy composite and labor.

In the final stage the aggregate of labor, capital and electricity, KLE_t^s, and composite material good M_t^s are used as input factors in the production of the sectoral basic good, Y_t^s, according to the following production technology:

$$KLEM_t^s = \left[(1 - \theta_{M,t}^s)^{\frac{1}{\epsilon_M^s}} (KLE_t^s)^{\frac{\epsilon_M^s - 1}{\epsilon_M^s}} + (\theta_{M,t}^s)^{\frac{1}{\epsilon_M^s}} (M_t^s)^{\frac{\epsilon_M^s - 1}{\epsilon_M^s}} \right]^{\frac{\epsilon_M^s}{\epsilon_M^s - 1}} \tag{14}$$

$$Y_t^s = e^{\xi_t^Y} \times KLEM_t^s \times (K_t^{PUB})^{\epsilon_{KP}} \tag{15}$$

where $\theta_{M,t}^s$ represents the share of materials in the production process of the basic good and ϵ_M^s is the elasticity of substitution between materials and the labor-capital-electricity composite good. Note that production of sectoral goods benefits from public capital, K_t^{PUB}, i.e. capital accumulated in the public sector. Moreover ξ_t^Y is an economy-wide productivity shock that we use to calibrate the dynamic properties of our model. The elasticity of product to public infrastructure is ϵ_{KP}.

Intermediate goods and raw materials. Aggregate intermediate material M_t^s used in sector s is produced using CES technology from a composite of fuels $FUELS_t^s$ and a composite of all other intermediate inputs MO_t^s:

$$M_t^s = \left[(\theta_{FLS,t}^s)^{\frac{1}{\epsilon_{MF}}} (FUELS_t^s)^{\frac{\epsilon_{MF} - 1}{\epsilon_{MF}}} + (\theta_{MO,t}^s)^{\frac{1}{\epsilon_{MF}}} (MO_t^s)^{\frac{\epsilon_{MF} - 1}{\epsilon_{MF}}} \right]^{\frac{\epsilon_{MF}}{\epsilon_{MF} - 1}} \tag{16}$$

where $\theta_{FLS,t}^s$ and $\theta_{MO,t}^s$ set the share of fuels and other materials in the intermediate input, with $\theta_{FLS,t}^s + \theta_{MO,t}^s = 1$, while ϵ_{MF} denotes the elasticity of substitution of the goods in question. The composite MO_t^s is produced using Leontief technology from materials $M_{i,t}^s$ purchased form all the basic goods sectors:

$$M_{i,t}^s = \theta_{i,t}^s MO_t^s \tag{17}$$

where $\theta_{i,t}^s$ with $\sum_{i \in S} \theta_{i,t}^s = 1$ defines the shares of intermediate good i in overall material consumption in sector s. Please note that this specification allows for the introduction of energy material input into the composite MO. During calibration, parameters are set so that energy only enters the composite of the ENG and RMP sectors, in order to replicate the high volatility of these two energy inputs observed in the data.

We now discuss the division of the intermediate input of the raw materials sector into specific raw material inputs, such as coal, oil, gas, etc. Let RM denote the set of raw materials. This set includes fuels denoted by fls. Raw materials intermediate good $M_{RMP,t}^s$ is produced using Leontief technology from raw materials (apart from fuels) $M_{j,t}^s, j \in RM - fls$:

$$M_{j,t}^s = \theta_{j,t}^s M_{RMP,t}^s, \qquad j \in RM - fls \tag{18}$$

with $\sum_{j \in RM} \theta^s_{j,t} = 1$, where $\theta^s_{j,t}$ denotes share of j-type raw material in overall raw material use in sector s. The remaining raw material inputs, which are fuels, are used to produce a fuels intermediate input composite, $FUELS^s_t$, given by the CES aggregator:

$$FUELS^s_t = \left[\sum_{k \in FLS} (\theta^s_{k,t})^{\frac{1}{\epsilon^s_{FLS}}} (M^s_{k,t})^{\frac{\epsilon^s_{FLS}-1}{\epsilon^s_{FLS}}} \right]^{\frac{\epsilon^s_{FLS}}{\epsilon^s_{FLS}-1}} \tag{19}$$

where FLS is the set of fuels, $M^s_{k,t}$ denotes input of k-th type of fuel, $\theta^s_{k,t}$ is the share of k-th fuel type in fuels intermediate input composite, and ϵ^s_{FLS} denotes the elasticity of substitution between different fuels in sector s.

Intermediate goods import. Intermediate sectoral material input, $M^s_{i,t}$, $i \in T$, $T = S \cup (RM - \{fls\}) \cup FLS$, is a composite of goods produced in home and abroad according to the Armington aggregator:

$$M^s_{i,t} = \left[(\theta^s_{iH,t})^{\frac{1}{\epsilon^i_H}} (M^s_{i,H,t})^{\frac{\epsilon^i_H-1}{\epsilon^i_H}} + (1-\theta^s_{iH,t})^{\frac{1}{\epsilon^i_H}} (M^s_{i,F,t})^{\frac{\epsilon^i_H-1}{\epsilon^i_H}} \right]^{\frac{\epsilon^i_H}{\epsilon^i_H-1}} \tag{20}$$

where $M^s_{i,H,t}$ and $M^s_{i,F,t}$ denote the amounts of intermediate goods of type $i \in T$ produced respectively home and abroad used to produce materials for use in sector s. These variables are set by parameter $\theta^s_{iH,t}$, while ϵ^i_H is the elasticity of substitution between home and foreign goods of type i. For simplicity we assume that elasticity of substitution between i-th intermediate goods produced in home and foreign country is the same for all sectors s that use i-th good. Electricity composite, ENG^s_t, is given by the same Armington aggregator:

$$ENG^s_t = M^s_{ENG,t} \tag{21}$$

Let us underline that parameters $\theta^s_{i,M,t}$ for $i \in S$ and $\theta^s_{E,t}$ as well as parameters $\theta^s_{iH,t}$ for $s, i \in S$ allow us to fully represent the inter-sectoral flows exhibited in the I/O matrix including the disaggregation of home produced and imported goods.

CO_2 emission. Emissions of greenhouse gasses are modeled on a sectoral level (in firms) as well as in households. In the first case CO^s_2 is produced as a byproduct when intermediate goods are utilized and sectoral aggregate is yielded. Formally:

$$CO2^s_t = \theta^s_{H,CO2,t} \times Y^s_t + \sum_{j \in T} \theta^s_{j,CO2,t} \times (M^s_{j,H,t} + M^s_{j,F,t}) \tag{22}$$

where $\theta^s_{j,CO2,t}$ determines the amount CO_2 generated in sector s by using j-type materials produced in home or foreign country. We assume that only fuels generates CO_2, i.e. $\theta^s_{j,CO2,t} \neq 0$ for $j \in FLS$. CO_2 can be also produced by chemical processes other than fuel combustion. We assume that such CO_2 production is proportional to amount of product produced in given sector and is controlled by the parameter $\theta^s_{H,CO2,t}$. Similarly, the amount of CO_2 emitted by households is equal to:

$$CO2^{CNS}_t = \sum_{j \in T} \theta^{CNS}_{j,CO2,t} \times M^{CNS}_{j,t} \tag{23}$$

Investment decisions. Let $K_t^{A,s}$ denote the book value of fixed assets of a firm. The value of accumulated assets, $K_t^{A,s}$, is given by the following equation:

$$K_t^{A,s} = (1-\delta_K^s)K_{t-1}^{A,s} + P_t^I I_t^s \tag{24}$$

where δ_K^s is the rate of capital depreciation, which may differ across sectors due to the specific characteristics of the fixed assets. Current book value of capital is equal to previous fiscal book value less depreciation increased by value of new investments, $P_t^I I_t^s$, where P_t^I is the price of investment goods.

Book capital differs from production capital due to investment frictions. Physical capital accumulation is governed by a stochastic time-to-build mechanism. We assume that firms make initial investment plans IN which enter into pending investment projects IP^2 or ready investment projects IP^1. Moreover, pending investment projects stochastically transform at rate $(1-\gamma^s)$ into ready investment projects IP^1 that enter the capital stock. This can be written as follows:

$$\begin{aligned} IP_t^{s,2} &= (1-\gamma^s)(IP_{t-1}^{s,2} + IN_t^s) \\ IP_t^{s,1} &= \gamma^s(IP_{t-1}^{s,1} + IN_t^s) \end{aligned} \tag{25}$$

Total investment in physical capital $I_t^{s,K}$ is equal to:

$$I_t^{s,K} = IP_t^{s,2} + IP_t^{s,1} \tag{26}$$

Capital stock in sector s is then augmented by the pool of ready investment projects. Investment influences the accumulation of production capital in age 0 in the following way:

$$K_t^s = (1-\delta_K)K_{t-1}^s + IP_t^{s,1} \tag{27}$$

We assume that firms incur a cost MC_t^s resulting from capital utilisation measured in units if investment good that which is equal to:

$$MC_t^s = \alpha_{us}\left((u_t^s)^{\beta_u} - 1\right) \tag{28}$$

with α_{us} determined by condition, that steady state capital utilization is equal to 1. Finally, the cost resulting from investment frictions CAC_t^s is given by the following equation:

$$CAC_t^s = \frac{\eta\left(\frac{I_t^{s,K}}{K_t^s} - \frac{I^{*K}}{K^*}\right)^2}{2} \tag{29}$$

where the asterisk is used to denote the steady state values of variables.

Endogenous technological progress. Memo III model includes a simplified research and development process. We assume that technological change affects in an endogenous way one particular feature of capital, namely energy intensity. This is a simplification but allows calibration of the energy intensity evolution, which is central to the practical use of the model - modeling of economic effects of mitigation actions. Kowal (2014a) discusses further the way of modeling of endogenous technological progress in the Memo III model.

Capital installed in a given period is characterized by a vector of features belonging to set F. We consider the following features: required energy intensity, required fuel intermediate inputs, intensity of CO_2 emission. The average level of capital feature $i \in F$, denoted by $X_{i,t}^s$ is given by:

$$K_t^s X_{i,t}^s = (1-\delta_K)K_{t-1}^s X_{i,t-1}^s + I_t^s Z_{i,t}^{sE} \tag{30}$$

where $Z_{i,t}^{sE}$ is the current technological frontier for i-th feature in sector s. Therefore $X_{i,t}^{s}$ is an aggregate of current and historic technological frontiers with the same aggregation scheme as in the case of effective investment in current period.

We assume, that firms can spend investment goods on research, which improves given feature of capital. The total investment in physical capital is

$$IC_t^s = I_t^{s,K} \times \left(1 + \sum_{i \in F} \chi_{it}(Z_{i,t}^{sE})\right) \tag{31}$$

where $\chi_{iT}(Z)$ is a function describing cost of creating new capital good with i-th feature equal to Z, given by

$$\phi_{it}(Z) = \alpha_i^Z \left(\frac{Z}{Z_{it}}\right)^{\beta_i^Z} \tag{32}$$

with Z_{it} denoting economy-wide technological frontier in i-th dimension. The total investment demand of sector s is equal to investment in physical capital and the costs related with investment frictions and capacity utilization:

$$I_t^s = IC_t^s + MC_t^s + CAC_t^s \tag{33}$$

Finally, we assume, that parameters governing shares of input factors and CO2 emission intensities are affine functions of current average level of capital features, i.e:

$$\theta_{ENG,t}^s = \theta_{ENG}^s(X_t^s) \qquad \theta_{M,t}^s = \theta_M^s(X_t^s)$$
$$\theta_{i,t}^s = \theta_i^s(X_t^s) \qquad \theta_{i,H,t}^s = \theta_{i,H}^s(X_t^s)$$
$$\theta_{j,CO2,t}^s = \theta_{j,CO2}^s(X_t^s)$$

where $i \in T$ and X_t^s is a vector of features with elements $X_{j,t}^s$, $j \in F$.

Price setting. Producers of sectoral goods have monopolistic power and are price setters. Producers take the demand function resulting from equation 42 as given. They face resource constraint in the form:

$$Y_t^s = \bar{Y}_t^s \left(\frac{P_t^s}{\bar{P}_t^s}\right)^{-\rho^s} \equiv Y_t^s \tag{34}$$

taking aggregate demand $\bar{Y}_t^s$ and aggregate price $\bar{P}_t^s$ as given.

Financial frictions. Firms face the following financial friction. We assume that a fraction of capital is financed externally in the sense that firms keep leverage ratio LR_t^s at an exogenously given level

$$LR_t^s = \frac{K_t^{As}}{K_t^{As} - B_t^s} \tag{35}$$

where B_t^s denotes the amount of debt held by firms in sector s. Net flow of goods resulting from debt management is

$$\Psi_t^{s,D} = \frac{B_t^s}{R_t} - B_{t-1}^s \tag{36}$$

where R_t denotes nominal price of debt. Debt is supplied by saving goods producers.

Budget constraint. Temporary cash-flow of a firm is equal to income from selling goods, $P_t^s Y_t^s$ less investment expenditure, $P_t^I I_t^s$, labor force expenditure, $N_t^s W_t^c$, cost of purchasing intermediate goods and electricity, CM_t^s, imposed taxes $T_t^{CO2,s}$, EXC_t^s and cost of debt. Formally:

$$\begin{aligned}\Pi_t^{s,B} &= P_t^s Y_t^s - N_t^s W_t^s - P_t^I I_t^s - CM_t^s \\ &- T_t^{CO2,s} - EXC_t^s + \Psi_t^{s,D}\end{aligned} \tag{37}$$

where N_t^s represents total labor demand reported by sector s and P_t^I is the price of the investment good.

Dividends paid to households equal $\Pi_t^{s,B}$ minus corporate income tax

$$\begin{aligned}\Pi_t^s &= \Pi_t^{s,B} - CIT_t^s \\ CIT_t^s &= \tau_t^{CIT} \Pi_t^{s,B}\end{aligned} \tag{38}$$

where τ_t^{CIT} is the corporate income tax rate.

Cost of intermediate inputs is

$$CM_t^s = \sum_{i \in T} P_t^{iH} M_{i,H,t}^s + \sum_{i \in T} P_t^{i,IM} M_{i,F,t}^s \tag{39}$$

where $P_t^{i,IM}$ is the price of i-th type of imported intermediate good expressed in home currency.

Taxes paid by the firm are defined as follows:

$$T_{CO2,t}^s = \tau_t^{CO2} \times CO2_t^s \tag{40}$$

$$EXC_t^s = \tau_t^{E,s}(Y_t^{sH} + Y_t^{sF}) \tag{41}$$

where τ_t^{CO2} and $\tau_t^{E,s}$ are efficient tax rates of CO2 and excise tax accordingly. Note that the base of excise tax is the volume of the good sold and not its value.

1.3.3 Final sectoral basic goods producers

The final sectoral basic good $\bar{Y}_t^s$ in sector s sold is a composite made of a continuum of intermediate goods $Y_t^s(i)$ produced by firms described in (1.3). The final firm produces the final good using the Dixit-Stiglitz aggregator.

The final good producers buy intermediate goods, package them into $\bar{Y}_t^s$ then sell them in a perfectly competitive market. They maximize profits:

$$\bar{\Pi}_t^s = \bar{P}_t^s \bar{Y}_t^s - \int_0^1 P_t^s(i) Y_t^s(i) di \tag{42}$$

$$s.t.\ \bar{Y}_t^s = \left(\int_0^1 (Y_t^s(i))^{\frac{\rho^s}{\rho^s - 1}} di \right)^{\frac{\rho^s - 1}{\rho^s}} \tag{43}$$

$\bar{P}_t^s$ and $P_t^s(i)$ denote the price of the final sectoral good and intermediate sectoral good respectively, while parameter ρ^s sets the markup. In symmetric equilibrium $Y_t^s(i) = \bar{Y}_t^s$, $P_t^s(i) = \bar{P}_t^s$ for any $i \in (0, 1)$.

1.3.4 Sectoral goods importers

We introduce two-sided search market between foreign sellers of imported goods and home buyers as in Mathä, Pierrard (2009). Let $T_t^{s,IM}$ be the number of contracts between buyers and sellers in $s \in T$ at period t, a contract meaning that both parties agree to exchange one unit of goods. These contracts terminate and the pairs separate at an exogenous rate $0 < \chi < 1$. The contract duration is, thus, on average given by $d = 1/\chi$. This results in a continuous depletion of the stock of contracts, and thus trade volume, and consequently a need to refill it. In order to do so, foreign sellers provide search effort $S_t^{s,IM}$ (marketing or advertising expenditures) to find new buyers; and home buyers provide a search effort $D_t^{s,IM}$ (by purchasing agents) to find new sellers. The number of new matches between sellers and buyers is increasing and concave in the search efforts, and assumed to be generated by a standard Cobb-Douglas matching function:

$$M_t^{s,IM} = \bar{m}^s (S_t^{s,IM})^\gamma (D_t^{s,IM})^{1-\gamma} \tag{44}$$

where $\bar{m}^s > 0$ and $0 < \gamma < 1$. The trade volume evolves according to

$$T_t^{s,IM} = (1-\chi) T_{t-1}^{s,IM} + M_t^{s,IM} \tag{45}$$

Aggregate imported goods $\bar{Y}_t^{s,IM}$ are produced by home buyers (based on linear technology) using final sectoral goods produced by sellers in foreign country. Firms maximize discounted profits

$$\max \widetilde{\Pi}_0^{s,IM}, \qquad \widetilde{\Pi}_t^{s,IM} = \bar{\Pi}_t^{s,IM} + E_t\{\Lambda_{t+1} \widetilde{\Pi}_{t+1}^{s,IM}\}. \tag{46}$$

where inter-temporal profits $\bar{\Pi}_t^{s,IM}$ satisfy

$$\bar{\Pi}_t^{s,IM} = (P_t^{s,IM} - q_t^f P_t^{s,f,SEARCH}) \times T_t^{s,IM} - SCB_t^{s,IM} \tag{47}$$

where $P_t^{s,f,SEARCH}$ is a price of s-type good produced in foreign country expressed in foreign currency negotiated between sellers and buyers. Home buyers sells its product on perfectly competitive market at price $P_t^{s,IM}$. Variable $SCB_t^{s,IM}$ denotes buyers' search cost on product market given by

$$SCB_t^{s,IM} = \frac{scb^{s,IM}}{2} (D_t^{s,IM})^2 \times P_t^{TRADE} \tag{48}$$

Search on product market requires goods produced in trade sector in home country at price P_t^{TRADE} and depends quadratically on search effort $D_t^{s,IM}$, parameter $scb^{s,IM}$ scales the search cost. Buyers take as given the rate at which search effort leads to a new match, therefore from buyers perspective dynamics of trade volume is given by

$$T_t^{s,IM} = (1-\chi) T_{t-1}^{s,IM} + q_t^{sD,IM} D_t^{s,IM} \tag{49}$$

where probability $q_t^{sD,IM}$ is

$$q_t^{sD,IM} = \frac{M_t^{s,IM}}{D_t^{s,IM}} \tag{50}$$

Buyers' optimization problem implies, that value from additional unit of trade volume at given price of foreign goods $P_t^{sf,SEARCH}$ is

$$VB_t^{s,IM} = P_t^{s,IM} - q_t^f P_t^{s,f,SEARCH} - SCB_t^{s,IM} + (1-\chi) E\{\Lambda_{t+1} VB_{t+1}^{s,IM}\} \tag{51}$$

and the first order condition for the search intensity is

$$\frac{scb^{s,IM} D_t^{s,IM}}{q_t^{sD,IM}} \times P_t^{TRADE} = P_t^{s,IM} - q_t^{s,IM} P_t^{s,f,SEARCH} \tag{52}$$

$$+ (1-\chi) E\{\Lambda_{t+1} \frac{scb^{s,IM} D_{t+1}^{s,IM}}{q_{t+1}^{sD,IM}} \times P_{t+1}^{TRADE}\} \tag{53}$$

Optimization problem of foreign sellers is similar. Foreign sellers produce imported goods based on linear technology using final sectoral goods produced in foreign country and sell its product to home buyers. Firms maximize discounted profits

$$\max \widetilde{\Pi_0}^{s,IM,f}, \qquad \widetilde{\Pi}_t^{s,IM,f} = \bar{\Pi}_t^{s,IM,f} + E_t\{\Lambda_{t+1}^f \widetilde{\Pi}_{t+1}^{s,IM,f}(i)\}. \tag{54}$$

where Λ_{t+1}^f is foreign discount factor, and inter-temporal profits $\bar{\Pi}_t^{s,IM,f}$ satisfy

$$\bar{\Pi}_t^{s,IM,f} = (P_t^{s,f,SEARCH} - P_t^{s,f,IM}) \times T_t^{s,IM} - SCS_t^{s,IM} \tag{55}$$

where $P_t^{s,f,IM}$ is an exogenous price of s-type good produced in foreign country expressed in foreign currency. Sellers buy s-type sectoral good produced in foreign country on perfectly competitive market. Variable $SCS_t^{s,IM}$ denotes sellers' search cost given by

$$SCS_t^{s,IM} = \frac{scs^{s,IM}}{2} (S_t^{s,IM})^2 \times P_t^{TRADE,f} \tag{56}$$

Similarly as in case of home buyers, search on product market requires goods produced in trade sector in foreign country at price $P_t^{TRADE,f}$ and depends quadratically on search effort S_t^s, finally parameter scs^s scales the search cost. Sellers also take as given the rate at which search effort leads to a new match, and dynamics of trade volume from sellers perspective is

$$T_t^{s,IM} = (1-\chi) T_{t-1}^{s,IM} + q_t^{sS,IM} S_t^{s,IM} \tag{57}$$

with probability $q_t^{sS,IM}$ given by

$$q_t^{sS,IM} = \frac{M_t^{s,IM}}{S_t^{s,IM}} \tag{58}$$

Value from additional unit of trade volume for foreign sellers satisfies

$$VS_t^{s,IM} = P_t^{s,f,SEARCH} - P_t^{s,f,IM} - SCS_t^{s,IM} + (1-\chi) E\{\Lambda_{t+1} VS_{t+1}^{s,IM}\} \tag{59}$$

with the first order condition for the search intensity

$$\frac{scs^{s,IM} S_t^{s,IM}}{q_t^{sS,IM}} \times P_t^{TRADE,f} = P_t^{s,f,SEARCH} - P_t^{s,f,IM} \tag{60}$$

$$+ (1-\chi) E\{\Lambda_{t+1} \frac{scs^{s,IM} S_{t+1}^{s,IM}}{q_{t+1}^{sS,IM}} \times P_{t+1}^{TRADE,f}\} \tag{61}$$

Finally, price $P_t^{s,f,SEARCH}$ is negotiated in Nash bargaining process in order to maximize

$$\max_{P_t^{s,f,SEARCH}} \left(VS_t^{s,IM}\right)^{\lambda} \left(VB_t^{s,IM}\right)^{1-\lambda} \tag{62}$$

where $0 < \lambda < 1$ is the seller bargaining power.

Price $P_t^{s,IM}$ is determined on perfectly competitive market by the clearing condition

$$\bar{Y}_t^{s,IM} = T_t^{s,IM} \tag{63}$$

1.3.5 Sectoral goods exporters

Exports are modelled using the same search mechanism as in the case of imports. All the equations governing the exporting firms are symmetrical to those described in section 1.3.4, with the buyer becoming the seller and vice versa. The main differences can be observed in the definitions of intertemporal profits of the two parties engaging in trade. The home exporting firms, referred to as the sellers, maximize discounted profits, with intertemporal profits $\bar{\Pi}_t^{s,EX}$ given by the following equation:

$$\bar{\Pi}_t^{s,EX} = (P_t^{s,SEARCH} q_t^f - \bar{P}_t^s) \times T_t^{s,EX} - SCS_t^{s,EX} \tag{64}$$

where $T_t^{s,EX}$ denotes export trade volume, $SCS_t^{s,EX}$ is the cost of the home export goods seller and $P_t^{s,SEARCH}$ is the contract price which, similarly to the import firms, is given through Nash bargaining procedure which takes place between the home seller and the foreign buyer.

Foreign buyers also optimize a stream of discounted profits, with intertemporal profit given by:

$$\bar{\Pi}_t^{s,EX,f} = (P_t^{s,f,EX} - P_t^{s,SEARCH}) \times T_t^{s,EX} - SCB_t^{s,EX} \tag{65}$$

where $P_t^{s,f,EX}$ is an exogenous price of s-type good bought by foreign country, expressed in foreign currency, and $SCB_t^{s,EX}$ is denotes foreign buyers search cost. The remaining equations change according to the symmetry resulting from above two definitions of intertemporal profit.

1.3.6 Production of final goods

There are three distinct types of final goods specified in the model: consumption, CNS, investment, INV and government, GOV. Consumption goods are purchased by households, investment goods are used in the process of accumulation of private and public capital, while government goods are purchased by the government in order to provide public consumption. Let us denote the set of final goods by $\mathcal{F} = \{CNS, INV, GOV\}$.

Intermediate final good Y_t^f is produced by a firm in order to maximize discounted profits

$$\max \widetilde{\Pi_0}^f, \qquad \widetilde{\Pi}_t^f = \Pi_t^f + E_t\{\Lambda_{t+1}\widetilde{\Pi}_{t+1}^f\}. \tag{66}$$

where inter-temporal profits Π_t^f satisfy

$$\Pi_t^f = P_t^f Y_t^f - COST_t^f \tag{67}$$

where $COST_t^f$ denotes all costs of production. The firm operates a production function F_{PROD}^f using material inputs $M_{j,t}^f, j \in S + RM$:

$$Y_t^f = F_{PROD}^f \left(\{M_{j,t}^f, j \in S + RM\}\right) \tag{68}$$

where the function F_{PROD}^f has the same functional form as in paragraph 1.3.2, and $M_{j,t}^f$ are material composites described in the next paragraph.

Material input in final sectors, $M_{i,t}^f$, $i \in T$, is a composite of goods produced in home and foreign country according to the Armington aggregator:

$$M_{i,t}^f = \left[(\theta_{iH,t}^f)^{\frac{1}{\epsilon_H^i}} (M_{i,H,t}^f)^{\frac{\epsilon_H^i - 1}{\epsilon_H^i}} + (1 - \theta_{iH,t}^f)^{\frac{1}{\epsilon_H^i}} (M_{i,F,t}^f)^{\frac{\epsilon_H^i - 1}{\epsilon_H^i}} \right]^{\frac{\epsilon_H^i}{\epsilon_H^i - 1}} \tag{69}$$

where $M^f_{i,H,t}$ and $M^f_{i,F,t}$ denote the amounts of intermediate goods of type $i \in T$ produced respectively in home and foreign country used to produce goods in final sector f. Similarly $\theta^f_{iH,t}$ sets the share of home goods produced of type $i \in T$ used in sector f and ϵ^i_H is the elasticity of substitution between home and foreign goods of type i.

Total cost of producing intermediate final good is:

$$COST^f_t = \sum_{i \in T} P^{iH}_t M^f_{i,H,t} + \sum_{i \in T} P^{i,IM}_t M^t_{i,F,t} \tag{70}$$

Finally CO_2 emitted by final sectors is:

$$CO2^f_t = \sum_{j \in T} \theta^f_{j,CO2,t} \times (M^f_{i,H,t} + M^f_{i,F,t}) \tag{71}$$

where $\theta^f_{j,CO2,t}$ determines the amount CO_2 generated in sector f by using j-type materials produced in home or foreign country.

1.3.7 Electricity generation sector

We assume that electricity is produced from different kind of resources including hydroelectric, thermoelectric, fossil fuels. Electricity produced from different resources is assumed to be nearly perfect substitutes.

Each electricity type is assumed to be produced by individual sector with internal structure as in 1.3.1 and then aggregated assuming imperfect perfect substitution. Therefore total electricity production satisfies

$$\bar{Y}^{ENG}_t = \left(\sum_{j \in EGS} (\bar{Y}^{ENG,j}_t)^{\rho_{ENG}} \right)^{\frac{1}{\rho_{ENG}}} \tag{72}$$

where EGS is a set of electricity types, and ρ_{ENG} determines the elasticity of substitution between electricity types.

1.3.8 Raw materials sectors

Raw material $j \in (RM - \{fls\}) \cup FLS$ is produced by monopolistically competitive firms according to technology:

$$Y^j_t = S^j_t \tag{73}$$

where S^j_t is raw material input. Raw material j is produced in order to maximize discounted profits:

$$\max \widetilde{\Pi_0}^j, \qquad \widetilde{\Pi}^j_t = \Pi^j_t + E_t\{\Lambda_{t+1}\widetilde{\Pi}^j_{t+1}\}. \tag{74}$$

Inter-temporal profits are:

$$\Pi^j_t = P^j_t Y^j_t - F_t(S^j_t) P^{RMP,H}_t - EXC^j_t \tag{75}$$

where $F_t(S^j_t)$ is the cost of production expressed in terms of product of raw material sector with price P^{RMP}_t, and EXC^j_t denotes excise tax paid by j-th raw material sector. The cost of producing i-th raw material is:

$$F_t(S^j_t) = (S^j_t)^{\theta^j_t} (\bar{S}^j_t)^{\delta^j_t} \tag{76}$$

where $\bar{S}_t^j$ is exogenously given geological layer, and $\theta_t^j \geq 1$. Excise tax is levied on the volume of goods sold:

$$EXC_t^j = \tau_t^{E,j} Y_t^j \tag{77}$$

1.4 Labor market

1.4.1 Dynamics of employed and non-employed

The labor market is modelled using the search and matching framework based on Mortensen (1989) and Pissarides (2000) results. Such a framework involves and employers who post vacancies and job seekers who send job offers. The matching process of the two parties is not perfect hence the number of filled vacancies, J_t, is lower than demand of employers and supply of employees. In the model we make a clear distinction between unemployed U_t and inactive IN_t, which together make the pool of the nonemployed NE_t. We assume that the inactive are marginally attached to the labor market and send job offers with a constant, 'base' low intensity, while the unemployed search with an endogenously determined high intensity. The division between the unemployed and inactive is calibrated by exogenous, imposed flows between the two groups.

The behavior of the labor market is defined as follows. At the beginning of period t a fraction δ of job matches are exogenously severed:

$$N_t^0 = (1 - \delta_N) \times N_{t-1} \tag{78}$$

We define the number of persons who send job offers with high and low intensity as follows:

$$U_t^0 = U_{t-1} + \delta_N \times N_{t-1} \tag{79}$$

$$IN_t^0 = IN_{t-1} \tag{80}$$

Please note that such a setup allows for persons who have just lost their jobs to search for employment in the same period. The total number of job offers posted by all job seekers is given by the equation:

$$O_t = \frac{1 - e^{-(e_t + \bar{e})\Psi_t}}{\Psi_t} \times U_t^0 + \frac{1 - e^{-\bar{e}\Psi_t}}{\Psi_t} \times IN_t^0 \tag{81}$$

where $\bar{e}$ is the constant, 'base' search intensity, e_t is the endogenous search intensity of the unemployed, and Ψ_t is the intensity of accepting job offers. At the same time firms post V_t vacancies, and the resulting number of new job matches is given by the equation:

$$J_t = \vartheta_t^m V_t^{1-\lambda_J} O_t^{\lambda_J} \tag{82}$$

We can now define the intensity of accepting job offers as:

$$\Psi_t = \frac{J_t}{O_t} \tag{83}$$

Similarly, the probability of filling a vacancy, Φ_t is:

$$\Phi_t = \frac{J_t}{V_t} \tag{84}$$

We can now define the equation for the flow of employment as follows:

$$N_t = N_t^0 + J_t = (1 - \delta_N) \times N_{t-1} + J_t \tag{85}$$

The pool of the unemployed and inactive are calibrated to the data by transition probabilities of becoming unemployed θ^{UU} or inactive θ^{UI} after losing a job or being previously unemployed, and the same transition probabilities (θ^{IU} and θ^{IU} respectively) after being inactive. We can now define the full model job transition matrix:

Table 1: Transition probabilities on the labor market

	N	U	IN
N	$(1-\delta_N) + \delta_N \Psi_t$	$\delta_N(1-\Psi_t)\theta^{UU}$	$\delta_N(1-\Psi_t)\theta^{UI}$
U	Ψ_t^U	$(1-\Psi_t^U)\theta^{UU}$	$(1-\Psi_t^U)\theta^{UI}$
IN	Ψ_t^I	$(1-\Psi_t^I)\theta^{IU}$	$(1-\Psi_t^I)\theta^{II}$

where Ψ_t^U and Ψ_t^I denote the probabilities of finding a job for an unemployed and for a (marginally attached) inactive person.

1.4.2 Matching firm

Households offer aggregated labor supply N_t to a perfectly competitive firm serving as an intermediary in the labor market – the matching firm. The firm maximizes expected discounted profit of the form:

$$\max E_0 \widetilde{\Pi_0}^L, \qquad \widetilde{\Pi}_t^L = \Pi_t^L + E_t\{\Lambda_{t+1}\widetilde{\Pi}_{t+1}^L\}. \tag{86}$$

where Π_t^L is temporary profit at time t defined in the following way:

$$\Pi_t^L = \sum_{s \in \mathcal{S}} W_t^s N_t^s - W_t N_t. \tag{87}$$

where N_t is the households' labor supply, W_t the offered wage, while N_t^s and W_t^s are the realized demand for labor and wage paid in sector s accordingly. Moreover:

$$N_t = \omega_N \times \left(\sum_{s \in \mathcal{S}} \omega_N^s (N_t^s)^{\frac{\epsilon_L - 1}{\epsilon_L}} \right)^{\frac{\epsilon_L}{\epsilon_L - 1}} + \upsilon_V \times V_t \tag{88}$$

$$N_t = (1 - \delta_N) \times N_{t-1} + \Phi_t V_t. \tag{89}$$

where parameters ω^s mirror the preferences of workers and impose the structure of labor supply in each sector while ϵ_L is the elasticity of substitution of these preferences. Moreover, parameter υ_V sets the cost of vacancy measured by the cost of work of recruiting employees who do not create any value added directly. The recruitment cost is equal to $CV_t = W_t \upsilon_V V_t$. In other words only $N_t - \upsilon_V \times V_t$ of employees produce basic goods and employees involved in the recruitment process earn CV_t. Parameter Ψ_t determines the probability of filling an open vacancy, and is treated by the matching firm as exogenous. Note that similarly to the household problem, the labor market intermediator also does not take into account employment dynamics in its optimization problem. Parameter ω_N is set in such a way that equilibrium condition $N_t = \sum_{s \in \mathcal{S}} N_t^s$ is satisfied.

1.4.3 Negotiation of wage and work time

Let V_t^E, V_t^U, V_T^I V_t^F denote respectively: (i) value in terms of consumption good for employed person, (ii) value in terms of consumption good for unemployed person,(iii) value in terms of consumption good for inactive person, (iv) value for firm from matching. We have

$$V_t^E = (1-\tau_t^W)(1-\tau_t^S)W_t + E_t\Lambda_{t+1}\left(\delta_N V_{t+1}^U + (-\delta_N)V_{t+1}^N\right) \tag{90}$$
$$V_t^F = X_t - W_t + (1-\delta^N)E_t\Lambda_t\{V_{t+1}^F\} \tag{91}$$

and

$$\begin{aligned} V_t^U &= -\Xi_t(e_t) + \Psi_t^U V_t^N + (1-\Psi_t^U)\left(b_t^U + E_t(\Lambda_{t+1}\theta^{UU}V_{t+1}^U + \theta^{UI}V_{t+1}^I)\right) \\ V_t^I &= \Psi_t^I V_t^N + (1-\Psi_t^I)\left(b_t^U + E_t(\Lambda_{t+1}\theta^{IU}V_{t+1}^U + \theta^{II}V_{t+1}^I)\right) \end{aligned} \tag{92}$$

where X_t is marginal productivity of one unit of labor, taken by employed persons and firms as given, $\Xi_t(e_t)$ is the labor search cost and Λ_t is stochastic discount factor. Variable b_t^U denotes marginal utility increase due to engaging in home production given by

$$b_t^U = \frac{\partial \tilde{C}_t}{\partial NE_t} \tag{93}$$

Finally, the value from posting a vacancy in terms of households' utility is

$$V_t^J = -\upsilon_V W_t + \Phi_t V_t^F \tag{94}$$

The optimality of vacancy posting requires that:

$$V_t^J = 0 \tag{95}$$

In each period t employees negotiate their wages with employers in the Nash bargaining procedure. Formally:

$$W_t = \arg\max_{W_t}(V_t^E - V_t^U)^{\upsilon}(V_t^F)^{1-\upsilon} \tag{96}$$

subject to $V_t^E - V_t^U \geq 0$, $V_t^F \geq 0$. The solution takes the form

$$\left(\upsilon + (1-\upsilon)(1-\tau_t^W)(1-\tau_t^S)\right) \times W_t = \upsilon X_t + \frac{1-\upsilon}{\lambda_t} \times \left(V_t^U - \beta E_t\{V_{t+1}^U\}\right) \tag{97}$$

1.4.4 Search intensity

Unemployed persons will increase their search intensity e_t until $\frac{\partial V_t^U(e_t)}{\partial e_t} = 0$. This results in the condition:

$$\Xi_t'(e_t) = \frac{\upsilon}{1-\upsilon} \times (1-\delta^N) \times \beta\varsigma\Psi_t \times V_t^F \tag{98}$$

1.5 Saving goods producer

Saving goods, B_t are produced using government bonds issued in home country as well as foreign countries according to technology:

$$B_t = \left(\phi_B(B_t^{h,h})^{\rho_B} + (1-\phi_B)(B_t^{h,f})^{\rho_B}\right)^{\frac{1}{\rho_B}}$$

where $B_t^{h,i}$ is the home demand for bonds emitted in country $i \in \{H, F\}$ and ρ_B determines the elasticity of substitution between bonds emitted in different countries and ϕ_B determines the share of bonds issued in home country. This introduces imperfect competition between bonds emitted in different countries.

Firms producing saving goods maximize discounted stream of temporal profits:

$$\tilde{\Pi}_t^B = \Pi_t^B + E_t \Lambda_{t+1} \tilde{\Pi}_t^B$$

where Π_t^B is the temporal profit and Λ_t is the discount factor determined by households' utility aggregator. Temporal profits, Π_t^B are given by:

$$\Pi_t^B = \frac{B_t}{R_t} - B_{t-1} - \left(\frac{B_t^{h,h}}{R_t^h} - B_{t-1}^{h,h} + \frac{B_t^{h,f}}{R_t^f \times RP_t} - \frac{q_t^f}{q_{t-1}^f} B_{t-1}^{h,f} \right)$$

where R_t is the price of the saving good, $R_t^{h,c}$ is the price of bonds issued in country c, $B_t^{h,c}$ is the home demand for bonds issued in country c and q_t^f is the real exchange rate, i.e. price of one unit of foreign numeraire good in terms of home numeraire good, and RP_t denotes risk premium.

The risk premium for holding foreign debt is defined as follows:

$$\ln RP_t = -\phi \frac{B_t^f - B^f}{GDP_t} \tag{99}$$

where B^f is the steady state level of foreign debt of domestic households and GDP_t is the gross domestic product of the whole economy.

1.6 Government

The government accrues a tax revenue from consumption, VAT_t, labor, PIT_t, corporate incomes, CIT_t, excise EXC_t and other taxes, τ_t^{CO2}, where:

$$VAT_t = VAT_t^H \tag{100}$$

$$PIT_t = \tau_t^W (1 - \tau_t^S) \times W_t \times N_t \tag{101}$$

$$CIT_t = \sum_{s \in T} CIT_t^s \tag{102}$$

$$\tau_t^{CO2} = \sum_{s \in S} \tau_t^{CO2,s} \tag{103}$$

$$EXC_t = \sum_{s \in T} EXC_t^s \tag{104}$$

The income is spent on purchase of public goods, $P_t^{GOV} G_t$, public investment, $P_t^{INV} GI_t$, transfers T_t to households and finally settling the debt B_{t-1}. It means that budget constraint of the government takes the following form $G_t^I = G_t^E$ where

$$G_t^E = P_t^{GOV} G_t + P_t^{INV} GI_t + T_t + \frac{1}{\pi_t} B_{t-1} \tag{105}$$

$$G_t^I = VAT_t + PIT_t + CIT_t + EXC_t + \tau_t^{CO2} + \frac{B_t}{R_t} \tag{106}$$

where T_t is given exogenously. Moreover, equations

$$P_t^{GOV} G_t = \omega^{GE} \times GDP_t \times e^{\xi_t^G} \tag{107}$$

$$P_t^{INV} GI_t = \omega^{GI} \times GDP_t \times e^{\xi_t^{GI}} \tag{108}$$

relate government consumption and public investment to the level of GDP. Variables ξ_t^G, ξ_t^{GI} are exogenous stochastic processes describing the discretionary part of governmental expenditure policy that we use to calibrate the dynamic behavior of the model. Additionally, public capital stock evolves according to

$$K_t^{PUB} = (1 - \delta_K^{PUB}) K_t^{PUB} + GI_t \tag{109}$$

Public expenditure are set as an exogenous share of GDP:

$$\frac{P_t^{GOV} G_t}{GDP_t} = \bar{G} \times \exp(\zeta_{G,t}) \tag{110}$$

The pension system is modeled in simplified way. We assume that the household pays a contribution from their labor income:

$$SSC_t = \tau_S W_t N_t \tag{111}$$

The pension system is in equilibrium in every period, i.e. pension payouts from the pension system (PEN_t) is equal to pension system's income:

$$PEN_t = SSC_t \tag{112}$$

1.7 Small open economy

Sectoral exports and imports, EX_t^s, IM_t^s are given by:

$$EX_t^s = \bar{P}_t^{s,F} \bar{Y}_t^{s,F} \qquad IM_t^s = \bar{P}_t^{s,IM} \bar{Y}_t^{s,IM} \tag{113}$$

where $s \in T$. Total export and import, EX_t, IM_t are given by:

$$EX_t = \sum_{s \in T} EX_t^s \qquad IM_t = \sum_{s \in T} IM_t^s \tag{114}$$

Current account and capital account balances are defined as:

$$CA_t = EX_t - IM_t, \qquad KA_t = \frac{B_{t-1}^{h,f}}{\pi_t^f} \frac{q_t^f}{q_{t-1}^f} - \frac{B_t^{h,f}}{R_t^f \times RP_t} \tag{115}$$

Equilibrium on the currency market requires that:

$$CA_t + KA_t = 0 \tag{116}$$

1.8 Market equilibrium

As it was already said, all prices in the model are relative to the price of the consumption good (being a *numeraire*), hence the assumption $P_t^{C,c} = P_t = 1$ does not affect the generality of the results of the model.

The discount factor Λ_t is given by

$$\Lambda_t = \beta \frac{\lambda_t}{\lambda_{t-1}} \tag{117}$$

where λ_t is the lagrange multiplier associated with the budget constraint of the representative household.

We also need to specify market clearing conditions which impose that supply and demand are equal in the goods, labor and international trade markets. For the basic goods markets we have the following equilibrium condition for each sector $s \in S$:

$$\bar{Y}_t^{sH} = \sum_{f \in \mathcal{F}} M_t^f + \sum_{i \in T} M_{i,H,t}^s \tag{118}$$

$$\bar{Y}_t^{sF} = EXV_t^s \tag{119}$$

The first equation specifies that the volume of goods produced for the home sector is equal to the sum of demands of the final sector firms and the intermediate demands (material inputs). The second equation specifies that the volume of goods produced for export is equal to the volume of exports. For the final goods we have the following equilibrium conditions:

$$Y_t^{INV} = \sum_{s \in S} I_t^s + GI_t \tag{120}$$

$$Y_t^{GOV} = G_t \tag{121}$$

Equilibrium in the consumption good sector is ensured due to the fact that the price of the consumption good is a reference to all prices in the model. The pension funds place their funds in risk free assets, yielding the following equilibrium condition on the asset market:

$$A_t + FR_t + FE_t = 0; \tag{122}$$

The equilibrium condition for the bonds market is as follows:

$$B_t = \sum_{i \in S} B_t^s + B_t^C \tag{123}$$

The last equilibrium condition is the clearing of the international market:

$$CA_t + KA_t = 0. \tag{124}$$

The remaining market clearing conditions, such as the clearing for labor, home and foreign bonds, money or deposits are implicitly stated in the model description.

1.9 Solution procedure

In this the preceding subsections we presented only the optimization problems of agents in the model, which describes behavior of the economy. Agents' decisions are determined by these optimization problems according to the optimal control theory. In this way we obtain a set of equations in the general form:

$$0 = E_t f(y_{t-1}, y_t, y_{t+1}, \epsilon_t) \tag{125}$$

where y_t denotes the vector of endogenous variables, ϵ_t is the vector of shock variables, and E_t is the expectations operator under information set known in period t.

The problem (125) can be solved using perturbation method described by Judd (1996). In this way we obtain agents' optimal controls given by rules describing decisions in period t as a function of state variables (variables taken as given). These optimal controls must be stable, i.e. the economy must converge to the steady state after any disturbances. Such solution guarantees that all transversality conditions (not described in the model specification) are satisfied.

According to the perturbation approach for solving the problem (125) we use the following procedure:

1. Find the deterministic steady state, y^*, satisfying $0 = E_t f(y^*, y^*, y^*, 0)$.

2. General solution takes the form

$$u_t = P(u_{t-1}, \epsilon_t)$$
$$y_t = y^* + R(u_{t-1}, \epsilon_t)$$

3. We concentrate only for the first order approximation of the solution. Such approximation can be fond by linearizing the function f around the steady state. The linearized model can be represented as $0 = A_1\tilde{y}_t + A_2\tilde{y}_{t+1} + A_3E_t\tilde{y}_{t+1} + V\epsilon_t$, where $\tilde{y}$ is deviation of the variable y_t from the steady state y^*.

4. Find solution in the form

$$\begin{aligned} u_t &= Pu_{t-1} + Q\epsilon_t \\ \tilde{y}_t &= Ru_t + S\epsilon_t \end{aligned} \tag{126}$$

   where u_t is a vector of state variables. Unknown matrices P, R, Q, S satisfy equations

$$\begin{aligned} 0 &= A_1R + (A_2 + A_3)RP \\ 0 &= A_1S + V \\ 0 &= A_2S + A_2RQ \end{aligned} \tag{127}$$

All necessary transformations required to obtain model equation in the form (125) from agents' optimization problems and all required numerical calculations are performed by the Forma Toolbox developed in the Institute for Structural Research by Paweł Kowal.

2 Model calibration

2.1 Introduction

Following DSGE methodology, the parameters can be divided into three main classes: (1) parameters determining the steady state levels of certain variables, (2) structural parameters describing properties of production technologies (elasticities of substitution) and other structural parameters (such as degree of habit formation) and (3) parameters determining the exact form of exogenous stochastic shocks. In the following subsections we discuss the first two groups of parameters and the calibration of CO_2 emissions. In this section we present a format of input data required to calibrate the model.

2.2 Model steady state properties

The first class of parameters is responsible for setting the steady state values of main macroeconomic variables of the model. The list of variables that needs to be set, along with parameters linked with them, includes:

- value of import relative to GDP,
- structure of domestic intermediate demand and import intermediate demand: $\theta^s_{i,t}$ and θ^s_{iH},

- structure of final goods wrt to domestic and import materials: $\theta^f_{i,H}$ and $\theta^f_{i,F}$,
- relative size of consumption, investment and public good in GDP: ω^X,
- share of labor in final production, long run employment rate: ω^s_N and δ^N,
- structure of tax income: τ^X,
- structure of CO2 emissions: $\theta^f_{j,CO2}$.

As most of the model parameters determine its steady state, their values are implicitly imposed by the values of directly observable variables. Most links between observable variables and parameters can be directly obtained from standard databases (national account, labor market, I/O matrices, and so on). Determination of values of the parameters associated with directly observable variables is performed by replacing the initial theoretical model by its calibration-adjusted version. All parameters belonging to the first class mentioned above (i.e. parameters setting the level of variables in the steady state) become special variables that we call *calibrators*, i.e. variables which determine the level of the steady state value of specified observed variable and only when the perturbation part of the solution is calculated they are treated as constants. Each calibrator is associated with a variable which is being calibrated. Determination of steady state means to find such a value for a given calibrator (in this case treated as variable) that the value of an observed variable associated with this calibrator becomes equal to the value suggested by data (in the steady state). For example, job destruction rate δ_N is set in such a way that the number of employed agents N_t (which is equal to rate of employment due to normalization of the workforce) is equal to the number found in the data. Values of all calibrators are calculated by numerical solver. The relation of main non-sector aggregate variables with model calibrators are given in table 1.

Table 2: Parametrization of steady state values of main macroeconomics variables - format of input table

variable	interpretation	unit	calibrator
π	inflation	%	ϕ^R
N^s	employment	%	δ_N
SSC	social sec. contr.	% quaterly GDP	τ^S
VAT	value added tax	% quaterly GDP	τ^V
CIT	capital tax	% quaterly GDP	τ^C
PIT	wage tax	% quaterly GDP	τ^W
DIV	property gov. income	% quaterly GDP	τ^D
EXC	excise tax	% quaterly GDP	τ^E

The structure of intermediate demand, calibrated by θ^s_{iH} (domestic part) and θ^s_{iF} (import part) is provided in tables 3 and 4. Please note that this table is not symmetric. The asymmetry results from the fact that on the final use side the Raw Materials Production sector has been disaggregated into specific raw material goods - oil, gas, coal, mining.

The structure of final demand, which calibrated by the parameters θ^f_{iH} (domestic part) and θ^f_{iF} (import part) is provided in tables 5 and 6. Please note the disaggregation of the Raw Materials Production sector into the specific materials.

Table 3: Structure of domestic intermediate use - format of input table

	AGR	CST	ENG	FIN	IND	PUB	RMP	SRV	TRD	TRN
AGR	39.1	0.4	0.0	0.0	3.5	0.8	0.6	4.3	0.5	0.2
CST	0.9	2.7	0.2	0.7	0.9	3.6	0.4	9.0	5.4	1.4
ENG	1.1	0.5	11.2	0.2	1.8	1.9	5.4	1.1	1.1	0.6
FIN	2.8	2.3	1.0	3.9	2.4	3.6	2.7	3.6	3.8	3.3
IND	5.6	9.7	0.1	0.6	12.5	3.6	6.9	3.0	4.4	0.8
PUB	0.6	0.3	0.1	0.4	0.6	5.8	0.6	2.9	1.3	1.0
mining	0.6	3.9	0.0	0.0	2.9	0.0	15.6	0.0	0.0	0.0
coal	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
gas	0.0	0.6	0.0	0.0	0.6	0.5	0.1	0.4	0.1	2.3
oil	3.9	5.2	0.4	0.0	0.9	0.9	1.7	1.4	1.2	6.0
SRV	6.9	6.2	1.6	3.4	5.4	6.8	10.6	6.5	10.8	7.5
TRD	6.3	3.9	0.3	0.5	6.7	2.5	4.4	3.8	3.9	6.1
TRN	4.1	1.7	1.0	1.3	5.1	2.7	5.5	2.6	15.5	14.0

Example with artificial data.

Table 7 contains the structure of the compensation of employees (COMP), the value of investment in each sector (INV), the sectoral structure of value added (VA), the total import of each sector (IMP) and the structure of employment. Please note, that contrary to the final use data, the Raw Materials Production sector is treated as a single sector.

2.3 CO2 emission

The amount of CO2 emitted in each sector is calibrated based on the consumption of fuels in given sector. Table 8 presents structure of fuels by sectors.

In the first step we assume that CO2 emission intensity of given fuel is the same in all sectors. This emission intensity is calibrated based on data presented in table 9 in order to match CO2 emitted by given fuel in the model with total CO2 emission generated by given fuel:

In the second stage of CO2 calibration we introduce a sector specific factor which modifies CO2 emission intensity of fuels. These factors are set in order to exactly match CO2 emitted by given sector in the model with data. Table 10 presents implied CO2 emission in given sector.

Table 11 presents final results.

2.4 Structural parameters

Table (12) presents baseline calibration of structural parameters of the model. In the baseline calibration we assume no price stickiness, therefore all parameters controlling nominal sector of the model are ignored.

Values of most parameters are standard. We set value of discount factor to 0.99, which is consistent with a steady-state real interest rate of 1 percent (per quarter).

Cost of posting vacancies to GDP is set to 0.3%, firms bargaining power in the Memo III model is 0.5.

Table 4: Structure of imported intermediate use - format of input table

	AGR	CST	ENG	FIN	IND	PUB	RMP	SRV	TRD	TRN
AGR	6.4	0.0	0.0	0.0	0.5	0.0	0.1	0.5	0.0	0.1
CST	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0
ENG	0.0	0.0	0.2	0.0	0.0	0.0	0.1	0.0	0.0	0.0
FIN	0.1	0.1	0.2	3.2	0.1	0.1	0.4	0.1	0.3	0.4
IND	3.7	3.2	0.0	0.1	13.7	3.2	4.9	2.3	6.2	0.5
PUB	0.0	0.0	0.0	0.0	0.0	0.4	0.0	0.0	0.0	0.0
mining	0.0	0.1	0.0	0.0	0.5	0.0	7.3	0.0	0.0	0.0
coal	0.0	0.0	0.7	0.0	0.1	0.0	0.1	0.0	0.0	0.0
gas	0.0	0.2	0.6	0.0	0.2	0.1	0.1	0.1	0.0	0.7
oil	1.7	1.8	1.5	0.0	4.0	0.5	15.1	0.3	0.6	5.6
SRV	0.0	0.0	0.0	0.0	0.0	0.2	0.4	4.3	0.0	0.5
TRD	0.7	0.0	0.0	0.0	1.4	0.1	0.5	0.2	1.8	0.0
TRN	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	11.7

Example with artificial data.

Table 5: Structure of domestic final demand - format of input table

	AGR	CST	ENG	FIN	IND	PUB	mining	coal	gas	oil	SRV	TRD	TRN
CNS	40.2	0.4	7.7	7.4	19.9	42.0	0.0	0.0	1.2	6.6	49.5	42.2	30.2
GOV	0.3	0.0	0.2	0.0	0.0	59.3	0.0	0.0	0.0	0.0	1.0	0.0	0.0
INV	1.5	62.0	0.7	0.0	4.2	0.0	2.5	0.0	0.2	0.8	0.1	7.4	0.3
EXP	38.2	0.0	0.1	1.2	28.9	0.9	75.3	0.0	1.0	3.1	3.4	11.4	24.0

Example with artificial data.

Cheron, Langot (2004) chosen similar values equal to 0.5% and 0.6 respectively. We set probability of filling vacancies to 0.9 according to Andolfatto (1996). Blanchard and Diamond (1989) estimated the elasticity of the matching rate with respect to number of vacancies posted estimated to 0.6 for the United States. We take higher value 0.8 which helps us to match volatility of unemployment rate. Employment-employment transition rate, unemployed persons search cost are set in order to match employment and unemployment moments. We assume high substitution between different types of labor. Imperfect substitution between differentiated labor is not structural in nature. In this situation we would like to not introduce important distortions in the model through this channel.

Value of Intertemporal elasticity of substitution, parameter governing habit formation, and elasticity of capital depreciation with respect to capital utilization are set in accordance with Smeth, Vouters (2003). They estimated habit formation parameter as 0.541 with standard error 0.077, elasticity of capital utilization as 1.169 with standard error 0.075, and risk aversion parameter as 1.607 with standard error 0.292. The value of intertemporal elasticity of substitution in the Memo III model implies relative risk aversion coefficient equal to 1.5. Parameter governing the risk premium in the model is set as in Selaive, Tuesta (2003).

There is a lot of uncertainty about elasticity of substitution between home and foreign goods. In their original work, Backus et al. (1994) set this parameter in DSGE model equal to 1.5 referring to a study by Whalley (1985). More recently, Hooper et al. (2000) report estimates for G7 countries in a range between 0.1 and 2. Heathcote and Perri (2002) show that with lower values the Backus et al. model can better reproduce

Table 6: Structure of imported final demand - format of input table

	AGR	CST	ENG	FIN	IND	PUB	mining	coal	gas	oil	SRV	TRD	TRN
CNS	5.0	0.0	0.0	0.0	25.2	8.4	0.0	0.0	2.5	4.6	0.0	0.0	0.1
GOV	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
INV	0.0	0.0	0.0	0.0	26.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Example with artificial data.

Table 7: Structure of supply - format of input table

	AGR	CST	ENG	FIN	IND	PUB	RMP	SRV	TRD	TRN
COMP	16.3	24.6	2.0	11.3	15.8	60.4	10.6	25.7	27.9	16.5
INV	10.3	10.2	3.2	5.4	9.3	18.0	12.2	16.9	9.8	10.4
VA	44.9	44.6	14.1	23.6	40.2	78.4	53.1	73.5	42.5	45.4
IM	12.7	0.1	0.3	4.8	100.6	8.8	38.1	5.3	4.8	12.0
N	0.100	0.146	0.007	0.018	0.116	0.208	0.033	0.103	0.197	0.073

Example with artificial data.

Table 8: Fuels consumption in basic prices - format of input table

	AGR	CST	ENG	FIN	IND	PUB	SRV	TRD	TRN	Households
Coal	0.0	0.0	3.7	0.0	1.3	0.0	0.0	0.0	0.0	0.0
Natural gas	0.0	0.3	3.5	0.0	4.5	0.5	0.3	0.1	4.3	3.8
Oil	4.8	2.5	12.6	0.1	41.9	1.2	1.0	1.2	23.3	11.5

Example with artificial data.

business cycle properties observed in the data. They also estimate a value of 0.9 for this elasticity.

We also assume quite a low substitution between home produced and imported (intermediate) goods used in production. This is now a standard assumption in current generation of DSGE models which include intermediate inputs channel. Low substitution between home produced and imported goods allows for explaining higher relative volatility of export and import wrt to GDP in a response to technology shock.

However, empirical studies suggest a greater than unity elasticity between goods produced in different countries. See for example McDaniel et al. (2002). These two facts can be reconciled assuming two stages of aggregation of foreign and home goods. In the first stage goods produced in different countries are aggregated to a composite of imported goods. Substitution at this stage is high. However firms operating in home country cannot easily substitute imported goods composite with goods produced in home country at the second stage of aggregation due to strong integration of technological processes between sectors of economy. This assumption also implies high elasticity of foreign demand on home produced goods (i.e. export) with respect to price.

Empirical study conducted by Okagawa and Ban (2008) shows that elasticity of substitution between capital and energy is very low, the null hypothesis of zero substitution was not rejected in 14 of 19 considered industries. Similarly, Kuper and Soest (2002) estimated zero capital-energy elasticity for Dutch economy. In the Memo III model we also assume very low capital-energy substitution at level 0.05.

Table 9: CO_2 emission by source - format of input table

	Coal	Oil	Gas	Total
CO_2 emission (million tones)	16.5	47.5	3.9	67.9

Example with artificial data.

Table 10: CO_2 emission by sectors - format of input table

	Energy	Industry	Transport	Households	Transport + Households	Other
Data	35.9	22.8	29.5	7.0	36.5	4.7

Example with artificial data.

Table 11: CO_2 emission by sectors - steady state values

	AGR	CST	ENG	FIN	IND	PUB	SRV	TRD	TRN	Households
CO_2 emission	0.17	0.84	24.38	0.00	15.46	1.18	0.67	0.34	13.39	11.41
Share	0.17	1.18	35.90	0.00	22.80	1.68	1.18	0.50	19.64	16.86

Source:

Result of model calibration for artificial data.

Elasticity of substitution between fuels and other materials is based on Koschel (2000) analysis, which shows, that Morishima elasticity of substitution between fuels and other material goods composite is very low, from 0.025 in industry sector to 0.05 in service sector.

Elasticity of substitution between capital and labor is set to 0.95. Lower value destroys long-term properties of the model, low capital-labor elasticity implies large employment increase on the growth path of the economy.

Estimated elasticity of substitution varies greatly. Stern (2009) presented a meta-analysis of empirical elasticities and shown that estimations of macroeconomic coal-oil elasticity of substitution varies form zero to 0.6 with mean between studied at 0.2. In case of oil-gas elasticity of substitution estimations varies from -0.2 to 1.0 with mean 0.2. Coal-gas substitution is much stronger, varies from 1.2 to 3.2 with mean 2.3. We assume low interfuels substitution at level 0.2.

Elasticity of substitution between material inputs and capital-energy-labor composite is set in order to match relative volatility of export and import with respect to GDP. In the baseline calibration we set value of this elasticity at level 0.2. Similar value (0.3) is used for example in Bucher's (2011) CGE model.

Table 12: Values of structural parameters

parameter destription	parameter	value
firms bargaining power	$1-\upsilon$	0.5
cost of posting vacancies to GDP	$V_t W_t / GDP_t$	0.003
employment-employment transition rate	$1-\delta^N$	0.97
probability of filling vacancies	Φ	0.9
substitution between labour types	ϵ_L	8.5
vacancy share in matching function	$1-\lambda_J$	0.8
intertemporal elasticity of substitution	$1/\sigma$	0.66667
discount factor	β	0.99
unemployed persons seach cost	ψ_u	0.02
substitution between market and home produced goods	ϵ_{CH}	0.8
public capital externality	ϵ_{KP}	0.05
habit formation	h	0.5
risk premium	ϕ	0.01
elasticity of capital utilization	β_u	1.1
substitution between home and foreign goods in foreign country	ρ^F	1.5
substitution between home and foreign goods in home country	ϵ_H	0.8
substitution between capital and energy	ϵ_E	0.05
substitution between capital-energy and labour	ϵ_{KE}	0.95
substitution between capital-energy-labour and materials	ϵ_M	0.2
substitution between fuels	ϵ_{FLS}	0.2
substitution between fuels composite and other materials	ϵ_{MF}	0.1

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