On the job search and financial frictions

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Motivation

- Situation in the labour markets is at the forefront of current discussions:
  - high unemployment in Spain, Greece, Portugal,
  - good situation in the labor market in the US.
- Great Recession brought attention to financial markets and their influence on macroeconomy.
- Growing literature combining financial and labour markets
- Most of the labor models are based on DMP framework but more new research on on-the-job search (OJS)
- Growing literature on introducing heterogeneity in DSGE.

What we do

- We build a model based on BM, MPV
  - *exogenously* heterogeneous firms,
  - on-the-job search labour market.
- We solve it using novel numerical method
  - projection within perturbation.
- We introduce financial market
  - working capital
- We study how the steady-state is affected by costly borrowing.
- We analyze model dynamics.

What we find

- Costly borrowing has following effect on steady-state distribution of firms:
  - Financing vacancies shifts size distribution of firms to the left: **more small firms**
  - Financing wages shifts size distribution of firms to the right: **more larger firms**

- Extent of poaching affected by the presence of financial markets
  - Costly borrowing decreases cross-firm flows more than flows from unemployment

- Following the productivity shock, the density of
  - Low productivity firms decreases
  - High productivity firms increases

Model

Labour market outline

- There is a distribution of firms which differ in productivity
- Unemployed and employed job seekers are matched with vacancies
    - Employed only move to jobs with higher productivity (and wage)
- MPV wage posting replaced by Nash bargaining

Model details, notation

- Firms distributed according to cdf $\Gamma(p)$ on interval $[\underline{p}\ \overline{p}]$, pdf is $\gamma(p)$
- Type-*p* firm produces output with labour using linear technology: $A_t p$
- Type-*p* firm posts vacancies with intensity: $v_t(p)$
- Total vacancies are given by: $VAC_t = \int_{\underline{p}}^{\overline{p}} v_t(p)\gamma(p)dp$
- Employed N_t and unemployed U_t send job offers with intensities: λ_e and λ_u

Model details, notation

- Number of **potential** job matches is: $M_t = \upsilon VAC_t^{1-\mu}(U_t\lambda_u + N_t\lambda_e)^{\mu}$

- Denote probability of finding job as Φ_t and probability of filling vacancy as Ψ_t

- If we denote $N_t(p)$ as the cdf of employment, we can define average firm size as:

$$L_t(p) = \frac{dN_t(p)/dp}{\gamma(p)}$$

Dynamics of average firm size

- Average firm size evolves according to

$$L_{t+1}(p) = (1-\delta)\left(1 - \Phi_{\mathrm{t}}^{\mathrm{N}} \frac{\overline{VAC}_t(p)}{VAC_t}\right) L_t(p) + U_t \Phi_{\mathrm{t}}^{\mathrm{U}} \frac{v_t(p)}{VAC_t} + (1-\delta) N_t \Phi_{\mathrm{t}}^{\mathrm{N}} \frac{v_t(p)}{VAC_t} \frac{N_t(p)}{N_t}$$

Probability of losing job or moving to a better firm

New hires from pool of unemployed

New hires from lower-*p* firms

- $\overline{VAC}_t(p) = \int_p^{\overline{p}} v_t(s)\gamma(s)ds$

Nash bargaining and Value functions

- Wage set to maximize

$$W_t(p) = \arg\max_{W_t(p)} (V_t^N(p) - V_t^U)^{\xi} \left(V_t^J(p) - V_t^V(p)\right)^{1-\xi}$$

- Value of unemployment

$$V_t^U = b + \beta E_t \left(\Phi_t^U \int_{\underline{p}}^{\overline{p}} \frac{v_t(s)}{VAC_t} V_{t+1}^N(s)\gamma(s)ds + (1 - \Phi_t^U)V_{t+1}^U \right)$$

- Value of vacancy

$$V_t^V(p) = -\Xi(VAC_t(p)) + \beta E_t \left(\left(\Psi_t^U + \Psi_t^N \frac{N_t(p)}{N_t} \right) V_{t+1}^J(p) \right)$$

Vacancy cost needs to be convex!

Solution method

Approach to solution method

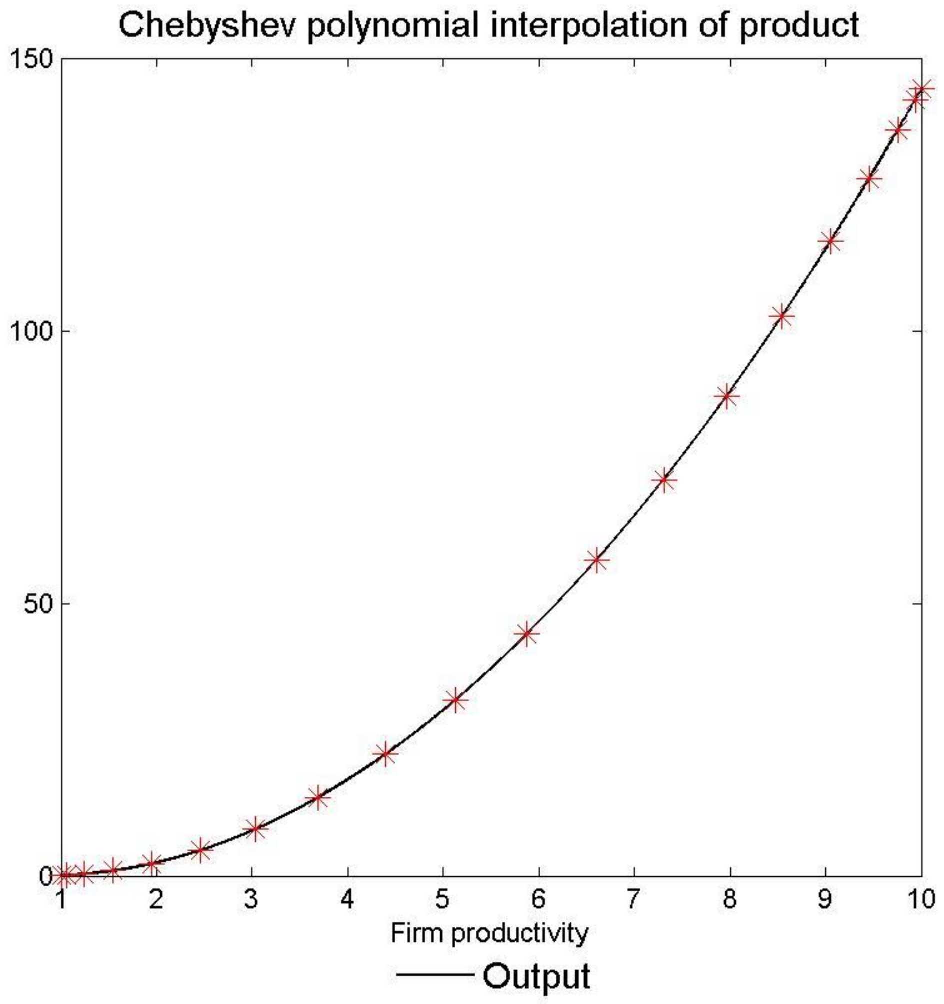
- Model is difficult to solve
  - We have several functions of productivity
  - We need to evaluate a number of nontrivial integrals
- We use Chebyshev polynomial approximation
  - We track values of functions in N points / nodes
- The method of evaluating integrals also works for dynamics

Approximation of functions of productivity

- For function $\mathrm{f}(p)$ approximation is:

$$\mathrm{P}_N^f(p) = \sum_{n=1}^{N} b_n^{\mathrm{f}} \times T_n\big(h(p)\big)$$

- Where: b_n^{f} - weights, T_n- Cheb. Polynomials, and $h(p)$ - linear transformation
- Inside the model we only need to know value of functions for nodes!



Solution method

- We need to calculate many integrals such as $VAC_{\mathrm{t}} = \int_{\underline{p}}^{\overline{p}} v_t(s)\gamma(s)ds$
- Using Chebyshev approximation:

$$\int_{-1}^{1} f(x)dx \approx \int_{-1}^{1} P_N^f(x)\, dx = \sum_{n=1}^{N} b_n^f \int_{-1}^{1} T_n(x)dx =$$

$$= \sum_{n=1}^{N} \frac{2}{N} \sum_{k=1}^{N} f(x_k)T_n(x_k) \int_{-1}^{1} T_n(x)dx$$

- Finally we have:

$$\int_{-1}^{1} f(x)dx \approx F_{1\times \mathrm{N}} \times W_{N\times N} \times T_{N\times 1}$$

Solution method

- If we need a more fancy integral, like: $\overline{V_t}(p) = \int_p^{\overline{p}} v_t(s)\gamma(s)ds$
- The modification the following:

$$\int_a^1 f(x)dx \approx \int_a^1 P_N^f(x)\,dx = \sum_{n=1}^{N} b_n^f \int_a^1 T_n(x)dx =$$

$$= \sum_{n=1}^{N} \frac{2}{N} \sum_{k=1}^{N} f(x_k)T_n(x_k) \int_a^1 T_n(x)dx$$

- Finally we have:

$$\int_{-1}^1 f(x)dx \approx F_{1\times\mathrm{N}} \times W_{N\times N} \times T_{N\times 1}^a$$

Solution method summary

- Calculating integrals boils down to scalar product of function values and parameters!

- Thanks to this we can calculate the steady state

- We can use standard methods to solve **dynamics** the model (Judd, Uhlig, or Dynare)

Results

Basic parameterization

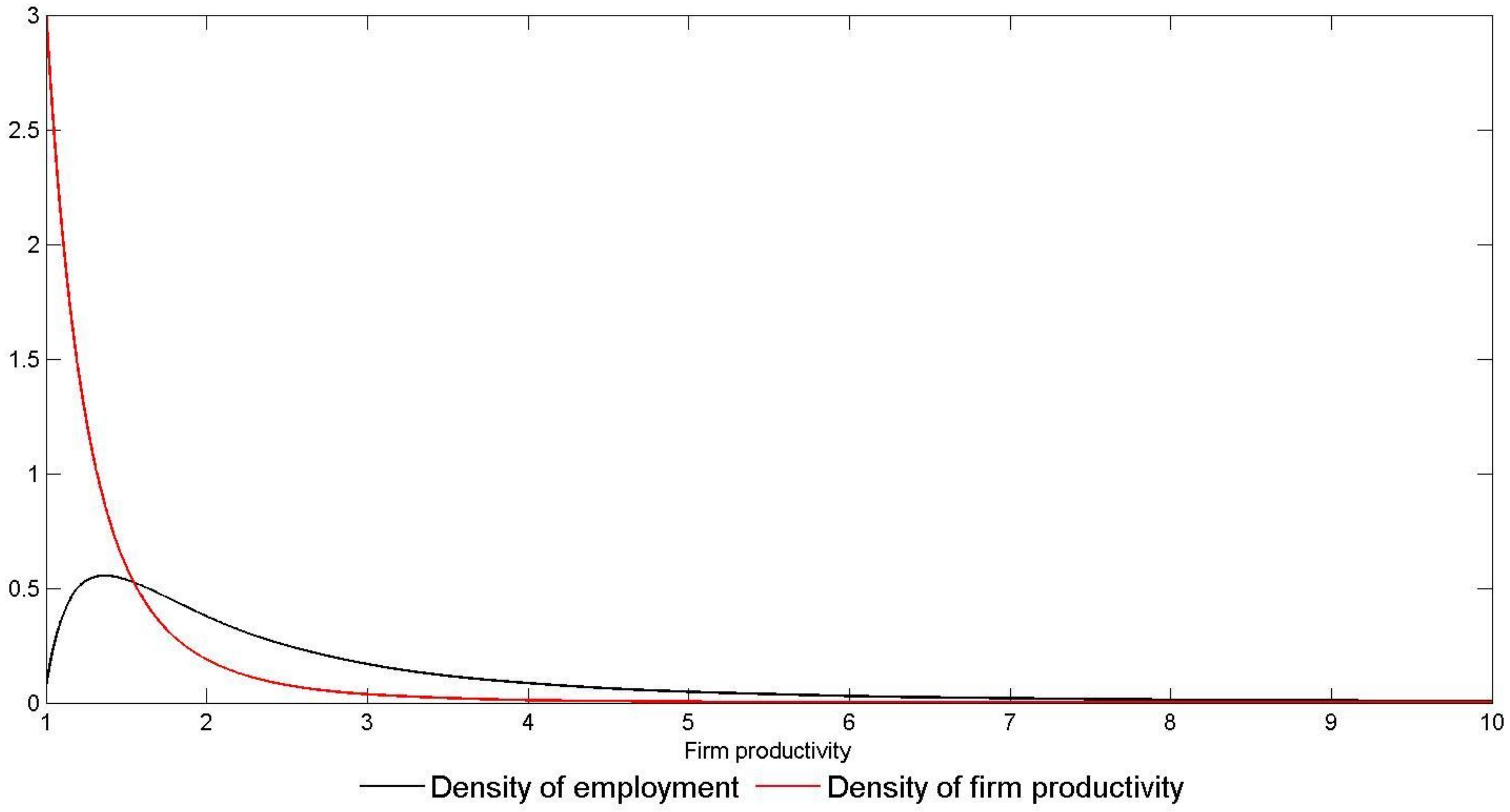
- Pareto distribution for productivity of firms
- Remaining parameters

parameter	interpretation	baseline value
β	discount factor	0.99
δ	job destruction rate	0.1
b	utility of unemployed	0
υ	matching function efficiency	0.5
μ	match elasticity wrt job offers	0.5
ξ	bargaining power	0.5
ν^{α}	linear vacancy cost	0.01
ν^{β}	quadratic vacancy cost	35
λ^{e}	search intensity of employed	0.1
λ^{u}	search intensity of unemployed	1

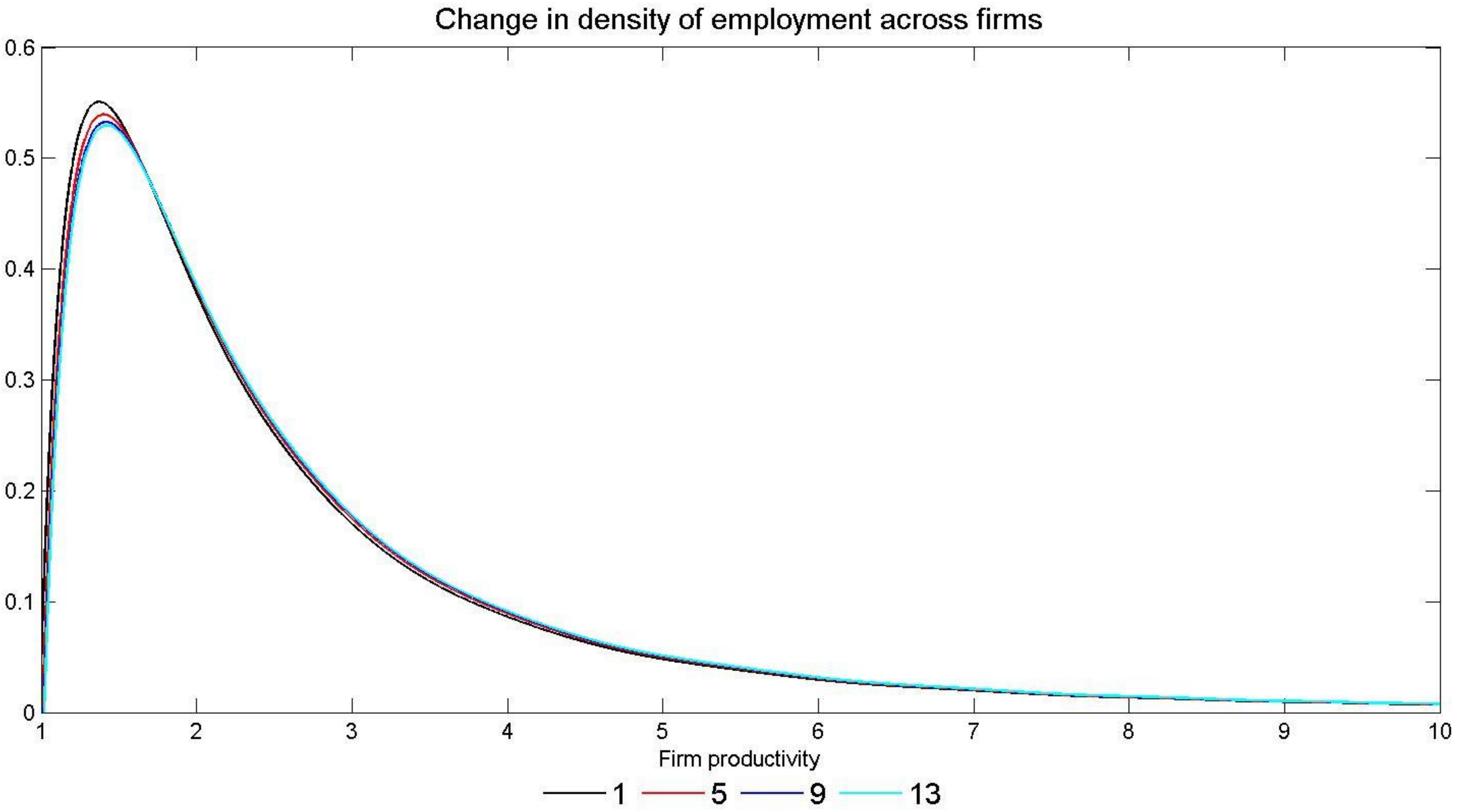
Basic results

- Consistent with BM and MPV
- Average firms size increases with productivity
- Wage increases with productivity

Density of firms and employment



Response of density function to technology shock



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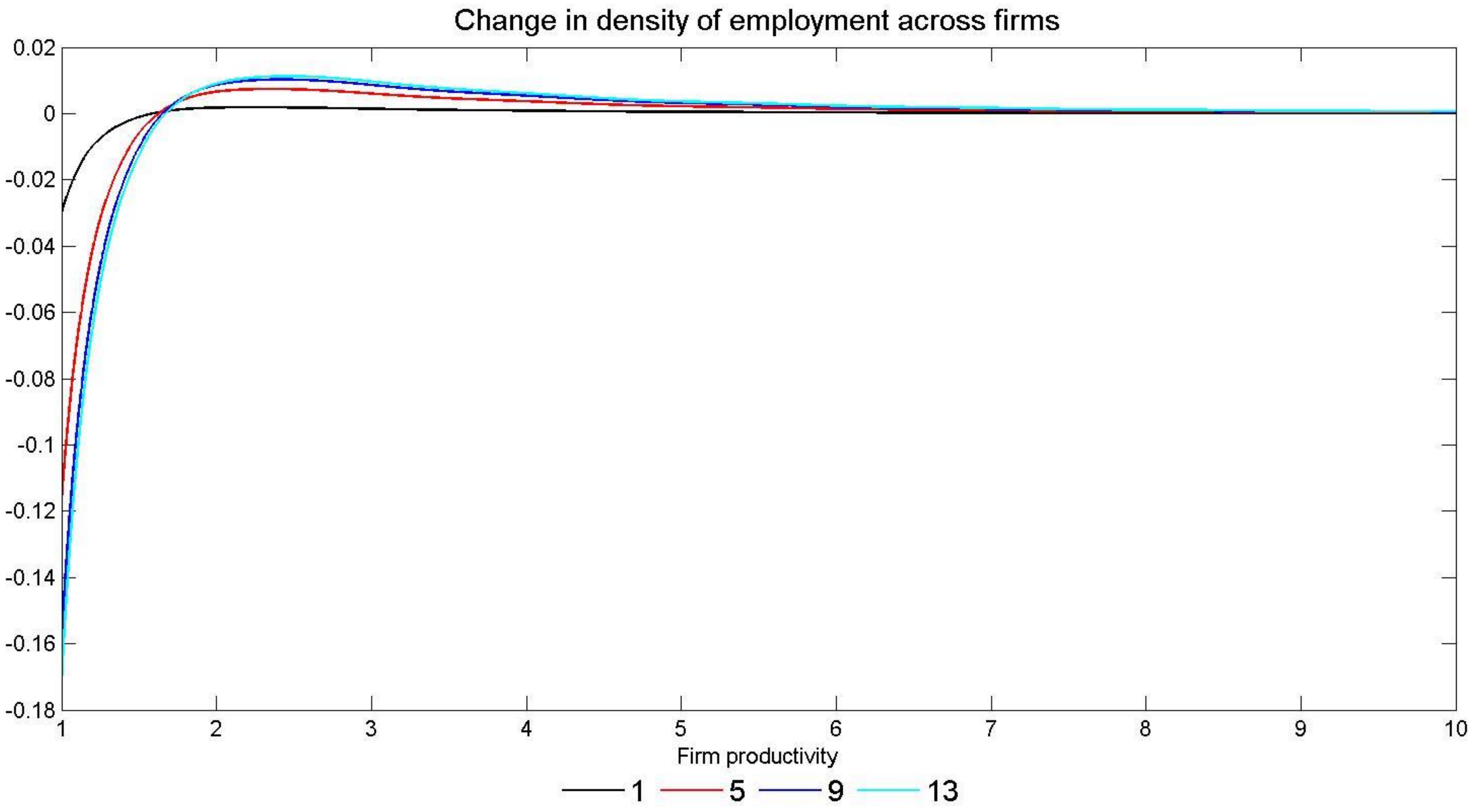


Working capital

Working capital

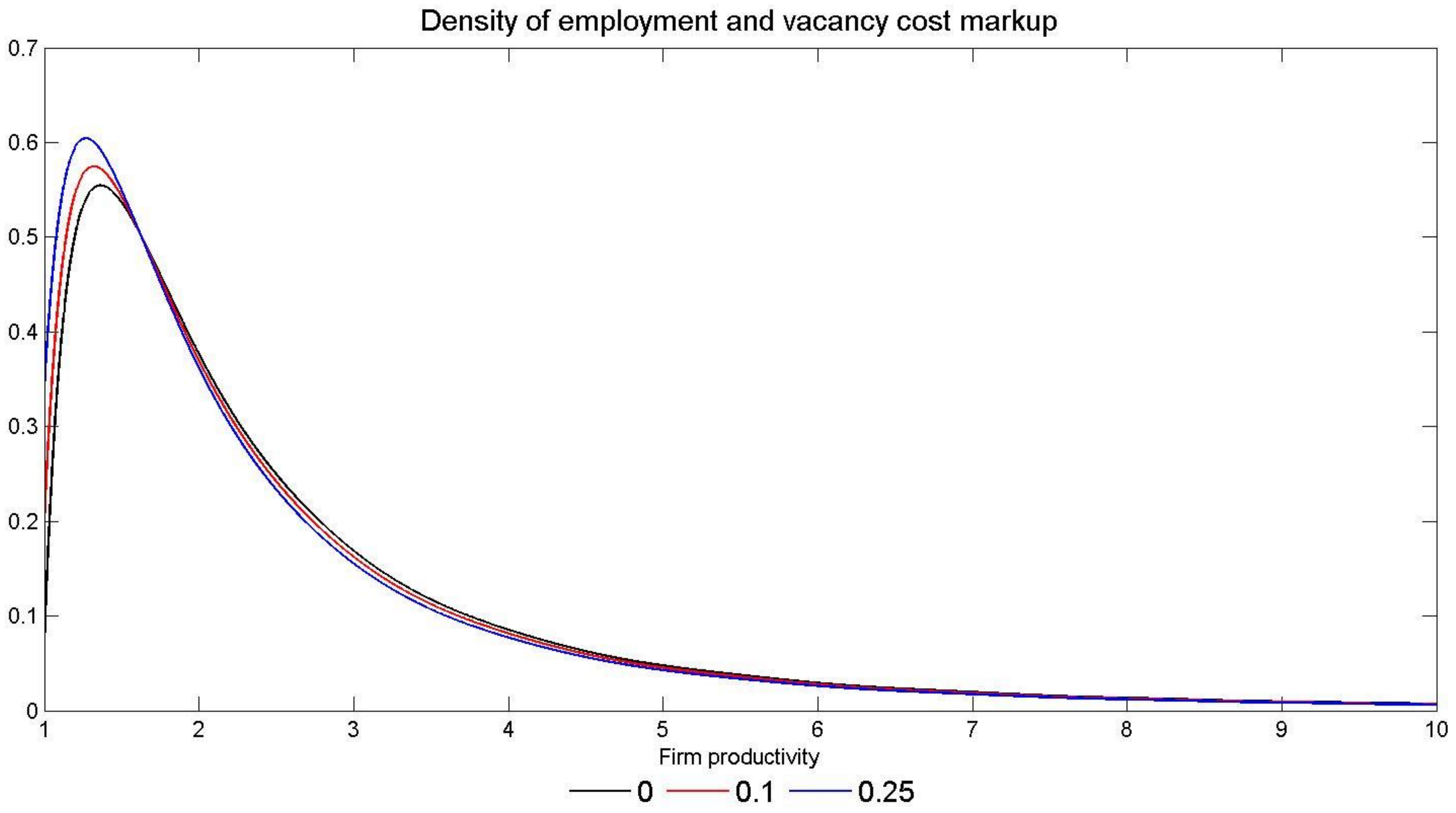
- We introduce working capital into the model
- Firms need to borrow to finance
  - Vacancy cost

$$V_t^V(p) = -\Xi(VAC_t(p))(1 + r_k^c) + \beta E_t\left(\left(\Psi_t^U + \Psi_t^N \frac{N_t(p)}{N_t}\right) V_{t+1}^J(p)\right)$$

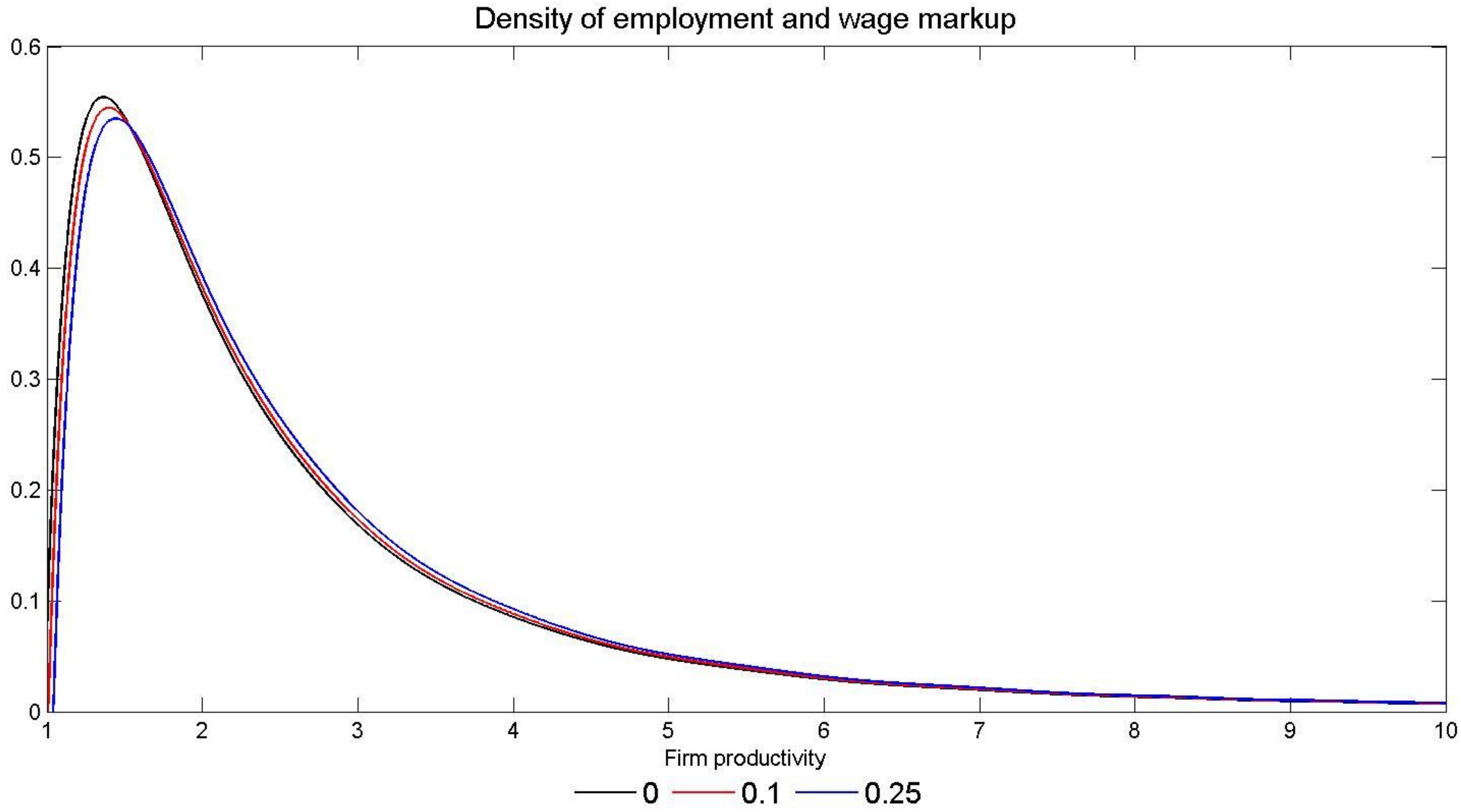
  - Wage bill

$$V_t^J(p) = W_t(p)(1 + r_k^w) + \beta E_t\left((1-\delta)\left(\frac{\Phi_t^N v_t(p)}{VAC_t} \int_p^{\overline{p}} \dots\right)\right)$$

Effect of vacancy cost markup



Effect of wage markup



Effect of working capital on new matches

	vacancy markup	wage markup	both
Total	98.1%	95.4%	94.2%
Job to Job	95.0%	88.2%	85.4%
From Unempl	98.6%	96.5%	95.6%

Directions for future research

- For this model we need to
  - bring the model closer to the data
  - consider implications in the GE
  - study dynamics (IRFs) of variables and distributions
- Thanks to flexible setup the model can be easily expanded
  - adding other frictions (borrowing constraints)
  - adding capital
  - endogenizing search intensity by job seekers
- Solution method can be used for other models

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Thank you for your attention!

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What can we calibrate the model to?

- Distribution of employment across firms
- Distribution of firm size
- Distribution of wages
- Ratio of biggest and smallest firm size
- Ratio of biggest and smallest firm product
- Scale of OJS