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Abstract

The timing of the response of CO2 emissions to a carbon tax depends crucially on the timing of response of energy demand to changes in energy prices. In this paper, we investigate the path of changing energy demand from the moment of a change in price until it reaches its new steady state. First, by applying the LeChatelier principle, we show that the response of energy demand in the short run must be smaller than in the long run if firms are only able to adjust their choices of technology in the long run. Then, using a putty-clay model with induced technological change, we show that the elasticity of demand approaches its long-run level exponentially at the rate that is determined by the capital depreciation rate and the growth rate of the economy. Thus, according to the model, it takes more than 8 years from the introduction of the carbon tax until half of the long-run effect of induced technological change on energy demand is realised in developed countries. We also examine the macroeconomic consequences of the long-run adjustment of energy demand. To this end, we incorporate the theoretical model into a large-scale multi-sector DSGE model. We find that the adjustment of energy demand reduces the negative impact of CO2 tax on GDP.

Keywords: induced technological change; rebound effect; general equilibrium model; mitigation costs

JEL Classification: O33, Q41, Q43, Q55

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Introduction

A carbon tax reduces emissions through two channels: by inducing a switch to carbon-free energy sources and by reducing energy consumption. The latter channel channel operates on the presumption that a carbon tax induces an increase in energy price that motivates consumers and firms to reduce their use of energy. In this paper we investigate what drives the depth of the resulting reduction in energy consumption.

The energy demand channel will play a large role primarily in the first decades of a low-carbon transition. In the later phase, when most of the energy is produced from low-carbon sources, the carbon tax will have a small effect on the price and use of energy. Conversely, today, when 81% of the world's total primary energy supply originates from fossil fuels, the impact of a carbon tax on energy price is substantial. We can expect that this increase in price will translate into a reduction in energy use. However, the size and timing of the response of demand to changes in price is unknown. Understanding this response is essential for an assessment of carbon tax effectiveness in terms of CO2 reduction potential, particularly in the early phase of a low-carbon transition.

The response of demand to a price increase is shaped by two effects. The first and immediate effect is the substitution of energy with other factors of production. For instance, firms could replace energy with labour or consumers could choose more energy-efficient public transport over individual modes of transport. These substitution possibilities are constrained by the existing economic structure that cannot be altered in the short run. The second effect is associated with the changes of economic structure in the long run. We label this effect 'price induced technological change'. Examples include the development or adoption of more energy-efficient methods of production by firms, or the modernisation and expansion of public transport networks, which incentivises commuters to use them instead of automobiles.

Intuition suggests that price-induced technological change should work in the same way as the substitution effect, that is, it should encourage firms to further economise on the input that became relatively expensive. Sue Wing (2006) and Gerlagh and Kuik (2014) provide a graphical argument showing that technological change induced by an increase in the price of the dirty input decreases demand for that input relative to other inputs. However, the effect of induced technological change on absolute demand for an input could differ from the effect on relative demand due to the rebound effect (see for instance Sorrell and Dimitropoulos 2008). While induced technological change decreases the amount of dirty input per unit of output (e.g. less petrol per km travelled), it also decreases the cost of output and thus encourages an increase in demand for output (more km travelled). The sign of the total effect of induced technological change on absolute demand for energy is therefore not obvious. A total negative effect is predicted in various settings, including models of energy-saving knowledge stock (Goulder and Schneider 1999, Popp 2004 and Bosetti et al. 2007) and directed technological change (Aghion et al. 2016, Hassler, Krusell and Olovson 2014, Andree and Smulders 2014, Witajewski-Baltvilks et al. 2017, Casey 2017). However, until now the literature has not reached a consensus as to whether this result holds in a general competitive setting.

Moreover, if the long-run response of energy is different to the short-run response, climate policy potential will only be fully realised in the long run. If energy demand falls in the long run due to higher prices, but the speed of the fall is slow, a carbon tax might need to be supplemented with policies incentivising fast adoption of zero-carbon energy sources. The speed with which the change in energy demand approaches its long-run level will therefore be crucial for evaluating past and future climate policy. This view is also supported by Gerlagh and Kuik (2014), who note that greater understanding of the dynamics of energy-saving technological change is needed to supplement existing literature.

This article contributes to the literature in three ways. First, we demonstrate that under a fairly general setting the long-run response of energy demand by a competitive firm must be larger than the short run response. Equivalently, the rebound effect associated with technological change cannot take its strong form (i.e. lead to an increase in energy demand) if technological change is induced by an increase in energy prices. Second, we show that if firms cannot adjust their choice of technology for vintages of capital installed in the past, then energy demand approaches its long-run level at the exponential rate given by the sum of the depreciation rate of capital and the growth rate. Third, we investigate the impact that long-run technological adjustments related to energy efficiency have on the dynamics of macroeconomic variables such as GDP, employment and wages. The quantitative results suggest that when firms adjust their technology, the drop in GDP is smaller after a carbon tax is introduced. However, this mechanism is also responsible for reducing the demand for the output of the mining sector and creates additional negative pressure on employment in this sector.

The first result is derived from a simple application of the LeChaterlier principle in the context of energy demand. The principle was originally formulated by Samuelson (1960). It states that the response of demand for any input to a change in prices must be larger in the unconstrained system than in the constrained system. The sole assumption behind the principle is that the firm is competitive and that it maximises profits.

The second result concerning the dynamics of the system is derived from an analytical dynamic model which combines the insight of the putty-clay vintage model and technology frontier framework. Following the putty-clay model (Johansen 1959 and Casey 2017), we assume that at the moment of installing a new vintage of capital, a firm has to decide on the amount of new capital needed and its energy efficiency. The firm cannot change its decision after the vintage is installed. In addition, we assume that at the moment of installing a vintage the firm can choose a technology. Following the technology frontier literature (Caselli and Coleman (2006), Jones (2005) and Growiec 2008, 2013),[1] the firm has to choose from technologies whose productivity parameters differ: it must choose between an energy-intensive and capital-saving technology, and an energy-saving and capital-intensive technology. As in Krusell (1998), the firm can choose the technology only for a new vintage and cannot change it afterwards. This is a departure from the typical assumption adopted in the literature (see e.g. Acemoglu et al. 2014 or Popp 2004) that every new innovation affects the characteristics of all (new and past) vintages of capital in the same way.

The third result on the macroeconomic consequences of long-run adjustment in demand for energy is derived by incorporating the framework of the analytical model described above in a large-scale, numerical multi-sector Dynamics Stochastic General Equilibrium model. We calibrate the model using input-output matrices for the Polish economy and conduct a case study analysis for Poland. The model accounts for general equilibrium effects of changes in prices, shifts of demand between 11 main sectors of the economy and carbon emissions.

The dynamic trajectory of technological change under climate policy (or resource scarcity) has been addressed in several studies on the directed technological change (DTC) (Acemoglu et al., 2012 Acemoglu et al. 2014, Aghion et al. 2016, Hassler, Krusell and Olovson 2014, Andree and Smulders 2014, Witajewski-Baltvilks et al. 2017). The technology frontier framework adopted in this paper differs from the DTC approach in several ways. First, it predicts that local carbon tax will lead to an adjustment of technologies in

[1] The technology frontier framework is built on the intuition found in the work on price-induced innovations Hicks (1932) and directed innovations (Kennedy (1964), Samuelson (1965) and Drandakis and Phelps (1966)). In our model, we will use the simplest CES framework proposed by Caselli and Coleman (2006).

any country, even if it is not on a technological frontier or it does not have a significant R&D sector. Second, it predicts that technology could adjust even if the long-run growth rate of the economy is zero. Finally, the framework does not require firms to be monopolists and hence it is easier to model in the Computable General Equilibrium (CGE) and Dynamic Stochastic General Equilibrium (DSGE)setting, which we carry out in the second part of the paper.

The remaining part of the paper as structured as follows. In section 2 we show how the Le Chatelier principle could be applied to predict the long-run response of energy demand, changes in energy efficiency and the rebound effect after an increase in the price of energy. In section 3 we explore the dynamics of energy consumption and differentiate between the immediate and the long-run effects of an increase in energy prices using a putty-clay model. In section 4 we demonstrate how the analitical framework developed in section 3 could be integrated in the numerical general equilibrium model, and in section 5 we present the quantiative predictions of the DSGE model on the macroeconomic effects of a carbon tax when firms are allowed to adjust energy efficiency in the long run.

The LeChatelier principle and demand for energy

In this section we demonstrate that in the long run, when the firm's optimisation problem is unconstrained, the response of energy demand to a change in the price of energy cannot be smaller than in the short run, when the firm's problem is constrained. If the constraint factor is a technology that determines the energy-intensity of production, the result implies that improvement in energy efficiency induced by an increase in price cannot lead to an increase in energy use. In other words, the rebound effect of a technological change induced by the price increase, cannot take its strong form.

In order to formalise this argument, consider a static model in which the firm operates production function $Y = F\left(E, z_1, ..., z_n, G_1, ..., G_m\right)$, where E denotes use of energy, z_1, z_2... denote the use of other inputs (such as capital, labour, materials and services) and $G_1..., G_m$ denote the technology parameters that determine the productivity of inputs. To simplify the exposition of the model, we will assume that the long run is equivalent to a situation where the choice of inputs is unconstrained and that the short run refers to a situation where the choice of technology is constrained. Adding constraints to the choice of other inputs would not change the main result. We denote the price of energy with p_E, the price of output with P and the price of other inputs with vector $\mathbf{w} = (w_1, ...w_n)$.

The vector characterising technology, $(G_1, ..., G_m)$ is determined by factors that are beyond the firm's control (previous knowledge stocks, spillovers) and the factors that the firm has control over (investment in R&D or by choosing certain technological features). Formally, we assume that $G_j = \Lambda_j(M_{j,} v_j)$, where $\mathbf{M} = (M_1, ..., M_m)$ denotes the variables exogenous to the firm and $\mathbf{v} = (v_1, ..., v_m)$ denote the choice variables. The dimension of the vectors are irrelevant for this analysis. If the choice of $\mathbf{v}$ is associated with costs (e.g. the costs of investment in R&D), that cost is added to the total cost of production. We use $\gamma\left(\mathbf{v}\right)$ to denote that cost. In addition, the firm's choice of technologies might be limited as in Samuelson's (1965) induced technological change model. We assume that the firm has to respect a constraint given by:

$$\Gamma\left(G_1, ..., G_m\right) = 0 \tag{1}$$

The presence of this constraint, or, indeed, any other constraints on the choice of inputs, does not change the main result of this section. We include it in order to be consistent with the model developed in section 3.

Finally, we assume that the firm is competitive, i.e. it takes the price of its output as well as the price of all inputs as given. The objective of the firm is to choose output and inputs, as well as vector $\mathbf{v}$, to maximise profit.

In the long run, the firm's unconstrained optimisation generates the supply curve $Y\left(P, p_E, \mathbf{w}\right)$ and demand curves $E\left(P, p_E, \mathbf{w}\right)$, $\mathbf{z}\left(P, p_E, \mathbf{w}\right)$ and $\mathbf{v}\left(P, p_E, \mathbf{w}\right)$, which describe the firm's optimal choices given prices. In the short run, when the firm cannot freely adjust its technology, the firm's constrained optimisation yields $Y^R\left(P, p_E, \mathbf{w}\right)$, $E^R\left(P, p_E, \mathbf{w}\right)$,$\mathbf{z}^R\left(P, p_E, \mathbf{w}\right)$ and $\mathbf{v}^R\left(P, p_E, \mathbf{w}\right)$.

Now we can proceed to the key results of this section, which we summarise in the following proposition:

Proposition 1

The demand curve for energy is steeper in the long run than in the short run,$\left|\frac{E(P,p_E,\mathbf{w})}{dp_E}\right| \geq \left|\frac{E^R(P,p_E,\mathbf{w})}{dp_E}\right|$, if in the short run the firm faces constraints on its choice of technology.

Proof follows the proof of the LeChatelier principle. See appendix A1 for the detailed derivations.

The proposition implies that the technological change induced by the change in energy price cannot lead to an increase in the use of energy. To see this, we can decompose the total effect of increase in energy price into:

$$\frac{dE}{dP_E} = \left.\frac{dE}{dP_E}\right|_{\mathbf{G}=\overline{\mathbf{G}}} + \sum \frac{dG_i}{dP_E}\frac{dE}{dG_i}$$

where $\frac{dE}{dP_E}$ is the response of energy demand to a change in price, $\left.\frac{dE}{dP_E}\right|_{\mathbf{G}=\overline{\mathbf{G}}}$ is the response of energy demand when efficiency is fixed at its level before price change and $\frac{d\varepsilon}{dP_E}\frac{dE}{d\varepsilon}$ is the effect of an increase in energy efficiency induced by an increase in price. Since, according to Proposition 1, the response of a constrained firm must be more negative than the response of an unconstrained firm ($\frac{dE}{dP_E} \leq \left.\frac{dE}{dP_E}\right|_{\mathbf{G}=\overline{\mathbf{G}}}$), it must be that $\sum \frac{dG_i}{dP_E}\frac{dE}{dG_i} \leq 0$.

The result in Proposition 1 implies also that the rebound effect associated with induced technological change cannot be strong, i.e. it cannot lead to an increase in energy demand. Recall that if price-induced technological change involves an increase in energy efficiency, we would observe both a drop in energy per output as well as an increase in production. When deriving the proposition, we allowed for both effects since we allowed the firm to freely adjust both its input and its output. If a firm decides to chose a technology that is more energy efficient, it is likely to increase its input. Such an increase would provide an upward pressure on the use of energy, i.e. leading to a rebound effect. However, according to the proposition, the rebound effect cannot dominate, since the total effect of induced technological change on energy demand must be negative. We discuss this point in greater detail in the subsequent section.

The rebound effect of efficiency improvement induced by energy price

Consider the framework proposed by Sorrell and Dimitropoulos (2008). Suppose that S is the amount of 'useful work' which, when combined with attributes of this work, translates into energy services demanded by firms or consumers. For instance, the useful work from private cars may be the distance travelled by consumers measured in vehicle kilometres. Useful work is derived from energy according to $S = E\varepsilon$, where ε is the efficiency of energy use. The demand for useful work depends on the energy cost of useful work given by $\frac{P_E}{\varepsilon}$, where P_E is the price of energy input. Sorrell and Dimitropoulos (2008) also allows energy efficiency to depend on the price of energy. Thus, energy demand is given by $E = \frac{S(P_E/\varepsilon(P_E))}{\varepsilon(P_E)}$

Taking logs and differentiating energy demand allows us to decompose the elasticity of energy demand with respect to energy prices ($\eta^E_{P_E} = \frac{\partial E}{\partial P_E}\frac{P_E}{E}$) into two components:

$$\eta^E_{P_E} = \eta^E_{P_E}|_{\varepsilon=\varepsilon^*} + \eta^E_{\varepsilon}\eta^{\varepsilon}_{P_E} \tag{2}$$

The first component, $\eta^E_{P_E}|_{\varepsilon=\varepsilon^*} = \frac{\partial E}{\partial P_E}|_{\varepsilon=\varepsilon^*}\frac{P_E}{E}$, is the elasticity of energy demand when efficiency is fixed at the level before the price change. It must be nonpositive due to the law of demand (MasCollel, Whinston and Green, 1995): when energy efficiency is constant, an increase in price of energy cannot increase demand for energy.

The second component $\eta^E_{\varepsilon}\eta^{\varepsilon}_{P_E} = \left(\frac{\partial E}{\partial \varepsilon}\frac{\varepsilon}{E}\right)\left(\frac{\partial \varepsilon}{\partial P_E}\frac{P_E}{\varepsilon}\right)$ is the effect of an increase in energy efficiency induced by increase in price. The sign of this component determines whether a drop in demand associated with the first component is reinforced or suppressed.

In general, the response of energy demand to energy efficiency (η^E_{ε}) could be positive or negative. Specifically, Sorrell and Dimitropoulos (2008) shows that $\eta^E_{\varepsilon} = -\eta^S_{P_S} - 1$, where $\eta^S_{P_S}$ is the elasticity of demand for service S with respect to its price. When the demand for services is elastic ($\eta^S_{P_S} < -1$, i.e. elasticity greater than unity in absolute terms), then the effect of energy efficiency on energy use is positive. One could, for instance, imagine that an increase in energy efficiency will motivate a firm to substitute services that do not require energy with services that require energy. When the demand for services is inelastic ($0 > \eta^S_{P_S} > -1$, i.e. elasticity is smaller than unity in absolute terms), the opposite is true. For instance, if a firm cannot easily substitute between services that require energy and those that does not, an increase in energy efficiency will motivate a firm to reduce the demand for energy.

Sorrell and Dimitropoulos (2008) does not specify whether the effect of a change in price on energy efficiency, $\eta^{\epsilon}_{P_E}$, is positive or negative. However, using the result in Proposition 1 we can show that it must have the opposite sign to the effect of energy efficiency improvement on energy demand, η^E_{ε}. We explain this using the following collorary of Proposition 1:

Corollary

Suppose that the decision regarding energy input and investment in energy efficiency is taken by a profit-maximising and price-taking firm. Then:

1. An increase in price always induces a change in energy efficiency that leads to a decrease in demand for energy.

(a) An increase in price induces an increase in energy efficiency if and only if an increase in energy efficiency decreases demand for energy.

(b) An increase in price induce a decrease in energy efficiency if and only if a decrease in energy efficiency decreases demand for energy.

2. An increase in energy price induces an increase in energy efficiency if and only if the demand for useful work of energy services is inelastic.

Proof

Proof of part 1:

2 can be restated as

$$\frac{dE}{dP_E} = \frac{dE}{dP_E}\bigg|_{\varepsilon=\varepsilon^*} + \frac{d\varepsilon}{dP_E}\frac{dE}{d\varepsilon}$$

where $\frac{dE}{dP_E}$ is the total effect of increase in price on energy demand, $\frac{dE}{dP_E}\Big|_{\varepsilon=\varepsilon^*}$ is the effect when efficiency is fixed at its level before the price change and $\frac{d\varepsilon}{dP_E}\frac{dE}{d\varepsilon}$ is the effect of an increase in energy efficiency induced by an increase in price.

From Proposition 1 we know that $\frac{dE}{dP_E} \leq \frac{dE}{dP_E}\Big|_{\varepsilon=\varepsilon^*}$. Thus, it must be that $\frac{d\varepsilon}{dP_E}\frac{dE}{d\varepsilon} = \eta^{\epsilon}_{P_E}\eta^{E}_{\varepsilon} \leq 0$.

This proves parts 1a and 1b.

Proof of part 2: Recall that $\eta^E_\varepsilon = -\eta^S_{P_S} - 1$. Thus, if $\eta^S_{P_S} < -1$, then $\eta^E_\varepsilon > 0$ and, using the result in part 1, $\eta^\epsilon_{P_E} \leq 0$. Similarly, if $\eta^S_{P_S} > -1$ then $\eta^\epsilon_{P_E} \geq 0$.

QED

Intuitively, if an improvement in energy efficiency decreases demand, an increase in the price of energy would motivate firms to improve energy efficiency. Conversely, if energy efficiency is associated with an increase in energy demand, the firm would never choose higher energy efficiency if the price of energy goes up.

An increase in price that leads to a drop in energy efficiency might not be obvious and requires further explanation. Recall from the previous discussion that the instance when an increase in energy efficiency is associated with falling demand for energy corresponds to the case when the demand for energy services is elastic. In this situation an increase in the price of energy motivates firms to abandon production processes based on energy services and instead switch to those that use services based on other inputs. In this case the firm would not be willing to waste resources on improving energy efficiency for the production process that is hardly utilised. For example, firms will not have an incentive to improve the efficiency of combustion engines in cars if consumers, following an increase in oil prices, do not use cars anymore.

The analytical model of dynamic demand

In this section we explore the dynamics of energy consumption and differentiate between the immediate and the long run effects of an increase in energy prices using a putty-clay model. The main assumption that drives our result is that the firm can adjust its technology (choosing between more or less energy-efficient production), however that adjustment can only be applied by the firm to new vintages of capital.

This introduces an inertia that slows down the adjustment of total energy consumption to changes in energy prices.

Set-up

Consider a representative firm with an objective to maximise a discounted stream of profits. In each instant of time the firm produces final output using a continuum of processes. We assume that new processes are constantly invented and become available for the firm. We will index the processes using the time of invention, t. Moreover we assume that at every instant of time a process could become obsolete and cannot be used by a firm anymore. The probability that this happens is given by $1-e^{-\delta}$. As we show later, parameter δ can be interpreted as the capital depreciation rate. Under these assumptions, at instance s, a process $t \leq s$ is available to the firm with probability $e^{-\delta(s-t)}$. Final output at time s, Y_s, is produced by integrating the product of processes, x_{ts}, and combining them with labour, L:

$$Y_s = L^{1-\alpha} \int_{-\infty}^{s} e^{-\delta(s-t)} \left(H_t x_{ts}\right)^{\alpha} dt \tag{3}$$

where H_t is the unit productivity of process t. Since $\left.\frac{dY}{dx_{tt}}\right|_{x_{tt}=0} = \infty$, the firm will immediately install every new process that becomes available.

More recent processes have higher productivity than processes invented earlier. Specifically, we assume that the productivity at the frontier grows at rate g_H. Thus, if H_t denotes the productivity of a process invented at time t, then $H_\tau = e^{g_H(\tau-t)} H_t$ for all processes invented at time τ. For notational convenience, we define, $\tilde{x} = \frac{x}{H^{\frac{1-\alpha}{\alpha}}}$, $\tilde{k} = \frac{k}{H^{\frac{1-\alpha}{\alpha}}}$ and $\tilde{e} = \frac{e}{H^{\frac{1-\alpha}{\alpha}}}$. We will refer to $\tilde{k}$ and $\tilde{e}$ as capital and energy per efficiency unit. The rate of growth of $\left(H^{\frac{1-\alpha}{\alpha}}\right)$, $g = \frac{1-\alpha}{\alpha} g_H$, will define the growth rate of the economy.

Each process uses two factors of production: energy and one factor which cannot be fully adjusted in the short run. To ease the narrative, we refer to this factor as capital. At time s, a process t uses Leontief technology to combine capital k_{ts} with a quantity of energy, e_{ts}, to generate the composite product x_{ts}:

$$x_{ts} = \min\left\{B_t k_{ts}, A_t e_{ts}\right\} \tag{4}$$

where A_t and B_t are the unit productivities of energy and capital for process t.

For process t, energy must be purchased at price p_{Es} at every instance of time $s \geq t$ when the process is in use. In constrast, capital for process t could be purchased only once, at time t, i.e. when the process is installed. Capital is expressed in units of final good, which is the numeraire, so the unit cost of capital is unity.

At every instance of time s the firm has to make three sets of choices: (i) about the capital devoted to process installed at current time s, (ii) about quantity of energy for all processes $t \leq s$ and (iii) about the choice of technology for processes installed in current time s. Following the literature on the technology frontier (see e.g. Caselli and Coleman, 2006), the firm can choose the parameters A_s and B_s from the set constrained by the condition

$$A_s^{\omega} + \gamma B_s^{\omega} = F. \tag{5}$$

The choice of technology with high A_s and low B_s would correspond to an energy-saving and capital-intensive production method for the new process s, while the choice with high B_s and low A_s would correspond to capital-saving and energy-intensive production.

The economy is also populated by consumers. The objective of a consumer at any instance of time is to maximise utility. In time 0, this maximisation problem is given by

$$U = \int_0^\infty e^{-\rho s} \ln (C_s)\, ds$$

where C_s is consumption in instant s and $\ln (C_s)$ is instantenous utility from that consumption. The consumer faces the following budget constraint

$$\int_0^\infty e^{-rs} W_s ds + K_0 = \int_0^\infty e^{-rs} C_s ds$$

where W_s is the income of a consumer at time s. Both consumers and firms have access to a capital market with interest rate r.

Solution to the model

The solution to the model is presented in two steps. First we find the steady state of the model and we show that the firm's choice of the quantity of energy per efficiency unit for a new process does not change over time providing that the firm does not expect any changes in prices in the future. In the second step we use an aggregation exercise to show that total energy consumption approaches its new steady state at a constant rate.

Energy demand for each process

First, we investigate the firm's choice of inputs and technology parameters for a process at the instance of time when it is installed. We normalise the price of the final good to unity. Since the outputs of processes are aggregated in an additive way (we relax this assumption later), the firm's choices of inputs for one process is independent from the choices of inputs for the other processes. Thus the firm's choice of capital, energy and technology for process t can be derived from the solution to the following maximisation problem:

$$\max_{A_t, B_t, k_t e_{ts}} \int_t^\infty e^{-(r+\delta)(s-t)} L_s^{1-\alpha} H_t x_{ts}^{\alpha} ds - k_t - \int_0^\infty e^{-(r+\delta)(s-t)} p_{Es} e_{ts} ds$$

subject to (4) and (5).

For the remaining part of subsection 2.2.1 we will drop the subscript t for all choice variables since we will investigate the inputs for only one process. We will use x_s, e_s to denote the firm's choices of product and energy at time s. We will also use x_t, e_t and k to denote the firm's choice of product, energy and capital at the instance of time when the process is installed, i.e. at instance $s = t$. Since in this section we assume that price of energy is constant, we also drop the time index for p_E.

The optimality conditions require $x_t = Bk = Ae_t$. Furthermore the Leontief technology implies that for $s > t$, $x_s = Ae_s \leq Bk = x_t$. This allows us to restate the maximisation problem using the following

Lagrangean:

$$\begin{aligned}\max_{A,B,x_s} \int_t^\infty H^\alpha e^{-(\delta+r)(s-t)} L^{1-\alpha} x_s^\alpha ds - \frac{x_t}{B} \\ - \int_t^\infty p_E e^{-(r+\delta)(s-t)} \frac{x_s}{A} ds \\ + \lambda \left(A^\omega + \gamma B^\omega - F\right) \\ + \int_t^\infty \mu_s \left(x_t - x_s\right) ds\end{aligned}$$

with $\mu_t = 0$.

The FOCs with respect to x_s for $s > t$ are given by:

$$\alpha H^\alpha L^{1-\alpha} x_s^{\alpha-1} - \frac{p_E}{A} = e^{(\delta+r)(s-t)} \mu_s$$

and the FOC with respect to x_t are given by:

$$\alpha H^\alpha L^{1-\alpha} x_t^{\alpha-1} - \frac{p_E}{A} + \int_t^\infty \mu_s ds = \frac{1}{B}. \tag{6}$$

This system has a unique solution at $x_s = x_t$, $\mu_s = (\delta + r)\, e^{-(\delta+r)s} \frac{1}{B}$ for $s > t$ and $\mu_t = 0$ [2]

We use this to restate equation (6) as:

$$\alpha H^\alpha L^{1-\alpha} x_t^{\alpha-1} \frac{1}{\delta + r} = \frac{1}{B} + \frac{p_E}{A} \frac{1}{\delta + r} \tag{7}$$

This resembles a standard optimality condition: the discounted marginal return from each proccess must be equal to the discounted marginal costs of its operation.

Next, we proceed to the choice of technology. The FOC with respect to A and B are given by

$$- \frac{p_E}{A^2} \frac{x_t}{(r+\delta)} + \lambda \omega A^{\omega-1} = 0 \tag{8}$$

$$- \frac{1}{B^2} x_t + \lambda \gamma \omega B^{\omega-1} = 0. \tag{9}$$

Solving the three equations above together with (5) results in:

$$A = F^{\frac{1}{\omega}} \frac{p_E^{-\frac{\omega}{\omega+1}}}{\left[p_E^{\frac{\omega}{\omega+1}} + (r+\delta)^{\frac{\omega}{\omega+1}} \gamma^{\frac{1}{\omega+1}}\right]^{\frac{1}{\omega}}} p_E$$

$$B = \left(p_E \gamma\right)^{\frac{-1}{\omega+1}} A$$

and the marginal costs of production under optimised technology is given by:

[2] When evaluating the integral, note that $\mu_t = 0$, thus $\int_0^\infty \mu_s = (\delta + r) \left(\frac{1}{\delta+r} - 1\right) \frac{1}{B}$.

$$\frac{\delta+r}{B}+\frac{p_E}{A}=F^{-\frac{1}{\omega}}m \tag{10}$$

where $m=\left(p_E^{\frac{\omega}{\omega+1}}+\gamma^{\frac{1}{\omega+1}}\left(r+\delta\right)^{\frac{\omega}{\omega+1}}\right)^{\frac{\omega+1}{\omega}}$.

This combined with equation (7) allows us to find the steady state output of each process per efficiency unit:

$$\tilde{x}\equiv\frac{x_s}{(H)^{\frac{\alpha}{1-\alpha}}}=\frac{x_0}{(H)^{\frac{\alpha}{1-\alpha}}}=\frac{L\alpha^{\frac{1}{1-\alpha}}F^{\frac{1}{\omega(1-\alpha)}}}{m^{\frac{1}{1-\alpha}}}$$

and the steady state level of energy input per efficiency unit is given by:

$$\tilde{e}\equiv\frac{e_s}{(H)^{\frac{\alpha}{1-\alpha}}}=\frac{x_s}{(H)^{\frac{\alpha}{1-\alpha}}A}=$$

$$=\Theta m^{\frac{-(\omega+\alpha)}{(\omega+1)(1-\alpha)}}p_E^{\frac{-1}{\omega+1}} \tag{11}$$

where $\Theta=L\alpha^{\frac{1}{1-\alpha}}F^{\frac{\alpha}{\omega(1-\alpha)}}$.

To complete the analysis, we also derive the steady state capital per efficiency unit:

$$\tilde{k}\equiv\frac{k_s}{H^{\frac{\alpha}{1-\alpha}}}=\tilde{e}\frac{A}{B}=\tilde{e}\left(p_E\gamma\right)^{\frac{1}{\omega+1}}=\Theta m^{\frac{-(\omega+\alpha)}{(\omega+1)(1-\alpha)}}\gamma^{\frac{1}{\omega+1}}$$

Equilibrium growth rates and interest rate

To complete our characterisation of the steady state in this section we will derive the equilibrium interest rate as well as the growth rate of output.

Since there are no other processes in the economy involving labour, in general equilibrium the wage adjusts in order to set $L=\overline{L}$, where $\overline{L}$ is the total supply of labour. We normalise that supply to unity. The final output will then be given by:

$$Y_s=L^{1-\alpha}\int_{-\infty}^{s}e^{-\delta(s-t)}\left(H_t x_{ts}\right)^{\alpha}dt=$$

$$=\frac{H_s^{\frac{\alpha}{1-\alpha}}n\tilde{x}^{\alpha}}{\delta+g}.$$

Since $\tilde{x}$ is constant, final output grows at the rate given by $g=\frac{\alpha}{1-\alpha}g_H$. Similarly, aggregate capital, K_s, is given by:

$$K_s=\int_{-\infty}^{s}k_t dt=$$

$$= \frac{H_s^{\frac{\alpha}{1-\alpha}} n\tilde{k}}{\delta + g}$$

and aggregate energy consumption is given by:

$$E_s = \int_{-\infty}^{s} e_{ts} dt = \frac{H_s^{\frac{\alpha}{1-\alpha}} n\tilde{e}}{\delta + g}$$

both growing at the rate g.

The only two sources of income for households are wage and capital compensation. Due to constant returns to scale, profit is equal to zero. Total compensation for employees could be derived from the firm's first order conditions with respect to labour input, $wL = \alpha Y$. Since in equilibrium $L = 1$, it must be that w as well as total compensation for employees grows at the rate g. Total compensation for capital owners is given by rK. Since r is constant along the balanced growth path, the compensation grows at the rate g.

Consumption at every instance of time is given by income from labour and capital less purchases of capital for new processes:

$$C = w + rK - \tilde{k} H^{\frac{\alpha}{1-\alpha}}$$

Since all the terms grow at the constant rate, g, consumption must grow at the same rate, $g_C = g$.

Finally, from the optimisation of the consumer, we know that $r = \rho + g_C = \rho + g$.

Elasticity of demand for energy

The elasticity of demand for energy for one process can be found immediately from equation (11):

$$\epsilon = \frac{d \log e_s}{d \log p_{E,s}} = \frac{-1}{\omega + 1} - \frac{\omega + \alpha}{(\omega + 1)(1 - \alpha)} \frac{d \log m_s}{d \log p_{E,s}}.$$

The first term reflects the possibility of replacing energy with capital by choosing a more energy-saving technology. The second term reflects the drop in demand due to a fall in production following an increase in the marginal costs of production.

In order to find the elasticity of total energy demand, we integrate the energy used for each process:

$$E_s = H_s^{\frac{\alpha}{1-\alpha}} \int_{-\infty}^{s} \tilde{e}\left(p_{E,t}\right) e^{-(g+\delta)(s-t)} dt. \tag{12}$$

Suppose that in past, at instant of time T the price of energy increases from $\underline{p_E}$ to $p_E = \underline{p_E} + dp_{E,T}$ (and thus m increases from $\underline{m}$ to $\overline{m}$). Unless the change is large, the firm will not change input of energy into processes that were installed before T. This is because every time the firm starts a new process it has to ensure that its marginal product covers the costs of both input. Since afterwards the firm has to cover only the cost of energy (and does not have to pay for the capital input anymore), the firm will never change the energy input into a process unless the price of energy changes by more than the compensation for the capital input at the original level. We discuss the dynamics when the change in price is large in section 3.3.2.

For this reason, the aggregate demand for energy becomes

$$E_s = H_s^{\frac{\alpha}{1-\alpha}} \left(\int_{-\infty}^{T} e\left(\underline{p_E}\right) e^{-(g+\delta)(s-t)} dt + \int_{T}^{s} e\left(P_E\right) e^{-(g+\delta)(s-t)} dt \right)$$

The elasticity of aggregate energy demand at time s with respect to a permanent change of energy price at time T is therefore:

$$\frac{d \log E_s}{d \log p_{E,T}} = \frac{\int_T^s e^{t(\delta+g)} e\left(\underline{p_E}\right) dt}{\int_{-\infty}^{T} e^{t(\delta+g)} e\left(\underline{p_E}\right) dt + \int_T^s e^{t(\delta+g)} e\left(p_E\right) dt} \varepsilon.$$

For marginal changes in price this reduces to:

$$\frac{d \log E_s}{d \log p_{E,T}} = \left(1 - e^{-(s-T)(\delta+g)}\right) \varepsilon. \tag{13}$$

Proposition 2

After an increase in energy price, energy demand approaches its long-run level at the exponential rate, which is the sum of depreciation and growth rates.

Proof in the text.

In OECD countries, the depreciation of capital is at the level of 6% (Barro, Mankiw and Sala-I-Martin (1995), Nehru and Dhareshwar (1993), Iradian (2007)), while the annual growth rate oscilates around 2% (World bank dataset). This implies that the elasticity of demand reaches half of its long-run value after 8 years, 80% after 20 years and 95% only after 37 years. In China and India, which experienced a growth rate of 7%, half of the long-run value of the elasticity of demand is attained after 5 years.

Discussion

Using the results of the model, in this section we (i) discuss the main characteristics of the path of energy demand after a change in price and (ii) investigate the consequences of this for evaluating climate policy. The discussion also gives an opportunity to flag the key assumptions that drive the main predictions of the model and investigate how the predictions would change under an alternative set-up.

The rate of adjustment

According to Proposition 2, energy demand approaches its long-run level at a constant exponential rate, which is given by the sum of the depreciation and growth rate. The reason is that in our model, the optimal choice of technology and optimal level of energy for each process can be applied only to processes that are installed in the current instant of time, after a price change takes place. The technology of proccesses installed before that date is set and cannot be changed, however it is still optimal to continue production using older processes due to the sunk cost of capital. Energy consumption of previously installed proccesses is tied to the level of capital embeded in them. The size of that capital relative to total capital decreases due to two reasons: the depreciation rate and the growth rate of the economy. The size of these two forces determines how quickly energy demand approaches its new long-run level.

The rate of change would be slower if in each process energy were bundled not with capital, but with a factor that is scarce at the aggregate level, for example land or specialist labour. In this case, a firm would distribute the scarce factor across processes according to the productivity of each process. In the appendix A2 we show that the parameter governing the shape of this distribution is the growth rate of H.[3] If the growth rate is positive, employment in new processes will be larger than for older processes and the distribution is skewed. This skewness will have consequences for the path of energy demand after an increase in prices. For skewed distribution, energy demand by processes using old technology choices constitute a small share of total energy demand. In fact, in the appendix A2 we show that in this case energy demand approaches its long-run level at the rate given by g, which is the growth rate of economy. This rate is smaller than in the case when energy was bundled with capital.

The rate of adjustment predicted in Proposition 1, however, does not change if we allow for complementarity between processes. In the appendix A3 we show the solution of the model if equation 3 is replaced with

$$Y_s = L^{1-\alpha} \left(\int_{-\infty}^{s} (H_t x_{ts})^{\sigma\alpha} \, dt \right)^{\frac{1}{\sigma}}$$

. Here, parameter σ measures the complementarity between processes. We show that if the productivity of the process at the frontier grows at the rate g_H, then the optimal size of production by processes at the frontier, x_T, grows at the rate $g = \frac{\alpha}{1+\alpha} g_H$, which is independent of parameter σ. Total output will grow at exactly the same growth rate, g. Thus, as in the original model considered in section 3.2, the size of production using old technology choices will decrease at the rate given by $\delta + g$.

The rate of change could be faster than the one predicted in the model if the economy is not on a balanced growth path. According to Solow (1957), economies that are converging to their long-run equilibrium are characterised by faster growth of capital than the growth rate of the economy. In this case, the rate of change of energy demand will be also faster.

On the other hand, the change could be slower than the one predicted in the model if the technological change involves some time for preparation. As noted by Sue-Wing (2006), the adoption of new technologies is often preceeded by a development stage, diffusion and scale-up of operations. In this case, the time necessary for a drop in energy could be even longer than if new technologies are available to firms immediately. This last phenomenon is in fact supported by the data. Sue-Wing (2006) observed that the increase in fuel prices at the beginning of 1970s led to an increase in energy efficiency embodied in technical progress only at the beginning of 1980s.

[3] Additionally, the distribution could be affected by changes in energy prices, if those changes are large. A firm would additionally reduce employment in processes that were installed before the change in price. However, this effect is negligible if changes in energy prices are small - the case that we consider in this paper.

Short-run elasticity of demand

The prediction of the model implies that immediately after a change in prices, energy demand does not change ($\frac{d\log E_T}{d\log p_{E,T}} = 0$). There are two assumptions standing behind this result: Leontief production function for each process and marginal changes in energy price. Both assumptions were made in order to simplify the exposition of the model. Relaxing these assumptions could alter the firm's decisions regarding the choice of technology and energy demand immediately after the change in prices, as we discuss below. However, they cannot affect the rate at which energy demand approaches its long-run level.

The assumption of Leontief technology for each process implies that the firm does not have the possibility to substitute energy with capital within processes that are already installed. If the production function for each process were to allow such a substitution, the firm would be able to immediately decrease the demand for energy. In this case, short-run elasticity of demand for energy would be larger.

The assumption of marginal changes in energy price implies that the firm always operates every process created in the past at full capacity, until the capital installed for that process disintegrates. From the firm's perspective, the cost of investment in the process (CAPEX) is sunk and the only cost the firm considers after the investment is the cost of operation (OPEX), which in our model is the cost of energy. The firm compares this cost with the productivity of the process. The productivity of the process is determined at the moment of making the investment and the firm sets it at the level equal to the marginal cost of the process that includes both the levelised CAPEX cost and the OPEX cost. Thus the productivity of that process when it is utilised at full capacity is always larger than the OPEX costs. In other words, the firm is never willing to get rid of its assets when the change in the price of energy is small.

This assumption could be violated if achieving climate targets will require an aggresive climate policy. In this situation, a carbon tax will drastically increase the price of energy. If the change in the price of energy is large, the productivity of processes using old technology choices operated at full capacity could be lower than the OPEX cost. In this case the firm would be willing to decrease the use of energy for those processes. The firm would do this for the oldest processes that are still in operation, which are characterized by the lowest productivity (lowest parameter H_t). This would lead to a sharp drop in energy consumption immediately after a jump in the price of energy. However, after this initial reaction, the rate of replacement of old processes with new processes would be unchanged. Thus, as predicted in Proposition 1, energy demand would still approach its long-run level at the rate determined by the depreciation and the growth rate. In addition, since the choice of technology, energy and capital is independent from the costs of operation of past processes, the value of the long-run elasticity of demand would be unaffected.

Long-run elasticity of demand

According to the model, the long-run elasticity of energy demand to change in prices is given by $\frac{d\log E_\infty}{d\log p_{E,T}} = \epsilon = \frac{-1}{\omega+1} - \frac{\omega+\alpha}{(\omega+1)(1-\alpha)}\frac{d\log m_T}{d\log p_{E,T}}$. This expression highlights two effects that shape the long-run response of energy demand.

The first term, $\frac{-1}{\omega+1}$, reflects the possibility to adjust technology after a change in energy prices. The size of this effect is solely determined by parameter ω, which measures the curvature of the technology frontier (as defined in equation (5)). If ω is close to unity, a firm can easily switch between energy-saving and capital-saving technologies. If ω is very large, then a small improvement in energy efficiency of technology comes at the cost of a large drop in capital efficiency of a technology. Thus, if ω is large, a firm would be less willing to make large adjustments in its technology.

The second term, $\frac{\omega+\alpha}{(\omega+1)(1-\alpha)} \frac{d \log m_T}{d \log p_{E,T}}$, reflects a drop in energy demand caused by a drop in production after an increase in the marginal cost of the final good. The size of this effect will depend on how big a share of the cost of production could be attributed to the cost of energy. At the level of aggregated economy this share will be usually small (most of the costs of production are associated with labour or capital costs), thus the size of this effect is likely to be small.

The value of the long-run elasticity will be different to that predicted in the proposition if the short-run elasticity is different from zero. For instance, if each processs does not use the Leontief technology, but rather a CES aggregate of energy and capital, $x = \left((Ae)^{\psi} + (Bk)^{\psi}\right)^{\frac{1}{\psi}}$, then the long-run elasticity of demand for energy would be given by:

$$\frac{d \log E_{\infty}}{d \log p_{E,T}} = -\frac{1}{1-\psi} - \frac{-\psi}{1-\psi} \frac{-\psi}{1-2\psi+(\omega-1)(1-\psi)}.$$

(ignoring the effect through m).

A similar decomposition into the immediate effect of substitution (the first term on the right-hand side) and the effect of technological adjustment (the second effect on the right-hand side) was obtained by Gerlagh and Kuik (2014), although those authors derived it from different microfoundations.

The impact of a carbon tax on emissions when demand for energy dynamically adjusts

In the introduction we noted that the adjustment of energy demand to a change in energy prices has fundamental importance for the timing of the response of emissions to a carbon tax. To formalise this argument, consider a simplified Kaya decomposition: total emissions (CO2) could be decomposed into total energy (E) and carbon intensity of energy (CO2/EN):

$$CO2 = \frac{CO2}{E} E.$$

Totally differentiating this identity brings:

$$d \log CO2 = \left(\epsilon_{CO2/EN,\tau} + \epsilon_{p,\tau}\epsilon_{E,p}\right) d \log (\tau).$$

This equation in combination with (13) reveals that the full effect of a carbon tax on emissions could be realised only in the long run. In light of the discussion in section 3.3.1, this time horizon would be particularly long in developed countries but it could be considerably shorter in emerging economies, which are characterised by having faster growth rates.

Ignoring the difference between the short- and long-run responses of energy to a change in prices could generate a bias in the predictions of integrated assessment models. Those models that assume a large subsitution between energy and capital (e.g. REMIND and WITCH models) overestimate the response of energy to a change in prices (and so carbon taxes) in the short run. Those models that assume little substitution between energy and capital (e.g. TIMES-MSA - see Winning 2014) underestimate the response and the effectiveness of carbon tax (in terms of CO2 reduction) in the long run. It is therefore important to include in those models a mechanism which differentiates between the short- and long-run elasticity of energy demand.

Adaptation to Dynamic Stochastic General Equilibrium setting

In this section we demonstrate how the framework of dynamic energy demand could be incorporated in a numerical general equilibrium model. To do so, we propose a reduced form of the model developed earlier. In the derivations, we will assume that there are no decreasing returns to scale for individual processes, i.e. $\alpha = 1$.

Note, first, that we could use the expression for aggregate energy, (12), and the analogous expressions for capital and output, to derive a system of differential equations:

$$\frac{d\tilde{E}_s}{ds} = \tilde{e}_s - (\delta + g)\,\tilde{E}_s$$

$$\frac{d\tilde{K}_s}{ds} = \tilde{k}_s - (\delta + g)\,\tilde{K}_s$$

$$\frac{d\tilde{Y}_s}{ds} = \tilde{x}_s - (\delta + g)\,\tilde{Y}_s$$

where $\tilde{E} = E/H^{\frac{\alpha}{1-\alpha}}$.

The discrete version analogue of these equations are:

$$\tilde{E}_s = \tilde{e}_s + (1 - (\delta + g))\,\tilde{E}_{s-1}$$

$$\tilde{K}_s = \tilde{k}_s + (1 - (\delta + g))\,\tilde{K}_{s-1}$$

$$\tilde{Y}_s = \tilde{x}_s + (1 - (\delta + g))\,\tilde{Y}_{s-1}. \tag{14}$$

Then, following the vintage model by Krusell (1998), we define energy services, ME, and capital services, MK, as:

$$ME_s = A_s\tilde{e}_s + (1 - (\delta + g))\,ME_{s-1} \tag{15}$$

$$MK_s = B_s\tilde{k}_s + (1 - (\delta + g))\,MK_{s-1}. \tag{16}$$

Because $\tilde{x}_s = \min\left\{A_s\tilde{e}_s, B_s\tilde{k}_s\right\}$ and because in equilibrium $\tilde{Y}_{s-1} = ME_{s-1} = MK_{s-1}$, equation (14) could be rearranged to:

$$\tilde{Y}_s = \min\left(ME_s, MK_s\right). \tag{17}$$

The problem of the firm could be therefore reduced to the profit-maximising choice of A_s, B_s, $\tilde{e}_s$ and $\tilde{k}_s$ subject to (5), (15), (16) and (17). A number of CGE, DSGE or IAM models already include an analogue of equation (17) (either in the Leontief form or as the CES - see for instance the REMIND model or WITCH model). Thus it is sufficient to introduce equations (5), (15) and (16) as additional constraints in the system and mark A_t, B_t, $\tilde{e}_t$ and $\tilde{k}_t$ as control variables. In the DSGE models, ME_{t-1} and MK_{t-1} should be treated as state variables.

The adaptation of the module to a multi-sector model requires additional adjustments. In these models, when firms could substitute energy with capital, an increase in carbon tax and the resulting increase in energy price will usually stimulate a large flow of capital towards energy-intensive sectors in order to substitute for energy. If the model does not have any constraints on the flow of capital, these adjustments will take place immediately, in the first period after the introduction of the carbon tax. However, a large-scale flow of capital from one sector to another in a very short period of time is usually not realistic. To mitigate this problem, one could introduce capital adjustment costs in each sector.

Macroeconomic consequences of dynamic adjustment

In this section we illustrate the consequences of long-run adjustment in technology choices on the dynamics of GDP, employment, investment and energy use after the introduction of a carbon tax. For this purpose, following the procedure described in section 4, we incorporate the dynamic demand set-up into the MEMO model. MEMO is a macroeconomic multi-sector DSGE model described in Antosiewicz and Kowal (2016). The detailed implementation of the changes to the original model by Antosiewicz and Kowal (2016) are described in the appendix A4. The model has been designed in order to study the macroeconomic effects of climate policy in the short and medium run. The version used in this study has been calibrated using data for the Polish economy.

We performed the simulation under three scenarios. In the first scenario, firms do not have the possibility to adjust their technology. For this scenario we set a very high value of parameter ω, which implies that any deviation from the pre-tax choice of technology is not profitable for the firm. In the second scenario, we allow firms to choose their technology. Following the set-up from our theoretical model in section 3, in this scenario we take into account that firms cannot adjust technology for their entire capital stock at once. In one period, a firm can adjust its technology only for capital purchased in this period and for a small fraction (10%) of its previous capital (see the appendix A4 for details on how this is implemented; allowing for adjustment of technology for 10% of previous capital stock is necessary for ensuring numerical stability of the model). In the third simulation, we allow the firm to adjust its technology immediately for 90% of its capital stock. Table 1 summarises the values of key parameters used in the simulations. We will focus on a comparison of the first two scenarios and refer to the third scenario whenever it is useful for understanding how gradual technology adjustment impacts economic dynamics.

We will choose a very low value of elasticity in the short run, following our assumption in section 3. This is also supported by empirical studies. Okagawa and Ban (2008) show that the null hypothesis of zero substitution was not rejected in 14 of 19 considered industries. Similar results were found by Kuper and Soest (2002) and Liu (2004). The long-run elasticity of demand used in the model is 0.3. It is higher than the

	no tech adjustment	gradual tech adjustment	immediated tech adjustment
short-run elasticity of substitution between energy and capital	0.05	0.05	0.05
curvature of technological possibility constraint (ω)	99	2.5 [4]	2.5
implied long-run elasticity of substitution between energy and capital	0.05	0.3	0.3
fraction of capital allowed to adjust immediately (σ parameter in the appendix)	not applicable	10%	90%

Table 1. Values of selected parameters in the numerical model.

value of 0.2 estimated for electricity in Liu (2004), but it is also in the lower range of the long-run elasticities for Europe estimated by Koetse et al. (2008). Higher values of elasticities caused model instability.

In each simulation we introduce a constant carbon tax. The level of carbon tax is the same in each scenario and its revenue reaches approximately 2.8% of total GDP in the first period.

Figure 1 depicts the path of energy consumption under the three scenarios. When firms are not allowed to adjust their technology, the drop in consumption is relatively modest. This is driven primarily by a falling share of energy-intensive sectors as well as a fall in aggregate production (both caused by an increase in the price of energy). If technology can be adjusted, the ultimate long-run drop is more profound, since firms will switch to more energy-efficient methods of production. When firms can immediately change their technology for the entire capital stock, the drop in energy use becomes significant shortly after the introduction of the carbon tax. When firms can adjust their technology gradually, the initial effect is similar in size to the no-adjustment scenario. As time passes, firms increase their energy-efficient capital stock and energy use converges to its new long-run steady state. In the long run, results for the scenarios with gradual and immediate technology adjustment are the same.

The path of energy consumption has straightforward consequences for CO2 emissions and employment in the sector of raw materials. Since energy production is the main source of CO2 emissions in Poland, a drop in energy consumption is directly translated into a fall in CO2 emissions (Figure 2). This drop is evident in both scenarios; however, it is significantly larger, especially in the long run, in the scenario with gradual technology adjustment than in the scenario when adjustment is not possible. The same pattern can be seen for employment in the mining and quarrying sector (Figure 3). In Poland this sector is dominated by coal mining supplying intermediate input for the coal-dependent energy sector. The drop in demand for energy is thus inevitably reflected in the drop of employment in mining.

A change in the price of energy leads to changes in investment; however, the size and dynamics of this effect depends on the possibilities for technology adjustment. Since capital and energy are complementary factors of production, an increase in the price of energy results in a sudden drop in returns on investment, and thus, a lower volume of investment in the economy (Figure 4). This is most evident when firms are not able to adjust their technology. An option to adjust technology mitigates this effect in two ways: first, since firms can pick up an energy saving technology, higher energy prices have a lower effect on the productivity of capital and hence on the return on investment. Second, since firms make technology choices for the new

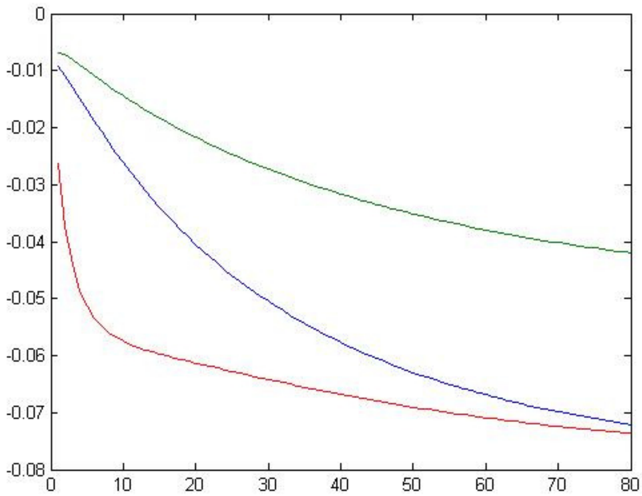

Figure 1. The log-deviations of energy consumption from the baseline (no carbon tax) with no technology adjustment (green line), gradual adjustment (blue line) and immediate adjustment (red line)

vintages of capital, they have an incentive to increase investment as this allows for the faster replacement of obsolete capital stock. If the firm had the opportunity to immdiately adjust its capital for the entire capital stock (the red line in Figure 4), the investment would be smaller.

Lower investment in the economy leads to a slower accumulation of capital and thus a lower output compared to the baseline with no carbon tax. Again, the size and dynamics of this drop depends on whether we take into account the gradual adjustment of technology. Since the decline of investment is smaller when firms are able to adjust their technology, in the long run the fall of output is smaller in that scenario. In the scenario with no technological adjustment in the first periods households anticipate increase in price of future consumption and thus decrease leisure and increase labour supply. This leads to a temporary increase in GDP. However the size of this effect is small and short-lived.

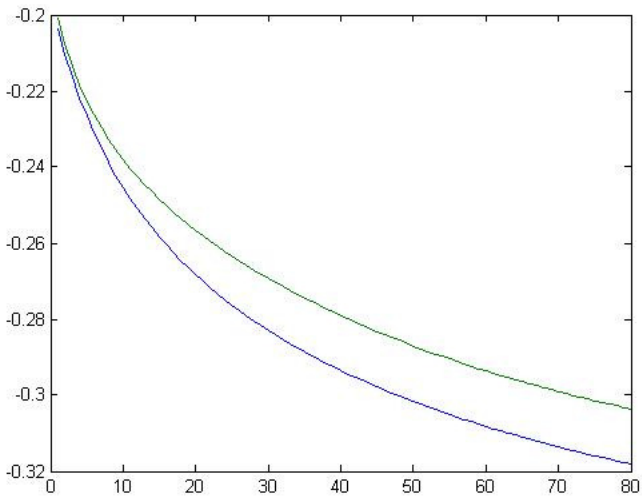

Figure 2. The log-deviations of CO2 emissions from the baseline (no carbon tax) with no technology adjustment (green line) and gradual adjustment (blue line)

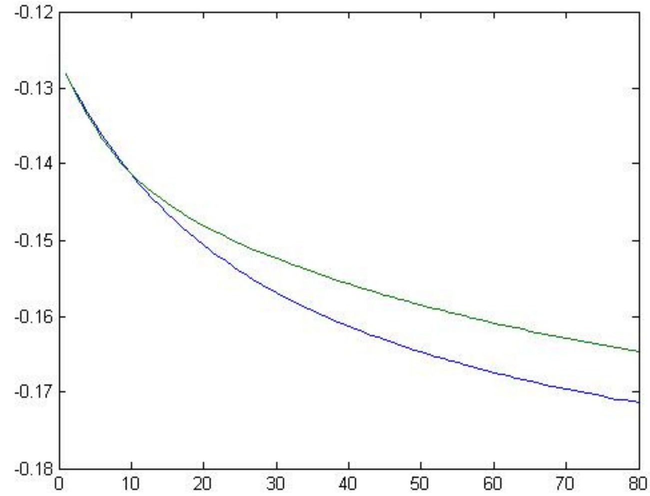

Figure 3. The log-deviations of employment in mining from the baseline (no carbon tax) with no technology adjustment (green line) and gradual adjustment (blue line)

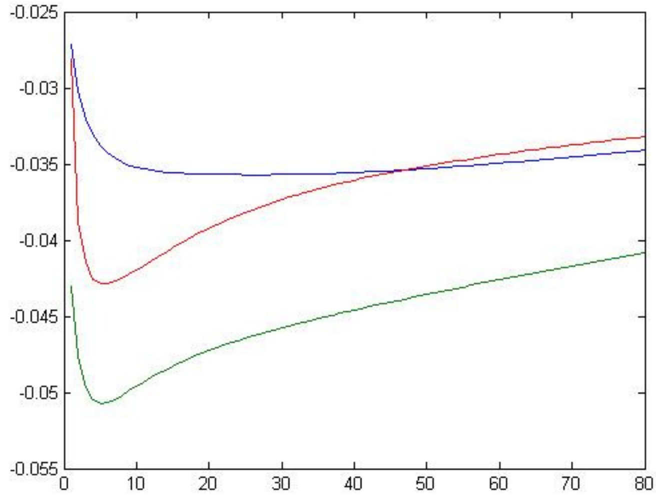

Figure 4. The log-deviations of investment from the baseline (no carbon tax) with no technology adjustment (green line), gradual adjustment (blue line) and immediate adjustment (red line)

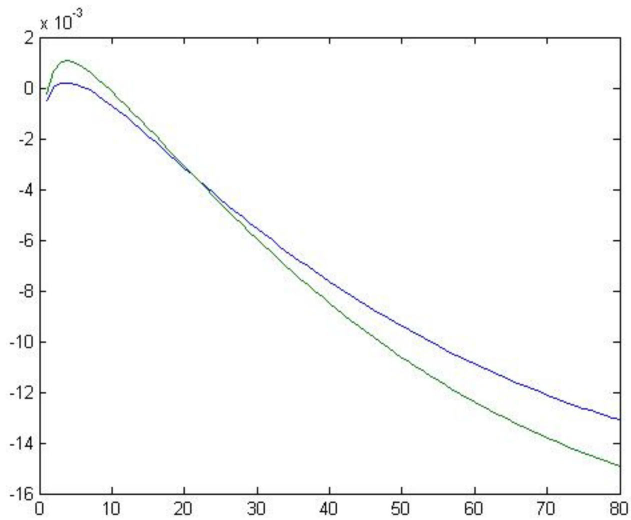

Figure 5. The log-deviations of GDP from the baseline (no carbon tax) with no technology adjustment (green line) and gradual adjustment (blue line)

Conclusion

Policies related to climate change adaptation or mitigation should be analysed both in the short and long run. Decarbonisation is a process which changes the deep structure of the economy and it is bound to last for at least a couple of decades. It is therefore useful to know how policies that we implement today will impact our economy 20 or 30 years from now and whether and when they have the potential to limit carbon emissions. On the other hand, climate change policies can have a significant short-term negative impact on our economy, which can limit their social acceptability. A comprehensive understanding of the effects of climate policies requires a modelling framework which takes into account the differences in economic adjustments which take place in both the short and long run.

In this paper we propose a vintage capital framework for analysing the dynamics of energy demand in which economic agents are constrained by the available technologies in the short run, but are able to adjust the technology they use in the long run. Most existing studies allow only for the substitution of inputs through the use of a CES function and those which incorporate technological adjustments assume that they apply to the entire capital stock installed in the economy. Not allowing for technological frictions in the short run would lead to overly optimistic estimates of the negative effects of climate policies. On the other hand, if we only allow economic agents in our model to substitute more expensive goods (such as energy from fossil fuels) with less expensive ones we will not estimate the full potential of a climate policy in the long run.

We highlight our results in a simple theoretical model where we show that the response of energy demand is stronger in the long run than in the short run, and that in the long run energy demand converges to a new steady state at an exponential rate. We next show the macroeconomic implications in a case study for the Polish economy. We do this by embedding the theoretical model of technological adjustments in a large-scale macroeconomic general equilibrium model. The results of the general equilibrium analysis confirm these intuitions. With technological adjustments, in the long run energy demand decreases at a greater rate due to the installation of more energy-efficient capital, and the drop in GDP is slightly larger. The drawback is that the deeper decrease in energy demand further reduces employment in the mining and quarrying sector.

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Appendix A1

In this appendix we present detailed derivations for the proof of proposition 1 from section 2.

The objective function in the maximisation problem of the firm is given by:

$$\pi\left(E, \mathbf{z}, \mathbf{v}, P, p_E, \mathbf{w}\right) = PF\left(E, \mathbf{z}, \mathbf{G}\left(\mathbf{v}\right)\right) - p_E E - \sum w_i z_i - \gamma\left(\mathbf{v}\right)$$

The Lagrengean for the firms' profit maximisation problem in the long run (i.e. with no constraints other than 1) is given by:

$$\max_{E,\mathbf{z},\mathbf{v}} \pi\left(E, \mathbf{z}, \mathbf{v}, P, p_E, \mathbf{w}\right) + \lambda \Gamma\left(\mathbf{G}\left(\mathbf{v}\right)\right)$$

First order conditions imply:

$$\frac{\partial \pi\left(E, \mathbf{z}, \mathbf{v}, P, p_E, \mathbf{w}\right)}{\partial E} = 0$$

$$\frac{\partial \pi\left(E, \mathbf{z}, \mathbf{v}, P, p_E, \mathbf{w}\right)}{\partial z_i} = 0$$

for every i and

$$\frac{\partial \pi\left(E, \mathbf{z}, \mathbf{v}, P, p_E, \mathbf{w}\right)}{\partial v_j} + \lambda \frac{\partial \Gamma}{\partial G_j} \frac{\partial G_j}{\partial v_j} = 0$$

for every j.

These first order conditions and constraint 1 define optimal choices, i.e. $E\left(P, p_E, \mathbf{w}\right)$, $\mathbf{z}\left(P, p_E, \mathbf{w}\right)$, $\mathbf{v}\left(P, p_E, \mathbf{w}\right)$ and $\mathbf{G}\left(\mathbf{v}\left(P, p_E, \mathbf{w}\right)\right)$, as well as $\lambda\left(P, p_E, \mathbf{w}\right)$ as a function of prices.

The optimal value function is then given by

$$\tilde{\pi}\left(P, p_E, \mathbf{w}\right) = \pi\left(E\left(P, p_E, \mathbf{w}\right), \mathbf{z}\left(P, p_E, \mathbf{w}\right), \mathbf{v}\left(P, p_E, \mathbf{w}\right), P, p_E, \mathbf{w}\right)$$

$$+\lambda\left(P,p_E,\mathbf{w}\right)\Gamma\left(\mathbf{G}\left(\mathbf{v}\left(P,p_E,\mathbf{w}\right)\right)\right).$$

Since $\mathbf{G}\left(\mathbf{v}\left(P,p_E,\mathbf{w}\right)\right)$ must satisfy constraint 1,

$$\tilde{\pi}\left(P,p_E,\mathbf{w}\right)=\pi\left(E\left(P,p_E,\mathbf{w}\right),\mathbf{z}\left(P,p_E,\mathbf{w}\right),\mathbf{v}\left(P,p_E,\mathbf{w}\right),P,p_E,\mathbf{w}\right).$$

Taking the total derivative:

$$\begin{gathered}\frac{d\tilde{\pi}}{dp_E}=-E+\frac{\partial\pi}{\partial E}\frac{dE\left(P,p_E,\mathbf{w}\right)}{dp_E}\\ +\sum\frac{\partial\pi}{\partial z_j}\frac{dz_j\left(P,p_E,\mathbf{w}\right)}{dp_E}\\ +\sum\left(\frac{\partial\pi}{\partial v_j}+\lambda\frac{\partial\Gamma}{\partial G_j}\frac{\partial G_j}{\partial v_j}\right)\frac{dv_j\left(P,p_E,\mathbf{w}\right)}{dp_E}\\ +\frac{d\lambda}{dp_E}\Gamma(G(v))\end{gathered}$$

Since $\frac{\partial\pi}{\partial E}$, $\frac{\partial\pi}{\partial z_j}{}'s$, $\left(\frac{\partial\pi}{\partial v_j}+\lambda\frac{\partial\Gamma}{\partial G_j}\frac{\partial G_j}{\partial v_j}\right)$ and $\Gamma(G(v))$ evaluated at the optimum are all equal to zero (see the first order conditions above), it must be that $\frac{d\tilde{\pi}}{dp_E}=-E\left(P,p_E,\mathbf{w}\right)$

Now consider the vector of prices $\left(\overline{P},\overline{p}_E,\overline{\mathbf{w}}\right)$. The choice of technology under these prices is given by $\overline{\mathbf{G}}=\mathbf{G}\left(\mathbf{v}\left(\overline{P},\overline{p}_E,\overline{\mathbf{w}}\right)\right)$. Suppose that, in the short run, the firm cannot change the choice of technology if the price changes (analogous derivations apply if one of the input z's is fixed). The problem is given by:

$$\max_{E,\mathbf{z},\mathbf{v}}\pi\left(E,\mathbf{z},\mathbf{v},P,p_E,\mathbf{w}\right)+\lambda\Gamma\left(\mathbf{G}\left(\mathbf{v}\right)\right)+\sum\mu_i\left(G_i(v_i)-\overline{G}_i\right)$$

with $\mu_i=0$ for those i which index elements of $\mathbf{G}$ that are unconstrained.

First order conditions imply:

$$\frac{\partial\pi\left(E,\mathbf{z},\mathbf{v},P,p_E,\mathbf{w}\right)}{\partial E}=0$$

$$\frac{\partial\pi\left(E,\mathbf{z},\mathbf{v},P,p_E,\mathbf{w}\right)}{\partial z_i}=0$$

for all i and

$$\frac{\partial\pi\left(e,\mathbf{z},\mathbf{v},P,p_E,\mathbf{w}\right)}{\partial v_j}+\lambda\frac{\partial\Gamma}{\partial G_j}\frac{\partial G_j}{\partial v_j}+\mu_i\frac{\partial G_j}{\partial v_j}=0$$

for all j.

Again, those first order conditions and the constraint 1 define optimal choices as a function of prices, i.e. $E^R(P, p_E, \mathbf{w})$, $\mathbf{z}^R(P, p_E, \mathbf{w})$, $\mathbf{v}^R(P, p_E, \mathbf{w})$ as well as $\lambda^R(P, p_E, \mathbf{w})$ and $\mu_i^R(P, p_E, \mathbf{w})$ for $i = 1, ..., m$

Now consider the function

$$\tilde{\pi}^R(P, p_E, \mathbf{w}) = \pi + \lambda^R \Gamma\left(\mathbf{G}\left(\mathbf{v}^R\right)\right) + \sum \mu_i^R\left(G_i\left(v_i^R\right) - \overline{G}_i\right).$$

Since $\mathbf{G}\left(\mathbf{v}^R(P, p_E, \mathbf{w})\right)$ must satisfy all the constraints of the maximization problem,

$$\tilde{\pi}^R(P, p_E, \mathbf{w})$$

$$= \pi\left(E^R(P, p_E, \mathbf{w}), \mathbf{z}^R(P, p_E, \mathbf{w}), \mathbf{v}^R(P, p_E, \mathbf{w}), P, p_E, \mathbf{w}\right).$$

Taking the total derivative:

$$\frac{d\tilde{\pi}^R}{dp_E} = -E^R(P, p_E, \mathbf{w}) + \frac{\partial \pi}{\partial E}\frac{dE^R}{dp_E}$$

$$+\sum \frac{\partial \pi}{\partial z_j}\frac{dz_j^R}{dp_E} + \sum\left(\frac{\partial \pi}{\partial v_j} + \lambda\frac{\partial \Gamma}{\partial G_j}\frac{\partial G_j}{\partial v_j} + \mu_j\frac{\partial G_j}{\partial v_j}\right)\frac{dv_j^R}{dp_E}$$

$$+\frac{d\lambda}{dp_E}\Gamma(G(v)) + \sum \frac{d\mu}{dp_E}(G_i(v_i) - \overline{G}_i)$$

Since $\frac{\partial \pi}{\partial E}$, $\frac{\partial \pi}{\partial z_i}$, $\left(\frac{\partial \pi(e,\mathbf{z},\mathbf{G})}{\partial G_i} + \lambda\frac{d\Gamma}{dG_i} + \mu_i\right)$, $\Gamma(G(v))$, and $G_i(v_i) - \overline{G}_i)$, evaluated at the $(P, p_E, \mathbf{w})$ are all equal to zero (see the first order conditions of the restricted problem), it must be that $\frac{d\pi}{dp_E} = -E^R(P, p_E, \mathbf{w})$.

Now, consider a function:

$$m(P, p_E, \mathbf{w}) =$$

$$= \pi\left(E^R(P, p_E, \mathbf{w}), \mathbf{z}^R(P, p_E, \mathbf{w}), \mathbf{v}^R(P, p_E, \mathbf{w}), P, p_E, \mathbf{w}\right)$$

$$-\pi(E(P, p_E, \mathbf{w}), \mathbf{z}(P, p_E, \mathbf{w}), \mathbf{v}(P, p_E, \mathbf{w}), P, p_E, \mathbf{w}) =$$

$$= \tilde{\pi}^R(P, p_E, \mathbf{w}) - \tilde{\pi}(P, p_E, \mathbf{w}).$$

Since $\pi(E(P, p_E, \mathbf{w}), \mathbf{z}(P, p_E, \mathbf{w}), \mathbf{v}(P, p_E, \mathbf{w}))$ maximise profit for a given level of prices, it must be that $m \leq 0$. Notice also that at $\left(\overline{P}, \overline{p}_E, \overline{\mathbf{w}}\right)$ a constrained firm is able to attain maximum level of profit of a firm that is not constrained, so $m\left(\overline{P}, \overline{p}_E, \overline{\mathbf{w}}\right) = 0$ is the local maximum of function m. Thus, it satisfies the following necessary second order conditions:

$$\frac{d^2\tilde{\pi}^R}{dp_E^2} - \frac{d^2\tilde{\pi}}{dp_E^2} = \frac{d^2 m}{dp_E^2} \leq 0$$

Using the previous result:

$$\frac{dE}{dp_E} \leq \frac{dE^R}{dp_E}$$

Thus the long-run demand for energy must be steeper than the short-run demand.

QED

Appendix A2

In the previous section we considered a set-up in which in each process energy was bundled with a factor of production that was unlimited in supply and could be purchased at a constant price (e.g. capital). Here we consider an alterantive setting where energy is bundled with a factor of production that is available only in finite supply. This could be, for instance, specialist labour l^{sp}(which should be ditinguished from conventional labour, L in equation (3).

We assume that that specialist labour is neccessary for the production of each process. Thus, equation (4) is replaced with:

$$x_i = \min\left\{B_t l_t^{sp}, A_t e_t\right\}.$$

The total supply of labour is constrained by unity, $\int_{-\infty}^{s} l_t^{sp} dt = 1$, where i an index for a process that became available at time i. Labour could be allocated in any process and can flow between the processes at any instance of time.

Since the allocation of a decentralised equilibrium must coincide with the decision of a benevolent central planner, it is sufficient here only to consider the problem of the planner.

The objective of a planner is to maximise output at every instance of time:

$$\max_{A_0, B_0 x_t} \left\{ \int_0^{\infty} e^{-\rho s} \int_{-\infty}^{s} H_t^{\alpha} e^{-\delta(s-t)} x_{ts}^{\alpha} dt ds \right.$$

$$- \int_0^{\infty} \lambda_s e^{-\rho s} \left(\int_{-\infty}^{s} e^{-\delta(s-t)} \frac{x_{ts}}{B_t} - 1 \right) dt ds$$

$$
- \int_0^\infty e^{-\rho s} \int_{-\infty}^s c e^{-\delta(s-t)} \frac{x_{ts}}{A_t} dt ds
$$

$$
\left. + \int_0^\infty \lambda_t \left(A_t^\omega + \gamma B_t^\omega - F \right) dt \right\}.
$$

First-order conditions with respect to x_t implies:

$$
x_{ts} = \alpha^{\frac{1}{1-\alpha}} H_t^{\frac{\alpha}{1-\alpha}} \left[\frac{\lambda_s}{B_t} + \frac{c}{A_t} \right]^{\frac{-1}{1-\alpha}}
$$

returning to the constraint:

$$
\int_{-\infty}^s e^{-\delta(s-t)} \frac{\alpha^{\frac{1}{1-\alpha}} H_t^{\frac{\alpha}{1-\alpha}} \left[\frac{\lambda_s}{B_t} + \frac{c}{A_t} \right]^{\frac{-1}{1-\alpha}}}{B_t} dt = 1.
$$

Providing the choice of A_i and B_i is the same across i's (which is verified later):

$$
x_{ts} = \alpha^{\frac{1}{1-\alpha}} H_t^{\frac{\alpha}{1-\alpha}} \left[\frac{\lambda_s}{B_t} + \frac{c}{A_t} \right]^{\frac{-1}{1-\alpha}} \tag{18}
$$

Now we return to the choices of A_t and B_t:

$$
\max_{A_0, B_0 x_i} \int_0^\infty e^{-\rho s} \int_{-\infty}^s H_t^\alpha e^{-\delta(s-t)} x_{ts}^\alpha dt ds
$$

$$
- \int_0^\infty \lambda_s e^{-\rho s} \left(\int_{-\infty}^s e^{-\delta(s-t)} \frac{x_{ts}}{B_t} - 1 \right) dt ds
$$

$$
- \int_0^\infty e^{-\rho s} \int_{-\infty}^s c e^{-\delta(s-t)} \frac{x_{is}}{A_i} dt ds
$$

$$
+ \int_0^\infty \lambda_t \left(A_t^\omega + \gamma B_t^\omega - F \right) dt
$$

First-order condtions with respect to A_i:

$$
\lambda_s e^{-\rho s} e^{-\delta(s-t)} \frac{x_{ts}}{B_t^2} = -\lambda_t \gamma \omega B_t^{\omega-1}
$$

Dividing one by the other gives:

$$
\frac{c\gamma}{\lambda_s} = \left(\frac{B_t}{A_t} \right)^{-1-\omega}
$$

Along the BGP, $\frac{x_{is}}{H_t^{\frac{\alpha}{1-\alpha}}}$ is constant, λ_s is constant and so the equation above together with $A_t^\omega + \gamma B_t^\omega = F$ implies that A_t and B_t are also constant. Since $\frac{x_{is}}{H_t^{\frac{\alpha}{1-\alpha}}}$ is constant, the dynamics are exactly the same as in the main model.

Appendix A3

One of the key assumptions in the derivations is that individual processes enter the production function in equation (3) linearly. This means that the productivity of each process is independent of the productivity in other processes. In this section of the appendix we consider a setting that permits complementarity between processes. In particular, we replace equation (3) with:

$$Y_s = L^{1-\alpha} \left(\int_{-\infty}^{s} (H_t x_{ts})^{\sigma\alpha} \, dt \right)^{\frac{1}{\sigma}}.$$

In this case, the firm's optimisation problem for a representative firm can be stated as:

$$\max_{\{A_{i0}, B_{i0} x_{is}\}_{i=-\infty, s=0}^{i=s, s=\infty}} \left\{ \int_0^\infty e^{-rs} L^{1-\alpha} \left(\int_{-\infty}^{s} e^{-\delta(s-t)} (H_t x_{ts})^{\sigma\alpha} \, dt \right)^{\frac{1}{\sigma}} ds \right.$$
$$- \int_0^\infty e^{-rt} \frac{x_t}{B_t} dt - \int_0^\infty e^{-rs} \int_{-\infty}^{s} e^{-\delta(s-t)} c \frac{x_{ts}}{A_t} dt ds$$
$$\left. + \lambda_i (A_i^\omega + \gamma B_i^\omega - F) + \int_0^\infty \int_{-\infty}^{s} \mu_{is} (x_t - x_{ts}) \, dt ds \right\}.$$

Note that while before the index s referred to both the time since the decision and the time since investment, now s refers only to the time since the decision. As before, the solution implies that $x_t \equiv x_{tt} = x_{ts}$. Thus the first-order conditions with respect to x_t brings for $t > 0$:

$$\frac{1}{r + \delta - (1-\sigma) g} L^{1-\alpha(2-\sigma)} Y_t^{1-\sigma} (H_t x_t)^{\sigma\alpha - 1} H_t \alpha$$
$$- \frac{1}{B_t} - \frac{1}{r+\delta} c \frac{1}{A_t} = 0.$$

The FOCs for A_t and B_t are the same as before, thus equation (10) still holds and the above could be expressed as:

$$x_t = Y_t^{\frac{1-\sigma}{1-\sigma\alpha}} H_t^{\frac{\sigma\alpha}{1-\sigma\alpha}} \frac{\alpha L^{1-\alpha(2-\sigma)} \frac{r+\delta}{r+\delta-(1-\sigma)g}}{F^{-\frac{1}{\omega}} \left[c^{\frac{\omega}{\omega+1}} + (r+\delta)^{\frac{\omega}{\omega+1}} \gamma^{\frac{1}{\omega+1}} \right]^{\frac{\omega+1}{\omega}}}.$$

Inserting this result into the final good production function gives:

$$Y_s = L^{1-\alpha}\left(Y_s^{\frac{1-\sigma}{1-\sigma\alpha}} H_s^{\frac{1}{1-\sigma\alpha}}\right)^{\alpha}\left(\int_{-\infty}^{0} e^{-\delta v}\left(e^{-g\frac{1-\sigma}{1-\sigma\alpha}v} e^{-g_H\frac{1}{1-\sigma\alpha}v}\right)^{\sigma\alpha} dv\right)^{\frac{1}{\sigma}}.$$

Therefore the growth rate of Y_s must be given by:

$$g = \frac{\alpha}{1-\alpha} g_H.$$

This implies that x_t must grow at the rate $\frac{\alpha}{1-\alpha} g_H$,

i.e. exactly the same as in the baseline model in section 3.2.

Appendix A4

The most important changes in the model with regard to the version described in Antosiewicz and Kowal (2016) is the new module responsible for the dynamic adjustment of technology. As in the previous version, the production function of a firm is the nested CES aggregate that combines labour, capital, energy and materials into final product. In the version from Antosiewicz and Kowal (2016), the first nest aggregates capital and energy. In the new version, the nest aggregates capital services and energy services according to:

$$Y_{KE,is} = c_{KE,i}\left(\left(\frac{KS_{is}}{share_{K1,i}}\right)^{\epsilon_{KE}} + \left(\frac{ES_{is}}{share_{K2,i}}\right)^{\epsilon_{KE}}\right)^{\frac{1}{\epsilon_{KE}}}$$

where $c_{KE,i}$ is the productivity parameter for sector i, $share_{K1,i}$ and $share_{K2,i}$ are the share parameters, ϵ_{KE} is the short-run elasticity parameter, KS_i denotes capital services, ES_i denotes energy services and Y_{KE} is the capital-energy composite.

Capital services are determined by the modified capital accumulation equation:

$$KS_{is} = (1-\delta_i)\,\sigma KS_{is-1} + (1-\delta_i)\,(1-\sigma)\,K_{is-1}B_{is} + I_{is}B_{is}$$

where δ_i is the depreciation rate, I_{is} is the total investment in sector i at time s, K_{is} is capital (defined by $K_{is} = (1-\delta_i)\,K_{is-1} + I_{is}$), B_{is} is the productivity of capital goods (endogenous in the model) and $1-\sigma$ is the share of capital accumulated before time s that could be adjusted to new technology set at time s.

Similarly, energy services are determined by:

$$ES_{is} = (1 - \delta_i)\, \sigma ES_{is-1} + FE_{is} B_{is}$$

where $FE_{is} = E_{is} - E_{is-1} + (1 - \sigma)\, E_{is-1}$ and E_{is} is the total consumption of energy. Notice that when $\sigma = 1$ and $\epsilon_{KE} \to -\infty$, then the set-up becomes identical to the set-up discussed in section 3. In the simulations we use two values of σ: 0.1 and 0.9. The value of 0.1 corresponds to the simulations in which technological adjustment is complete and immediate. The value of 0.9 corresponds to the the simulations in which technological adjustment takes time (as discussed in section 2). The simulations with values of σ above 0.9 or below 0.1 are not stable numerically. Similarly, we use $\epsilon_{KE} = -32$, which gives a close approximation of the model with $\epsilon_{KE} \to -\infty$.

In order to ensure the numerical stability of the model, we also add capital and energy adjustment costs, which have to be covered by firms in each sector:

$$CAC_{is} = c_{CAC} \left(\frac{I_{is}}{K_{is}} - \frac{I_{i0}}{K_{i0}} \right)^2 \frac{1}{2}$$

$$MAC_{is} = c_{CAC} \left(\frac{FE_{is}}{E_{is}} - \frac{FE_{i0}}{E_{i0}} \right)^2 \frac{1}{2}$$

where $c_{CAC}$is the parameter and I_{i0} ,K_{i0}, FE_{i0} and E_{i0} are the values of I_{is}, K_{is}, FE_{is} and E_{is} in the steady state.

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