

INSTITUTE FOR STRUCTURAL RESEARCH

**Assessing Effects of Joining Common Currency Area with  
Large-Scale DSGE model: A Case of Poland**

- preliminary version -



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# Assessing Effects of Joining Common Currency Area with Large-Scale DSGE model: A Case of Poland

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## Abstract

In this paper we present a large scale dynamic stochastic general equilibrium model, in order to analyze and simulate effects of Euro introduction in Poland. Presented framework is based on a two-country open economy model, where foreign acts as the Eurozone, and home as a candidate country. We have implemented various types of structural frictions in the open economy block, that generate empirically observable deviations from purchasing power parity rule. We consider such mechanisms as a division between tradables and nontradables, endogenous pricing-to-market with transportation costs, Balassa-Samuelson effect and non-homothetic production functions. Moreover model contains market imperfections accounting for deviations from UIP, therein especially important features like incomplete capital market with endogenous risk premium.

Apart from open economy block, model encapsulates numerous labor market frictions. The construction of labor market module is based on search model with endogenous choice in intensive and extensive labor supply supplemented by imperfect unemployment insurance scheme. We propose a production structure (horizontal and vertical) with CRESH technology in production sectors, which allows us to calibrate structural parameters directly on sectoral national accounts. Investments in the production process are restricted by time-to-build mechanism.

Central bank policy block provides framework for testing monetary policy in both fixed and floating exchange rate regime. The pricing mechanism is characterized by nominal frictions. We proposed merged n-period, finite horizon price contracts with Bertrand competition model including changes in the number of firms in business cycle.

Model investigates the effects of shocks directly connected with introducing Euro currency in short run including real transaction costs shock, price level shock and interest rates shock. Besides short-term effects we assess long-term impacts associated with real appreciation pressure.

**Keywords:** DSGE models, common currency area, open economy models, labor market search, large scale models

**JEL Classification Numbers:** E32, E60, F40, F43,

# 1 Introduction

This paper presents the large scale dynamic stochastic general equilibrium model (DSGE) for Polish economy and euro zone - EUROMOD. We describe the main building blocks of the model economy, solution method, calibration procedure and main results. Next, we provide the dynamic properties of the model - impulse responses, presenting the role of economic mechanisms implemented and the derivation of novel price contract.

The objective of this paper is twofold: to serve as a tool for policy analysis and to prepare the theoretical structure for forecasting key macro variables on a medium-term horizon. The latter project is still being developed. As a tool for policy analysis, model investigates quantitative effects of joining the common currency area for Poland. It allows for understanding not only the first-round effects of different shocks, but also the second round effects as it is a general equilibrium model with flow stock consistency. Moreover, deep parameters are, in principle, invariant to policy changes. Therefore, policy analysis is robust to policy shocks.

The EUROMOD is meant to characterize the main features of the Polish economy, a small open economy that exchange the most of its foreign trade with euro zone. The model is a large scale DSGE including over 1600 variables, some added for calibration purposes. Implemented mechanisms may be divided into four basic blocks: open economy specification, labor market and real of the economy structure, monetary sphere and long run implications mechanisms. In the open economy block we consider various mechanisms generating deviations from purchasing power parity including a division between tradables and nontradables, endogenous pricing-to-market with transportation costs, Balassa-Samuelson effect and non-homothetic production functions. Moreover model contains market imperfections accounting for deviations from UIP, therein especially important features like incomplete capital market with endogenous risk premium. On the real side of the economy we model the labor market extending the search mechanism. Household members choose between three states on the labor market: employment, unemployment and inactivity. However, we treat the activity vs. non activity choice as a quasi endogenous one. We supplement labor search structure with imperfect risk sharing mechanism, augmenting impulse responses of labor aggregates. We also control unemployed according to the length of unemployment in order to simulate unemployment benefits scheme properly. Investment are restricted by the time-to-build and ex post rigidities mechanism. In order to calibrate the model economy directly on the sectoral national accounts we model the complex (vertical and horizontal) production structure including five production stages. We use CRESH technology production, being a generalization of CES, in order to model nonunit elasticity between production inputs. At stage II, where firms act as price setters we investigate the Bertrand competition with finite number of firms structure. It allows us to merge the research and development sector with general production structure and as a result to implement endogenous firm creation mechanism. At the nominal side of the model we propose a novel price contract. We have merged the two common types of contracts, namely Calvo (1983) and Taylor (1980) and settled it in Bertrand competition. Our proposal is an extension of Murchison *et. al.*(2004) and Dotsey, King and Wolman (1999). The probability of setting price optimally is time dependent as in Taylor approach, however for some firms price contract may lasts infinitely long as in Calvo. Prices are set in Bertrand competition structure in which firms take into the account their impact on the price aggregate. Model incorporates money through *cash in advance* constraint. We

model the competitive banking sector and central bank conducting monetary policy in both fixed and floating exchange rate regime. We also investigate the long run properties of the model that are useful in measuring the effects of joining common currency area. The fundamental mechanism is currency appreciation channel. In EUROMOD it has three sources: Samuelson-Balassa effect resulting from different technology growth rates in sector, nonhomotetic production function in final consumption goods sector and home bias shock. As it occurs (see Appendix) the latter is necessary for the model to replicate strong appreciation of Polish currency trend between 1996-2006.

Implementing common currency in the model economy means at first switching exchange rate regime from floating to fixed one and then equalizing the interbanking interest rate. The latter is regulated by the central bank. In case of common currency area it is equal for both economies. Then, we investigate the four effects related to switching to Euro currency: reduction of transaction costs and convergence of interest rates on deposits and loans - a real reallocation shocks, price shock - shock to probability of setting price optimally and the long run impact of real appreciation channel. Our main result is that Polish economy will benefit in the long term from the Euro adoption mainly through transaction costs reduction and interest rates convergence. However the scale of benefit measured in GDP level compared to no entrance scenario will not exceed respectively 1.65 and 0.45 percent in the long run.

We also study the dynamic properties of the model by analyzing the impulse-responses functions to different shocks. We investigate the role of implemented mechanisms for shock transmission and the dynamics of the macroeconomic variables. This paper is organized as follows. The next section presents the general structure of the model including optimization problems of agents. Then in Section 2 we describe the calibration procedure. Section 3 discussed our main results. In Appendix A we provide dynamic properties of the model in both short and long run. Finally Appendix B includes derivation of optimal price in proposed novel price contract.

## 2 The model economy

### 2.1 Notation comments

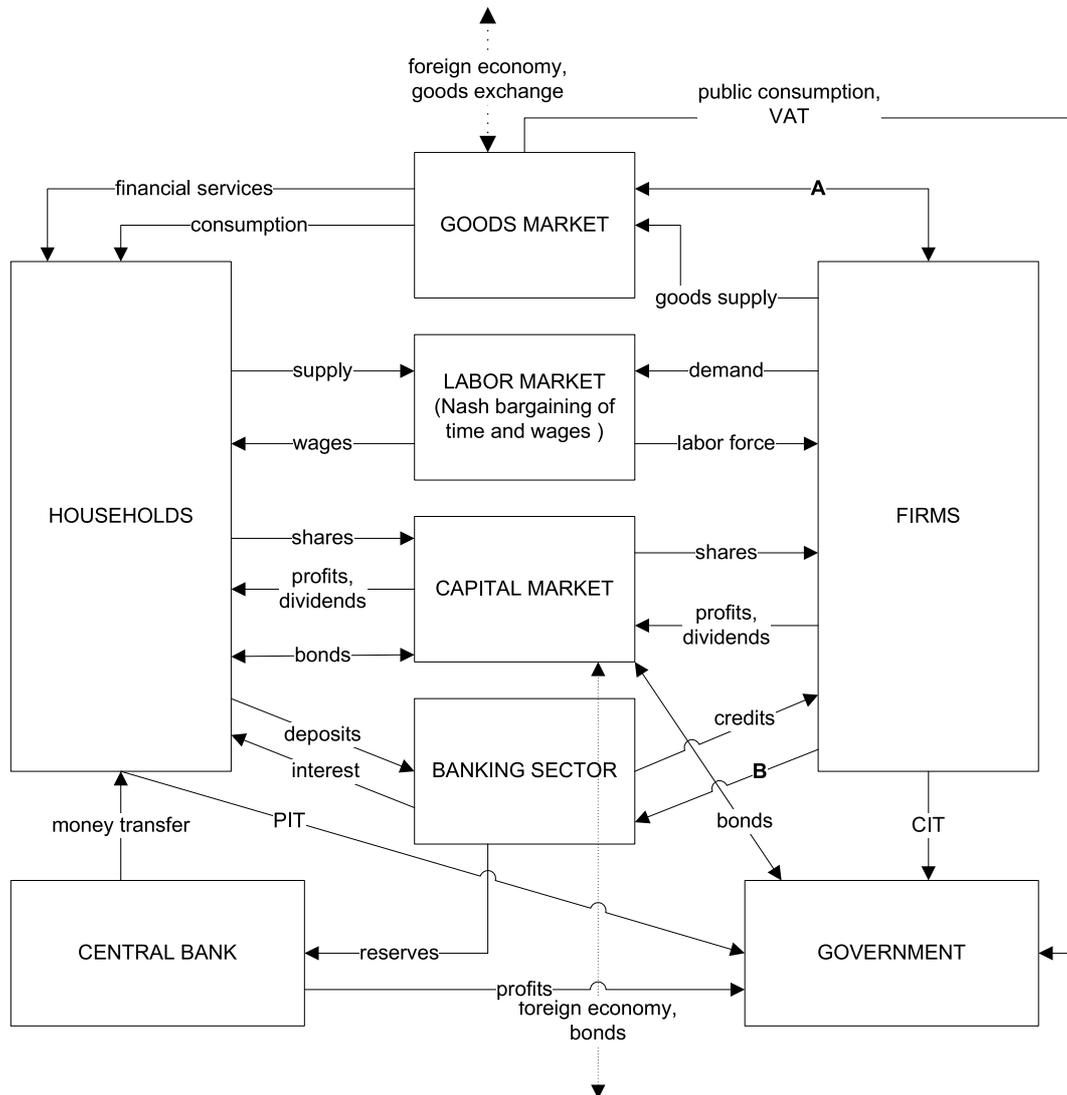
This is an open economy, two country, large scale dynamic stochastic general equilibrium model. Model economies are symmetric and described by the same mathematical objects. However, they differ in parametrization, scale or shares of certain sectors. We use the following notation. First of all, index  $t$  denotes variable is dependent on shocks realization including these in period  $t$ . Hence, information set at time  $t$  includes the realization of all shocks (home and foreign) that have occurred before  $t + 1$  period. Secondly, we index each variable and parameter with  $c \in \{H, F\}$ , where  $H$  denotes home economy,  $F$  denotes foreign economy. For variables directly determined by trade or capital relations (for instance for goods produced in  $H$  country and sold in  $F$ ) we use double indexation according to the scheme:  $X_t^{cf}$  assuming for any  $c, f \in \{H, F\}$  the first index ( $c$ ) denotes country of origin the and the second index ( $f$ ) destination country. Finally all variables (including prices) are expressed in real units, namely in ratio to respective *numeraire* such as total population (in case of variables expressed in the number of people) or the price of the reference goods (for variables being relative prices). Each country has its own set of reference units, constrained by the full formal symmetry, i.e. all variables being *numeraire*

differ between two countries only with index  $c \in \{H, F\}$ . Prices in goods and capital exchange are switched by real exchange rates.

## 2.2 General structure of the model

Model contains of eight main blocks: (1) Households (2) Production block (3) Banking sector (4) R&D sector (5) Labor market (6) Capital market (7) Government (8) Central Bank. The relations between the segments are summarized in the Figure 1:

**Figure 1: Basic segments of the model**

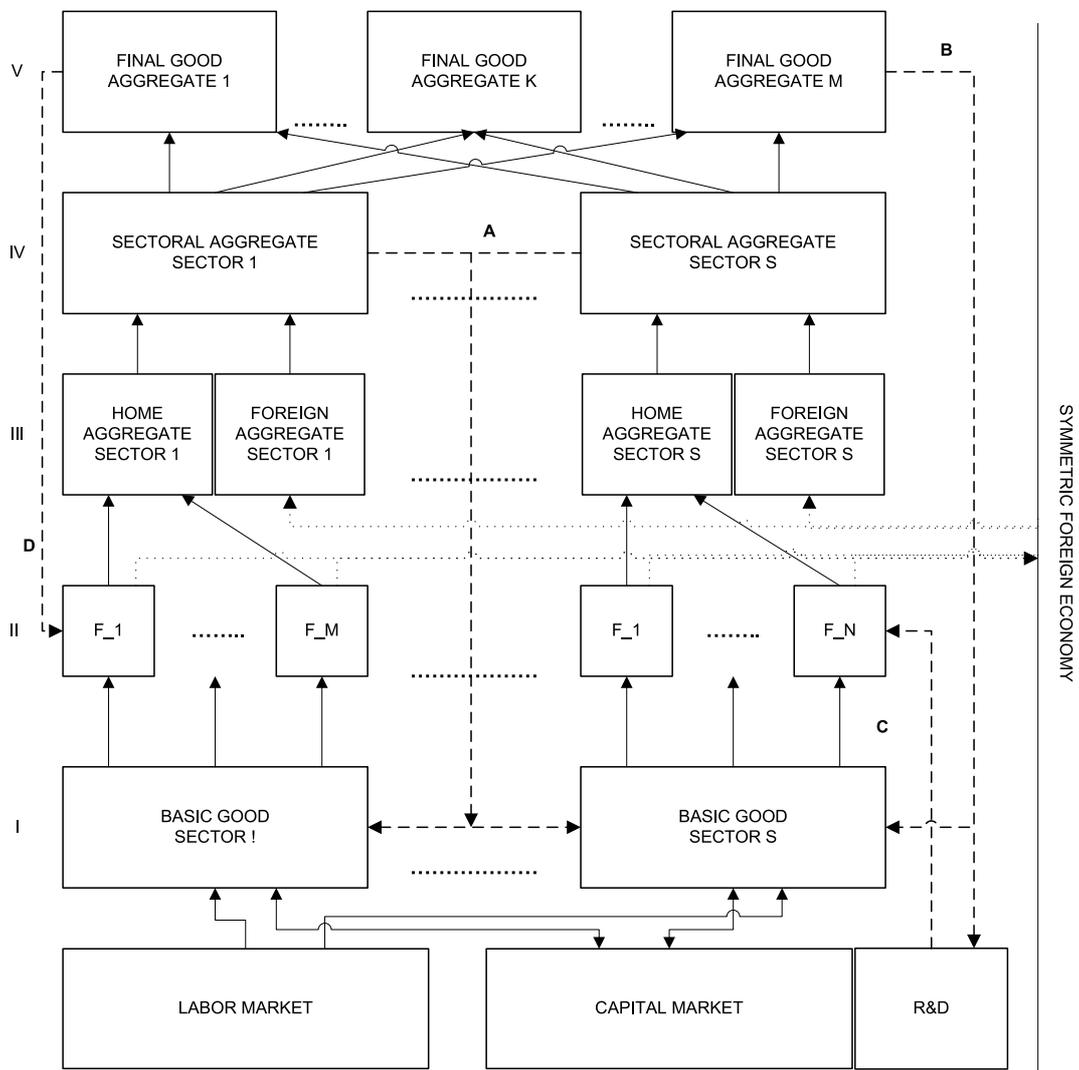


Note: A- intermediate, distribution, investment and financial goods B- financial services

Additional explanation is required regarding firms block. Firms not only produce output, but also exchange goods and services with foreign economy and generate the significant part of nominal and real frictions in the model. Detailed production structure is presented in Figure 2. There are few stages in production process. At the first stage

a basic, homogenous good is produced in all of the sectors. Next, product is being differentiated by the finite number of monopolistic firms (Bertrand competition), however each of them sets prices for both: home and foreign markets. Prices for both markets are rigid and distribution costs are also charged. Firms at second stage are also exporters and pay transaction costs on the currency market. Differentiated goods for home and foreign market from each sector are aggregated in one aggregated good contained of home components at third stage. Thereafter, firms at stage four taking aggregates bought home and foreign as inputs are producing one, international aggregated sectoral good. Hence, they are also importers. Their product is used to produce aggregated final goods at fifth stage and also as intermediate goods at first stage.

**Figure 2: Structure of the production process.**



Note: A-intermediate goods flow, B-investment goods flow, C-firm creation, D-financial and distribution goods flow

## 2.3 Households

We analyze the representative household. Since the labor market is non-walrasian each member of the household may be in one of three possible states on the labor market: employment, non employment and inactivity. Thus, in each period household contains of  $N_t^c$  employed,  $NE_t^c$  non employed and  $NA_t^c$  inactive members. This leads to identity:

$$L_t^c = N_t^c + NE_t^c + NA_t^c \quad (1)$$

where  $L_t^c$  denotes the total population. Each person is employed in one of the sectors  $s \in \mathcal{S}$ . The number of employed in sector  $s$  is indicated by  $N_t^{cs}$ . We assume people staying out of the labor market are inactive by definition. This group includes old age and disability pensioners  $NA_{tR}^c$  and long-term inactive people in working age (15-64). Long term inactive are not the only group of inactive in the model. We assume non employed, at the same time not being permanent long-term inactive, may be temporary inactive,  $NA_{tN}^c$ , or unemployed  $U_t^c$ . Additionally, we distinguish non employed according to the number of periods for that they are out of employment, denoting for  $\tau \in \mathcal{T}$  by  $NE_t^{c\tau}$  the number of individuals, who have not been employed for  $\tau$  periods. Thanks to these distinction we are able to control for instance the short term and long term unemployment rate, but also to model the discouragement effect. Based on above, the following identities must be held:

$$\begin{aligned} N_t^c &= \sum_{s \in \mathcal{S}} N_t^{cs} & NE_t^c &= \sum_{\tau \in \mathcal{T}} NE_t^{c\tau} \\ NA_t^c &= NA_{tW}^c + NA_{tR}^c & U_t^c &= \sum_{\tau \in \mathcal{T}} U_t^{c\tau} \\ NE_t^c &= NA_{tN}^c + U_t^c & NA_{tN}^c &= \sum_{\tau \in \mathcal{T}} NA_{tN}^{c\tau} \end{aligned} \quad (2)$$

Similarly to Givens (2007) we impose imperfect risk sharing mechanism on the labor market. To do so, we assume employed members of the household channel to non-employed and unemployed a part of their income called base consumption. However opposite to Givens (2007) we assume imperfect risk sharing mechanism results from disutility of household from the base consumption transfer, rather than from lowering the consumption due to social insurance contributions.

### 2.3.1 Employed

Household negotiates with firms contracts determining real wage  $W_t^{cs}$  for the effective unit of labor in sector  $s$  and the number of hours worked  $h_{tN}^{cs}$ . As a result, real labor income of the employed individual is  $W_t^{cs}h_{tN}^{cs}$ . Details of the negotiation process are included in "Labor market" section. Consumption of the employed in sector  $s$  household member at time  $t$  is given by:

$$C_{tN}^{cs} = C_{tN}^{cB} + (1 - \tau_t^{cL})h_{tN}^{cs} \times W_t^{cs}, \quad (3)$$

where  $h_{tN}^{cs}$ , denotes hours worked in sector  $s$ ,  $W_t^{cs}$  is real gross wage for unit of labor,  $\tau_t^{cL}$  is tax rate. Hence, leisure of employed member is equal to:

$$l_{tN}^{cs} = 1 - h_{tN}^{cs}. \quad (4)$$

Household sets extensive labor supply,  $N_t^{cs}$  (extensive margin), taking destruction rate, exogenous retirement process and the number of unemployed, who found a job in a given period into consideration. Hence evolution of labor supply is:

$$N_t^{cs} = (1 - \delta_{NA}^c) \times \left( (1 - \delta_N^{cs}) N_{t-1}^{cs} + v_{t-1}^{cs} \sum_{\tau \in \mathcal{T}} (1 - (1 - \Phi_{tN}^{cs})^{(\iota_{t-1}^{c\tau})^{e_U^c}}) U_{t-1}^{c\tau} \right). \quad (5)$$

where  $\delta_N^{cs}$  is exogenous separation rate in sector  $s$ , that may differ across countries and sectors.  $\delta_{NA}^c$  is the percentage of employed, unemployed and short term inactive who become long term inactive.  $\Phi_{tN}^{cs}$  is the job finding probability in sector  $s$  by the individual, who sent single job offer.  $\iota_{t-1}^{c\tau}$  is the number of sent offers by non employed for  $\tau$  periods in previous period.  $e_U^c$  is parameter generating decreasing productivity of job seeking process.  $v_{t-1}^{cs} \geq 0$  such that  $\sum_{s \in \mathcal{S}} v_{t-1}^{cs} = 1$  is the part of offers sent to sector  $s$ .

### 2.3.2 Unemployed and short term inactive

One can become inactive in two ways: by losing a job or through the flow from long term inactive part of population. There are also flows in opposite directions, non employed are finding a job or become permanently inactive. Moreover, there is a flow of young people to the labor market and a part of pensioners is passing away, in order to keep the population constant. Technically, the number of non employed for one period is given by:

$$NE_t^{c1} = (1 - \delta_{NA}^c) \times \sum_{s \in \mathcal{S}} (1 - \delta_N^{cs}) N_{t-1}^{cs} + \delta_R \times NA_{t-1R}^c \quad (6)$$

for these non employed for  $\tau \in \{2, \dots, T-1\}$  periods it is equal to:

$$NE_t^{c\tau} = (1 - \delta_{NA}^c) \times (NE_{t-1}^{c\tau-1} - \sum_{s \in \mathcal{S}} (1 - (1 - \Phi_{tN}^{cs})^{(\iota_{t-1}^{c\tau-1})^{e_U^c}}) \times v_{t-1}^{cs} NE_{t-1}^{c\tau-1}) \quad (7)$$

finally, the dynamics of the last cohort of non employed is:

$$NE_t^{cT} = (1 - \delta_{NA}^c) \sum_{\tau \in \{T-1, T\}} \left( NE_{t-1}^{c\tau} - \sum_{s \in \mathcal{S}} (1 - (1 - \Phi_{tN}^{cs})^{(\iota_{t-1}^{c\tau})^{e_U^c}}) \times v_{t-1}^{cs} NE_{t-1}^{c\tau} \right) + \delta_D \times NA_{t-1W}^c \quad (8)$$

where  $\delta_R \times NA_{t-1R}^c$  indicates the number of new labor market participants, equal to the number of pensioners, who passed away at time  $t$  (to keep the stationary population).  $\delta_D \times NA_{t-1W}^c$  denotes the number of long-term inactive, who become active at time  $t$ , augmenting the stock of non employed for at least  $T$  periods.

Only these, who actively seek for the job, among non employed, are unemployed. Between  $t-1$  and  $t$  job offers of non employed are sent randomly according to the Poisson process with intensity  $\iota_t^{c\tau}$ , at time  $t$  the number of these, who did not sent any offer (i.e. were inactive) is equal to:

$$NE_{tN}^{c\tau} = e^{-l_t^{c\tau}} \times NE_t^{c\tau} \quad (9)$$

as a result the quantity of unemployed, not having a job for  $\tau$  periods is:

$$U_t^{c\tau} = (1 - e^{-l_t^{c\tau}}) \times NE_t^{c\tau} \quad (10)$$

We assume unemployed, being out of job no longer than boundary value of  $\tau_U^c$  periods are entitled to unemployment benefit amounting to  $T_{tU}^c$ . The rest of unemployed and also non employed are not allowed to get the benefit. Job seeking is costly. The cost is expressed in consumption  $\zeta^c \times l_t^{c\tau}$ , hence non employed consume the base consumption plus unemployment benefit less job seeking costs:

$$C_{tNE}^{c\tau} = (C_{tNE}^{cB} - \zeta^c \times l_t^{c\tau}) + T_{tU}^c \times \frac{U_t^{c\tau}}{NE_t^{c\tau}}, \quad \tau \leq \tau_U^c \quad (11)$$

$$C_{tNE}^{c\tau} = (C_{tNE}^{cB} - \zeta^c \times l_t^{c\tau}), \quad \tau > \tau_U^c \quad (12)$$

We assume inactive part of population does not receive any benefits. However, job seeking does not imply decreasing leisure:

$$l_{tNE}^c = 1 \quad (13)$$

but at the same time results in losing a portion of consumption, proportionally to the number of sent offers.

### 2.3.3 Long term inactive population

The closing part of household members in the model is the long term inactive individuals group. We model a percentage of them  $\psi_R^c$  as old age and disability pensioners.

$$NA_{tR}^c = \psi_R^c NA_t^c \quad NA_{tW}^c = (1 - \psi_R^c) NA_t^c \quad (14)$$

We assume dynamics of permanent inactivity is exogenous for households, generated by the parameter  $\delta_{NA}^c$ . At the same time inactive consume the whole leisure, no matter whether they are pensioners or not. Pensioners, being by definition out of the labor market, finance their consumption  $C_{tR}^c$  from governmental transfers equal to  $T_{tNA}^c$  (per capita). Although, the consumption of the rest of inactive,  $C_{tNA}^c$ , is financed by the other household members through the base consumption. We assume the latter to be equal to the base consumption of the short term inactive or unemployed (non employed). Hence, we have formally:

$$C_{tR}^c = T_{tR}^c \quad C_{tNA}^c = C_{tNE}^{cB} \quad (15)$$

## 2.4 Utility

Household in each country  $c \in \{H, F\}$ , maximizes expected value of long live utility from consumption and leisure  $\mathcal{U}_0^c$ , discounted for the period  $t = 0$ :

$$\mathcal{U}_0^c = \sum_{t=0}^{\infty} \frac{(\beta_U)^t}{L_t^c} \times u_t^c, \quad (16)$$

where  $u_t^c$  is momentary utility derived at time  $t$  from leisure, goods and services consumption.  $L_t^c$  denotes the total number of individuals in household,  $\beta_U$  is a discount factor. Total momentary utility includes the utilities of respective types of individuals decreased by the disutility implied by imperfect risk sharing mechanism. The latter is the larger, the greater is the difference between base consumption of employed and non employed:

$$\begin{aligned} u_t^c = & \sum_{s \in \mathcal{S}} N_t^{cs} v_t^c(C_{tN}^{cs}, l_{tN}^{cs}) + \sum_{\tau \in \mathcal{T}} N E_t^{c\tau} v_t^c(C_{tNE}^{c\tau}, l_{tNE}^{c\tau}) + N A_{tR}^c v_t^c(C_{tR}^c, l_{tR}^c) + \\ & + N A_{tW}^c v_t^c(C_{tNA}^c, l_{tNA}^c) - \frac{\theta_B}{2} (C_{tN}^{cB} - C_{tNE}^{cB})^2. \end{aligned} \quad (17)$$

We assume, as common, momentary utility of each member  $v_t^c$  is a CRRA class function:

$$v_t^c(C_t, l_t) = \frac{1}{1 - \sigma_U} \times \left[ C_t \times (l_t)^{\omega_U} \right]^{1 - \sigma_U}. \quad (18)$$

Parameter  $\omega_U > 0$  measures consumption-leisure preferences, whereas  $\sigma_U > 0$  is the inverse of intertemporal elasticity of substitution of consumption. Parameter  $\theta_B$  controls the scale of risk sharing on the labor market between household members. For  $\theta_B = 0$  we have a perfect risk sharing mechanism as the base consumption will be chosen to equalize the marginal utilities from consumption of employed and unemployed, despite the differences in income. Then, for  $\theta_B \rightarrow \infty$  we have no risk sharing, the base consumption of employed and non employed is equal. Deviation from perfect risk sharing assumption is important due to labor market dynamics. Perfect risk sharing implies a number of dynamic properties being inconsistent with empirical data, for instance low unemployment volatility or similar level of consumption for employed and unemployed causing poor incentives for unemployed to look for the job.

## 2.5 Budget constraint

Representative household in  $c \in \{H, F\}$  country maximizes expected, discounted utility from consumption and leisure subject to budget constraint:

$$EXP_t^c = REV_t^c. \quad (19)$$

where  $EXP_t^c$  and  $REV_t^c$  indicate respectively expenditures and incomes at time  $t$ . Total households expenditures,  $EXP_t^c$ , consist of consumption expenditures  $P_t^c C_{tE}^c$  and the expenditures for asset holdings change: money holdings  $M_t^c$ , shares in firms  $d_t^c$  of  $V_t^c$  value each, domestic bonds  $B_t^{cc}$  and foreign bonds  $B_t^{cf}$ , whereas  $f = \{H, F\} - c$ . Money holdings include cash  $Q_t^c$  and bank deposits  $D_t^c$ , however time structure implies, division of money holdings between cash and deposits applies to its stock from previous period normalized by inflation,  $\pi_t^c$ , implying:

$$D_t^c = \frac{M_{t-1}^c}{\pi_t^c} - Q_t^c \quad (20)$$

We assume governments issue zero-coupon bonds with the nominal, risk-free rate of return  $R_t^{cB}$  and  $R_t^{fB}$ . Variables  $B_t^{cc}$  and  $B_t^{cf}$  are real stock of bonds held by household in country  $c$  issued respectively in home and foreign country, expressed in units of consumption goods in country  $c$ . Home and foreign bond markets are imperfectly integrated. Therefore in order to operate on the foreign bond market one has to include risk premium  $\phi_{tRP}^c$  related to debt to output,  $Y_t^c$ , ratio:

$$\phi_{tRP}^c = \exp\left[-\phi \times \frac{B_t^{cf}}{Y_t^c}\right]. \quad (21)$$

Imposing risk premium weakens the relation between home and foreign interest rates resulting from uncovered interest rate parity (UIP). Thus, this relation is dependent additionally on net foreign debt. Household allocates the income in: base consumption channeled to employed  $C_{tN}^{cB}$ , inactive  $C_{tNA}^{cB}$ , non employed  $C_{tNE}^{cB}$  and the change of assets holdings - money  $M_t^c$ , shares  $d_t^c$  and bonds  $B_t^{cc}$ ;  $B_t^{cf}$ . Technically we have:

$$\Delta M_t^c = M_t^c - Q_t^c - R_t^{cD} \times D_t^c \quad \Delta d_t^c = d_t^c - d_{t-1}^c \quad (22)$$

$$\Delta B_t^{cf} = \frac{B_t^{cf}}{R_t^{fB} \phi_{tRP}^c} - \frac{B_{t-1}^{cf}}{\pi_t^f} \times \frac{q_{t-1}^c}{q_t^f} \quad \Delta B_t^{cc} = \frac{B_t^{cc}}{R_t^{cB}} - \frac{B_{t-1}^{cc}}{\pi_t^c} \quad (23)$$

where  $R_t^{cD} = 1 + r_t^{cD}$  denotes rate of return on deposits. The relative price between home and foreign assets is determined by the real exchange rate  $q_t^f$  between  $c$  and  $f$  country. Real exchange rate states the price of unit of home consumption good expressed in units of foreign consumption goods. Summing up, total income side of the household may be written as:

$$EXP_t^c = P_t^c C_t^{cB} + \Delta B_t^{cc} + \Delta B_t^{cf} + V_t^c \Delta d_t^c + \Delta M_t^c \quad (24)$$

where  $C_t^{cB}$  indicates total base consumption equal the sum of particular households members base consumptions:

$$L_t^c C_t^{cB} = \sum_{s \in \mathcal{S}} N_t^{cs} C_{tN}^{cs} + \sum_{\tau \in \mathcal{T}} N E_t^{c\tau} C_{tNE}^{cB} + N A_{tW}^c C_{tNE}^{cB} \quad (25)$$

Notice, consumption complied in budget constraint differs from the total consumption of household,  $C_t^c$ , equal to:

$$L_t^c C_t^c = \sum_{s \in \mathcal{S}} N_t^{cs} C_{tN}^{cs} + \sum_{\tau \in \mathcal{T}} N E_t^{c\tau} C_{tNE}^{c\tau} + N A_{tW}^c C_{tNA}^c + N A_{tR}^c C_{tR}^c \quad (26)$$

The reason is the fact that within household particular members do not decide about capital engagement. These decisions are made collectively together with the ones about the base consumption for employed and non employed. On the right hand side of budget

constraint in  $c \in \{H, F\}$  there are revenues,  $REV_t^c$ , containing dividends from aggregated after tax profits  $d_{t-1}^c \times (1 - \tau_t^{cK})\Pi_t^{cA}$  increased by lump sum transfers from government,  $T_t^c$ .

$$REV_t^c = d_{t-1}^c \times (1 - \tau_t^{cK})\Pi_t^{cA} + T_t^c \quad (27)$$

Labor income, old age and disability pensions are excluded from the revenues since due to imperfect risk sharing they directly augment the consumption of employed, unemployed and pensioners. We assume the nominal value of market goods purchases is restricted by the stock of liquid assets held by household in a given period (*cash-in-advance* constraint):

$$L_t^c P_t C_t^c = \left[ \mu_M^c (Q_t^c)^{\varepsilon_M^c} + \nu_M^c (D_t^c)^{\varepsilon_M^c} \right]^{\frac{1}{\varepsilon_M^c}}, \quad (28)$$

where  $C_t^c$  is defined above, average consumption per household member. Taking  $\varepsilon_M^c < 1$  implies imperfect substitution between cash and bank deposits,  $\mu_M^c$  and  $\nu_M^c$  measure the importance of respectively cash and deposits in market transactions.

## 2.6 Firms

### 2.6.1 Stage I - basic, homogenous good production

#### Profit of basic good producer

In each country  $c \in \{H, F\}$  there is a set of production sectors, denoted by  $\mathcal{S}$ . In each sector  $s \in \mathcal{S}$  there operates perfectly competitive firm, producing basic, homogenous, sectoral good,  $Y_{tP}^{cs}$ , sold at price,  $P_{tP}^{cs}$ . We assume it is produced using capital, labor and fabric, intermediate goods. Each producer maximizes profit stream discounted for time  $t = 0$ :

$$\max E_0 \Pi_{0A}^{cs}, \quad \Pi_{0A}^{cs} = \sum_{t=0}^{\infty} \Lambda_t^c \Pi_t^{cs}, \quad (29)$$

Momentary profit,  $\Pi_t^{cs}$ , is equal to revenues from selling goods,  $P_{tP}^{cs} Y_{tP}^{cs}$ , less current operational and financial costs. Part,  $\phi_{LD}$ , of the operational costs,  $O_t^{cs}$ , is financed through short term loan  $LD_t^{cs} = \phi_{LD} \times O_t^{cs}$ , charged with banking interest rate,  $R_t^{cL} = 1 + r_t^{cL}$ . Loan is taken out at the beginning of each period and repayed at the end. As a consequence, total costs of firms at stage one include: operational costs not financed through the loan,  $(1 - \phi_{LD}) \times O_t^{cs}$ , and financial costs of loan with interests,  $R_t^{cL} \times \phi_{LD} \times O_t^{cs}$ . Thus:

$$\Pi_t^{cs} = P_{tP}^{cs} Y_{tP}^{cs} - (1 - \phi_{LD}) \times O_t^{cs} - R_t^{cL} \times \phi_{LD} \times O_t^{cs} \quad (30)$$

Current operational costs,  $O_t^{cs}$ , include: wages expenditures,  $W_t^{cs} h_{tN}^{cs} N_t^{cs}$ , expenditure for materials required to produce intermediate goods used in production process,  $\sum_{r \in \mathcal{S}} P_t^{cr} Y_{tr}^{cs}$ , and the costs of recruiting new employees  $P_t^{cC} J_{tC}^{cs}$ . Hence we get:

$$O_t^{cs} = W_t^{cs} h_{tN}^{cs} N_t^{cs} + \sum_{r \in \mathcal{S}} P_t^{cr} Y_{tr}^{cs} + P_t^{cI} I_t^{cs} + P_t^{cC} J_{tC}^{cs} \quad (31)$$

All costs, apart the wage ones, are expenditures for purchasing market goods and services: materials, investment or consumption.

### Production technology

Basic good producer,  $Y_{tP}^{cs}$ , operating in  $c$  country in sector  $s \in \mathcal{S}$  is using Cobb-Douglas type technology:

$$Y_{tP}^{cs} = A_t^{cs} (Z_t^{cs} N_t^{cs} h_{tN}^{cs})^{\alpha_L^{cs}} (Y_{tNL}^{cs})^{1-\alpha_L^{cs}} \quad (32)$$

whereas the level of unit labor productivity,  $Z_t^{cs}$ , increasing in line with technological progress, is exogenous and described by AR(1) process. Moreover,  $A_t^{cs}$  denotes temporary technological shock in  $s \in \mathcal{S}$  sector. The specification of  $A_t^{cs}$  i  $Z_t^c$  implies convergence process of the economy is exogenous. Growth rates of technology level may be different among sectors and countries. Implementing economic growth together with different laboriousness in sectors causes Samuelson-Balassa effect. As a result the higher laboriousness in sector, the faster is the technology growth. At the same time, complying differences among long term growth rates between sectors is augmenting Samuelson-Balassa effect, as the prices rise in sector with lower long term growth rate (for instance services), even if sectors are homogenous.

Apart from effective labor input,  $N_t^{cs} h_{tN}^{cs}$ , basic good production technology requires aggregate input,  $Y_{tNL}^{cs}$ , consisted of other production inputs i.e.: accumulated capital  $K_t^{cs}$  used with intensity,  $U_{tK}^{cs}$ , and intermediate goods  $X_t^{cs}$ . We assume aggregate input is given by homotetic, constant ratio of elasticity of substitution function - CRESH (see Hanoch (1977)), being a generalization of ,usual for business cycles models, CES aggregate. The idea behind using CRESH is to include richer substitution structure between production inputs. Technically,  $Y_{tNL}^{cs}$  aggregate is given in an implicit way:

$$0 = \frac{\alpha_K^{cs}}{1 - \rho_K^{cs}} \times \left[ \left( \frac{U_{tK}^{cs} K_t^{cs}}{Y_{tNL}^{cs}} \right)^{1-\rho_K^{cs}} - 1 \right] + \frac{\alpha_X^{cs}}{1 - \rho_X^{cs}} \times \left[ \left( \frac{X_t^{cs}}{Y_{tNL}^{cs}} \right)^{1-\rho_X^{cs}} - 1 \right] \quad (33)$$

Basic good production technology has CRS property and satisfies axioms of neoclassical production functions for any costs and values of parameters satisfying  $\alpha_K^{cs}, \alpha_X^{cs} > 0$  and  $\rho_K^{cs}, \rho_X^{cs} > 1$ .

### Intermediate goods production

In line with Iacoviello et. al (2007) and Christiano (1988) we assume basic good production in  $s$  sector requires using intermediate goods  $X_t^{cs}$ . Similar to basic goods production, we assume intermediate goods  $Y_{tX}^{cs}$  are produced with CRESH technology using aggregated sectoral goods produced by firms at stage III -  $r \in \mathcal{S}$ ,  $Y_{tr}^{cs}$ :

$$0 = \sum_{r \in \mathcal{S}} \frac{\beta_r^{cs}}{1 - \varrho_r^{cs}} \times \left[ \left( \frac{Y_{tr}^{cs}}{Y_{tX}^{cs}} \right)^{1-\varrho_r^{cs}} - 1 \right]. \quad (34)$$

Parameters  $\beta_r^{cs}$ ,  $r, s \in \mathcal{S}$  indicate relative shares of intermediate goods costs in  $r$  sector in total costs of intermediate goods in  $s$  sector. Moreover, parameters  $\varrho_r^{cs}$  determine elasticities of substitution between intermediate goods in certain sectors. We assume,  $X_t^{cs} = Y_{tX}^{cs}$ , hence firm does not accumulate inventories.

## Capital accumulation

In each sector capital accumulation is restricted by time-to-build mechanism integrated with ex post rigidities, in line with Edge 2000, Murchison *et al.* 2004). We assume complementarity between investment expenditures devoted to certain investment project in different time periods, which discourage firms to change the original investment plan. We model this feature assuming effective, i.e. the one increasing the level of capital, level of firm investment at time  $t$ ,  $I_t^{cs,E}$ , is CES aggregate comprising previous and current investment level:

$$I_t^{cs,E} = \left[ \sum_{j=0}^{\tau} ((\phi_j^{cs})^{\frac{1}{\theta_I^{cs}}} I_{t-j,t}^{cs})^{\frac{\theta_I^{cs}-1}{\theta_I^{cs}}} \right]^{\frac{\theta_I^{cs}}{\theta_I^{cs}-1}}. \quad (35)$$

where  $I_{t-j,t}^{cs}$  denotes expenditures at time  $t-j$  for investment project completed at time  $t$ . Parameter  $\theta_I^{cs}$  regulates the scale of complementarity between investment expenditures in certain time periods. As  $\theta_I^{cs} \rightarrow 0$ , investment expenditures are perfect complements and once scheduled the investment plan is fixed. In case  $\theta_I^{cs} \rightarrow \infty$  investment plans are perfect substitutes, that allows for revising them at any point in time. Parameters  $\phi_j^{cs}$  determine time structure of investment. Implementing *time-to-build* mechanism merged with *ex-post* rigidity helps to generate lagged reaction of investment to supply shocks. Effective level of investment,  $I_t^{cs,E}$ , which increases the capital used in production process differs from the total investment expenditures at time  $t$ . The latter is given by:

$$I_t^{cs} = \sum_{j=0}^{\tau} I_{t,t+j}^{cs}. \quad (36)$$

and thus, capital entering the books is accumulated according to:

$$K_{tM}^{cs} = (1 - \delta_{tK}^{cs})K_{t-1M}^{cs} + P_t^{cI} I_t^{cs} \quad (37)$$

and is different from the one which enters the production process,  $K_t^{cs}$ . Capital engaged in production process is equal to the sum of non depreciated effective capital from previous period increased by effective investment from period  $t$ :

$$K_t^{cs} = \left( 1 - \frac{\delta_{tK}^{cs}}{\sigma_K^c} \right) \times K_{t-1}^{cs} + (I_t^{cs,E})^{\sigma_K^c} \left( \frac{\bar{\delta}_K^{cs}}{\sigma_K^c} K_{t-1}^{cs} \right)^{1-\sigma_K^c}. \quad (38)$$

where parameter  $\sigma_K^c$  regulates the elasticity of production capital to current investment. Depreciation rate  $\delta_{tK}^{cs}$  depends on the intensity of capital use:

$$\delta_{tK}^{cs} = \widehat{\delta}_K^{cs} \left( \frac{X_t^{cs}/K_{t-1}^{cs}}{\bar{X}_I^{cs}/\bar{K}^{cs}} \right)^{\theta_K^{cs}} + \beta_K^{cs} \times \left( (U_{tK}^{cs})^{\phi_K^{cs}} - 1 \right). \quad (39)$$

parameters  $\theta_K^{cs}$ ,  $\beta_K^{cs}$  and  $\phi_K^{cs}$ , regulate the scope of capital use in steady state and its reaction to shocks.

## Workers seeking and recruiting costs

Basic goods producer opens vacancies  $V_t^{cs}$ , and looks for the workers to fill them. However at time  $t$  only these vacancies open at time  $t - 1$  are filled. Probability of filling vacancy  $\Psi_t^{cs}$  is exogenous for the firm. At the same time, job destruction and flow to inactive population of employees take place, according to the rates, respectively,  $\delta_N$  and  $\delta_N A$ . Consequently, producer's decision about production inputs is made subject to the evolution of employment equal to:

$$N_t^{cs} = (1 - \delta_N) \times (1 - \delta_{NA}) \times N_{t-1}^{cs} + \Psi_t^{cs} \times V_{t-1}^{cs} \quad (40)$$

Above equation is taken into consideration in negotiation process (see "Labor market" block). Moreover, opening vacancies and recruiting process are costly  $P_t^{cJ} J_{tC}^{cs}$ . We assume recruiting process requires purchasing  $J_{tC}^{cs}$  units of final consumption goods at price  $P_t^{cC}$ . The amount of these costs depends on the number of vacancies in previous period,  $V_{t-1}^{cs}$ , probability of filling vacancy,  $\Psi_{tN}^{cs}$  and the employment level  $N_t^{cs}$  in the following way:

$$J_{tC}^{cs} = \varpi_J^{cs} (\Psi_{tN}^{cs} V_{t-1}^{cs} - \varpi_J^c \times \bar{\Psi}_N^{cs} \bar{V}_N^{cs})^{\phi_J^c} \times (N_t^{cs})^{1-\phi_J^c}. \quad (41)$$

where  $\bar{\Psi}_N^{cs}$  and  $\bar{V}_N^{cs}$  denote steady state values. Factor  $(\Psi_{tN}^{cs} V_{t-1}^{cs} - \varpi_J^c \times \bar{\Psi}_N^{cs} \bar{V}_N^{cs})^{\phi_J^c}$  reflects the impact of necessity to find and train new workers on recruiting costs. Then, factor  $(N_t^{cs})^{1-\phi_J^c}$  according to the value of parameter  $\phi_J^c$  lets costs will be increasing (for  $\phi_J^c < 1$ ), stable (for  $\phi_J^c = 1$ ) or decreasing (for  $\phi_J^c > 1$ ) in line with the growth of total employment in the firm. Finally parameters  $\varpi_J^c$  and  $\varpi_J^{cs}$  allow for calibration of total spending for recruiting in economy and probability of filling vacancy in  $c$  country and  $s$  sector.

### 2.6.2 Stage II - basic good differentiation

#### Profit

There is a finite number of firms operating in Bertrand competition environment in each period  $t$  in every sector indexed with  $i$ . Each of them has a monopoly power resulting from being established by the household by purchasing unique product innovation from R&D sector. The number of firms varies and differs among sectors and country. For  $c \in \{H, F\}$  and  $s \in \mathcal{S}$  we denote it with  $F_{tN}^{cs}$  symbol. Each firm at this stage using basic good,  $Y_{tP}^{cs}$ , as input produces differentiated good,  $Y_t^{cs,i}$ , with the linear technology:

$$Y_t^{cs,i} = Y_{tP}^{cs,i}, \quad (42)$$

where  $Y_{tP}^{cs,i}$  is a demand for basic good from  $i$  firm. Hence the following identity must be held:

$$Y_{tP}^{cs} = \sum_i^{F_{tN}^{cs}} Y_t^{cs,i}, \quad (43)$$

Firm  $i$  maximizes discounted profit  $E_0 \Pi_{tA}^{cs,i}$ , given in recursive way:

$$\Pi_{tA}^{cs,i} = \Pi_t^{cs,i} + E_t \left\{ \Lambda_{t+1}^{cB} \Pi_{t+1,A}^{cs,i} \right\}, \quad (44)$$

where  $\Lambda_{t+1}^c = (1 - \delta_{t+1F}^{cs}) \frac{\lambda_{t+1}^c}{\lambda_t^c}$  indicates discount rate, whereas  $\delta_{t+1F}^{cs}$  denotes exogenous firm destruction probability.

### Home and foreign market price differentiation

Similar to Betts and Devereux (1996) we assume firms at stage II may charge different prices on home and foreign market, according to *pricing-to-market* mechanism. Domestic firm  $Y_t^{cs,i}$  sets home price for domestic market,  $P_t^{csc,i}$ , and foreign price,  $P_t^{csf,i}$ , for foreign market. Both prices are expressed in units of domestic consumption good. Profit maximization of  $i$  firm is subject to home and foreign demand functions, respectively  $Y_{tD}^{csc,i}$  and  $Y_{tD}^{csf,i}$ .

$$\begin{aligned} Y_{tD}^{csc,i} &= \frac{(P_t^{csc,i})^{-\epsilon^{csc}}}{\left[ \sum_{i=1}^{F_{tN}^{cs}} (P_t^{csc,i})^{1-\epsilon^{csc}} \right]^{\frac{-\epsilon^{csc}}{1-\epsilon^{csc}}}} \times \bar{Y}_t^{csc}, \\ Y_{tD}^{csf,i} &= \frac{(P_t^{csf,i})^{-\epsilon^{csf}}}{\left[ \sum_{i=1}^{F_{tN}^{cs}} (P_t^{csf,i})^{1-\epsilon^{csf}} \right]^{\frac{-\epsilon^{csf}}{1-\epsilon^{csf}}}} \times \bar{Y}_t^{csf}. \end{aligned} \quad (45)$$

As a result the following identity must be held:

$$Y_t^{cs,i} = Y_{tD}^{csc,i} + Y_{tD}^{csf,i}. \quad (46)$$

Domestic demand for differentiated goods produced in  $c$  country  $Y_{tD}^{csc,i}$ , and foreign demand for these goods,  $Y_{tD}^{csf,i}$ , are determined by the production function of firms at stage III, aggregating and producing homogenous sectoral good sold at given market. Setting price for  $r \in \{H, F\}$  market, firm  $i$  operating in  $c$  country in sector  $s$ , takes the total demand of aggregating firms ( $\bar{Y}_t^{csc}$ , or,  $\bar{Y}_t^{csf}$ ) into consideration. Moreover it includes its own influence on the price aggregate, taking the other firms prices as given (Bertrand competition). Hence her profit in each period,  $\Pi_t^{cs,i}$ , is given by:

$$\Pi_t^{cs,i} = (P_t^{csc,i} - \bar{P}_{tP}^{csc}) \times Y_{tD}^{csc,i} + (P_t^{csf,i} - \bar{P}_{tP}^{csf}) \times Y_{tD}^{csf,i}. \quad (47)$$

where  $\bar{P}_{tP}^{csr}$  is the price of basic good increased by the distribution costs. Distributions costs augment *pricing-to-market* mechanism, causing larger deviation from the law of one price at sectoral level. It holds even if the elasticities of substitution between home and foreign differentiated goods are equal. In other words, in line with Nevel and Rebelo (2000), Corsetti and Debola (2002) or Selaive and Tuesta (2003) we have imposed wedge between producer and consumer price, however in our specification, the wedge is levied on marginal cost of goods differentiating process:

$$\bar{P}_{tP}^{csr} = P_{tP}^{cs} + \kappa_t^{csr} \quad (48)$$

where distribution cost,  $\kappa_t^{csr}$  has the form:

$$\kappa_t^{csr} = P_t^{cD} \kappa^{csr} + \delta_{fr} P_t^{fD} \kappa^{csfr} q_t^{fc}, \quad (49)$$

however  $\delta_{fr}$  is Dirac delta symbol, taking value 1 in export case and 0 in domestic trade case. Parameter  $\kappa^{csr}$  regulates the number of home distribution good units sold at price  $P_t^{cD}$  required to transport the unit of domestic good in home country (for  $r = h$ ). Then  $\kappa^{csfr}$  denotes the number of additional foreign distribution good units sold at price  $P_t^{fD}$ , expressed in foreign goods units, required to transport exported good in foreign country.

## Price contracts

Pricing mechanism assumed in our model is an extension of Murchison *et. al.*(2004) and Dotsey, King and Wolman (1999). Proposed price contract is a generalization and amalgamation of two basic ideas dominating in the literature: Calvo (1983) contracts, in which time invariant price rigidity may last infinitely long and Taylor (1980) contracts, in which contracts last for specific period of time. The modification we are presenting increases the numerical complexity of the model, nevertheless contrary to standard Calvo and Taylor approaches allows for explaining a variety of empirical regularities, being the difficulty for models with less general approach to pricing mechanism. For instance Calvo pricing in standard model predicts the maximal response of inflation on the exogenous shock should take place in the first period after the shock (see Kiley 2002). As a consequence Philips curve estimations based on standard Calvo model imply unrealistic length of price contracts and the significant role of lagged inflation. Our structure of price contracts implies existence of lagged inflation in Philips curve even in case of lack of indexation mechanism (Dotsey 2002).

In each period  $t$  firm  $i$ , operating in  $c$  country and  $s$  sector may set optimal price for  $r \in \{H, F\}$  market,  $P_t^{csr,i}$ , or index the price from previous period according to the indexation rule,  $\Omega_t^{csr}$ . Firm setting optimal price at time  $t$  expects receiving in successive  $t+k$  periods signal,  $\xi_{t,t+k}^{csr,i}$  informing whether the price can be reoptimize or must be indexed. Hence, the firm faces the additional set of constraints in optimization problem:

$$\xi_{t,t+k}^{csr,i} (\bar{P}_{t+k}^{csr,i} \Omega_{t,t+k}^{cr} - \tilde{P}_t^{csr,i}) = 0, \quad (50)$$

where  $\xi_{t,t+k}^{csr,i}$  is a random variable equal 1 with probability  $\xi_k^{csr}$  and 0 with probability  $1 - \xi_k^{csr}$ . Variable  $\tilde{P}_t^{csr,i}$  denotes optimal real price for  $r$  market set by firm  $i$  operating in sector  $s$ , country  $c$  for period  $t$ . For  $\xi_{t,t+k}^{csr,i} = 1$  price is rigid, for  $\xi_{t,t+k}^{csr,i} = 0$  optimal price  $P_{t+k}^{csr,i}$  may be set. Nevertheless we assume distribution of  $\xi_{t,t+k}^{csr,i}$  is dependent on time that have passed since last indexation, namely on  $k$ . We assume, for  $1 \leq k < J$  periods, probability of  $\xi_{t,t+k}^{csr,i} = 1$  differs from probability of  $\xi_{t,t+k+1}^{csr,i} = 1$ , namely  $\xi_k^{csr} \neq \xi_{k+1}^{csr}$  (Taylor contracts part). For  $k > J$  we have  $\xi_{t,t+k}^{csr,i} = \xi_C^{csr}$ . (Calvo part). Once the price can not be reoptimized firm indexes current price according to the rule:

$$\Omega_t^{cc} = \frac{(\pi_{t-1}^c)^{\omega_\pi}}{\pi_t^c} \quad \Omega_t^{cf} = \frac{(\pi_{t-1}^c)^{\omega_\pi}}{\pi_t^c} \times q_t^f \times q_{t-1}^c \quad (51)$$

where  $\omega_\pi$  regulates the impact of past inflation, however in baseline version of the model it is equal to zero, meaning no past inflation indexation. Finally, the Lagrangian for firm at Stage II is given by:

$$L_0 = \sum_{t=0}^{\infty} \lambda_t \left[ \Pi_{tA}^{cs} - \Pi_t^{cs} - \beta E_t \Lambda_{t+1}^{cB} \Pi_{t+1A}^{cs} - \mu_t \left( \Pi_t^{cs} - \sum_{r \in \{H,F\}} (\tilde{P}_t^{csr} - \bar{P}_{tP}^{csr}) Y_{tD}^{csr} \right) - \sum_{r \in \{H,F\}} \eta_t^{csr} \left( Y_{tD}^{csr} - \left( \frac{\tilde{P}_t^{csr}}{P_t^{csr}} \right)^{-\epsilon} \bar{Y}_t^{csr} \right) - \sum_{r \in \{H,F\}} \varphi_t^{csr} \xi_t^{csr} \left( \tilde{P}_t^{csr} - \tilde{P}_{t-1}^{csr} \Omega_t^{cr} \right) \right]$$

After few pages of algebra, one can reduce the solution to several stochastic recursive equations and derive the optimal price in an implicit way (see Appendix).

### 2.6.3 Stage III - home and foreign sectoral aggregation

At Stage III in country  $c \in \{H, F\}$  certain domestic goods,  $Y_t^{cs,i}$ , and foreign goods  $Y_t^{fs,i}$ , where  $f \in \{H, F\} - \{c\}$ , are used in production of aggregated sectoral goods for home and foreign market, respectively  $Y_t^{csc}$  and  $Y_t^{fsc}$ . Both goods are produced with CES technology by perfectly competitive firms:

$$\bar{Y}_t^{csc} = \left[ \sum_{i=1}^{F_{tN}^{cs}} (Y_{tD}^{csc,i})^{\frac{\epsilon^{csc}-1}{\epsilon^{csc}}} \right]^{\frac{\epsilon^{csc}}{\epsilon^{csc}-1}}, \quad \bar{Y}_t^{fsc} = \left[ \sum_{i=1}^{F_{tN}^{fs}} (Y_{tD}^{fsc,i})^{\frac{\epsilon^{fsc}-1}{\epsilon^{fsc}}} \right]^{\frac{\epsilon^{fsc}}{\epsilon^{fsc}-1}}, \quad (52)$$

where  $Y_{tD}^{csc,i}$  i  $Y_{tD}^{fsc,i}$  denote, respectively, domestic demand for home differentiated good  $i$  and domestic demand for differentiated foreign good. Hence, cost minimization of production cost implies demand for goods  $Y_{tD}^{csc,i}$ ,  $Y_{tD}^{fsc,i}$ :

$$Y_{tD}^{csc,i} = \frac{(P_t^{csc,i})^{-\epsilon^{csc}}}{(P_t^{csc})^{-\epsilon^{csc}}} \times \bar{Y}_t^{csc}, \quad Y_{tD}^{fsc,i} = \frac{(P_t^{fsc,i})^{-\epsilon^{fsc}}}{(P_t^{fsc})^{-\epsilon^{fsc}}} \times \bar{Y}_t^{fsc}. \quad (53)$$

Substituting proper demand functions in production function of  $Y_t^{csc}$ ,  $Y_t^{fsc}$  goods, we get price indices  $P_t^{csc}$ ,  $P_t^{fsc}$ :

$$P_t^{csc} = \left[ \sum_{i=1}^{F_{tN}^{cs}} (P_t^{csc,i})^{1-\epsilon^{csc}} \right]^{\frac{1}{1-\epsilon^{csc}}}, \quad P_t^{fsc} = \left[ \sum_{i=1}^{F_{tN}^{fs}} (P_t^{fsc,i})^{1-\epsilon^{fsc}} \right]^{\frac{1}{1-\epsilon^{fsc}}}. \quad (54)$$

and as a result demand functions defined at stage II.

### 2.6.4 Stage IV - aggregated good production

At the next stage of production process in  $c \in \{H, F\}$  country aggregated sectoral good is produced,  $Y_t^{cs}$ . The price of the aggregated good is  $P_t^{cs}$  and is expressed in units of domestic consumption good. It is produced on perfectly competitive market using CES technology and taking domestic goods  $\bar{Y}_t^{csc}$  at price  $P_t^{csc}$  and foreign goods  $\bar{Y}_t^{fsc}$  at price  $P_t^{fsc}$  as inputs:

$$Y_t^{cs} = \left[ (\omega^{cs})^{\frac{1}{\epsilon^{cs}}} (\bar{Y}_t^{csc})^{\frac{\epsilon^{cs}-1}{\epsilon^{cs}}} + (1 - \omega^{cs})^{\frac{1}{\epsilon^{cs}}} (\bar{Y}_t^{fsc})^{\frac{\epsilon^{cs}-1}{\epsilon^{cs}}} \right]^{\epsilon^{cs}/(\epsilon^{cs}-1)}, \quad (55)$$

$$P_t^{cs} = \left[ \omega^{cs} (P_t^{csc})^{1-\epsilon^{cs}} + (1 - \omega^{cs}) (P_t^{fsc})^{1-\epsilon^{cs}} \right]^{1/(1-\epsilon^{cs})}.$$

Production cost minimization of the unit of  $Y_t^{cs}$  implies demand for goods  $\bar{Y}_t^{csc}$  and  $\bar{Y}_t^{fsc}$ :

$$\bar{Y}_t^{csc} = \omega^{cs} \left[ \frac{P_t^{cs}}{P_t^{csc}} \right]^{-\epsilon^{cs}} Y_t^{cs}, \quad \bar{Y}_t^{fsc} = (1 - \omega^{cs}) \left[ \frac{P_t^{cs}}{P_t^{fsc}} \right]^{-\epsilon^{cs}} Y_t^{cs}. \quad (56)$$

subject to which firms at stage III optimize profits.

### 2.6.5 Final goods aggregates

At the last stage of production process final goods are produced. Let the set  $\mathcal{T} = \{I, D, G, F\}$  indexes types of final goods in the model economy, excluding separate consumption good, that has been described individually. The elements of set are respectively: investment, distribution, public and financial goods. Aggregated final good  $k \in \mathcal{T}$ ,  $Y_t^{ck}$  is produced by perfectly competitive firms with CRESH technology using  $Y_{tr}^{ck}$ ,  $r \in \mathcal{S}$  sectoral goods as inputs:

$$0 = \sum_{r \in \mathcal{S}} \frac{\beta_r^{ck}}{1 - \varrho_r^{ck}} \times \left[ \left( \frac{Y_{tr}^{ck}}{Y_t^{ck}} \right)^{1 - \varrho_r^{ck}} - 1 \right], \quad (57)$$

whereas parameters  $\beta_r^{ck}$  for  $r \in \mathcal{S}$  denote shares of goods produced in  $r$  sector costs in total cost of production  $k \in \mathcal{T}$  goods. Parameters  $\varrho_r^{ck}$  determine elasticity of substitution between goods produced in certain sectors. Producers of  $k \in \mathcal{T}$  goods maximize profits (in equilibrium equal to zero):

$$\Pi_t^{ck} = P_t^{ck} Y_t^{ck} - \sum_{r \in \mathcal{S}} Y_{tr}^{ck} P_t^{cr}, \quad (58)$$

where  $P_t^{ck}$  is the price of aggregated goods  $k \in \mathcal{T}$ . We describe the aggregate final consumption good  $Y_t^{cC}$  separately. It is manufactured using certain sectoral goods  $Y_{tr}^{cC}$ ,  $r \in \mathcal{S}$  and following technology:

$$0 = \sum_{r \in \mathcal{S}} \frac{\alpha_{rC}^c}{1 - \delta_{rC}^c} \times \left[ \left( (Y_t^{cC})^{-\beta_{rC}^c} (Y_{tr}^{cC}) \right)^{1 - \delta_{rC}^c} - 1 \right]. \quad (59)$$

which is Direct Implicit Addilog (DIA) see Barnett 1981). For  $\beta_{rC}^c = 1$  DIA reduces to CRESH technology. Such technology allows to determine relative utility of households from consumption of goods from different sectors. Manufacturers of aggregated consumption good maximize profits, given by:

$$\Pi_t^{cC} = P_t^{cC} Y_t^{cC} - (1 + \tau_t^{cC}) \times \sum_{r \in \mathcal{S}} P_t^{cr} Y_{tr}^{cC} \quad (60)$$

$\tau_t^{cC}$  denotes the level of tax levied on consumption. We assume consumption goods producers set prices according to the *marginal cost pricing mechanism*, maximizing discounted for time  $t = 0$  profits stream:

$$E_0 \sum_{t=0}^{\infty} \Lambda_t^c \Pi_t^{cC}. \quad (61)$$

Due to pricing mechanism first order conditions are sufficient. They imply  $P_t^{cC}$  is the marginal cost of  $Y_t^{cC}$  good production.<sup>1</sup> Relaxing homoteticity condition in technology specification is second, apart from Balassa-Samualson effect, source of real exchange rate appreciation being a result of country's income surge. If income elasticity of services is more than one, income growth implies higher demand for services, their prices go up, and the appreciation occurs (see for example Bergstrand (1991)).

## 2.7 Labor market

### 2.7.1 Dynamics of employment and unemployment

We model labor market using search mechanism, based on Mortensen (1989) and Pissarides (1990) results. On one hand, employers create new jobs by opening vacancies to fulfill, on the other hand unemployed send job offers to the firms. Imperfect labor market matching process causes the number of vacancies filled in each period  $t$ ,  $J_t^{cs}$ , is lower than labor demand submitted by firms and labor supply of unemployed. We assume matching process evolves according to the following rule:

$$J_t^{cs} = \vartheta_t^m \frac{V_{t-1}^{cs} U_{tS}^{cs}}{\left( (V_{t-1}^{cs})^{\lambda_J^{cs}} + (U_{tS}^{cs})^{\lambda_J^{cs}} \right)^{\frac{1}{\lambda_J^{cs}}}} \quad (62)$$

where

$$U_{tS}^{cs} = (1 - \delta_{NA}^c) \times \nu_{t-1}^{cs} \times \sum_{\tau \in T} NE_t^{c\tau} (\iota_{t-1}^{c\tau})^{e_U^c} \quad (63)$$

is the total number of sent job offers at time  $t - 1$  by active non employed to firms from sector  $s$ . Consequently, the number of newly created jobs depends on the number of vacancies offered and the job search effort of unemployed. Matching function type that has been imposed, allows for non unit substitution between job offers and vacancies<sup>2</sup>. Additionally, contrary to standard Cobb-Douglas matching function the specified one guarantees probability of filling the vacancy  $\Psi_{tN}^{cs}$  and the probability of finding a job  $\Phi_t^{cs}$  are within  $[0, 1]$  range, and are defined as:

$$\begin{aligned} J_t^{cs} &= \Psi_{tN}^{cs} \times V_{tN}^{cs} \\ J_t^{cs} &= (1 - \delta_{NA}) \times \nu_{t-1}^{cs} \times \sum_{\tau} NE_{t-1}^{c\tau} (1 - (1 - \Phi_t^{cs})^{\iota_{t-1}^{c\tau}})^{e_U^c} \end{aligned} \quad (64)$$

Therefore they may be treated as mathematical probabilities. This eases extracting them from the data. Both, firms and household treat these probabilities as given in their decision problems.

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<sup>1</sup>In case of nonhomotetic production function first order conditions of profit maximization problem may not be sufficient. However, marginal cost pricing mechanism imposes the competition mechanism implicitly, that implies FOC's are also sufficient conditions.

<sup>2</sup>Note, that  $\frac{1}{J_t^{cs}} = \frac{1}{\vartheta_t^m} \times \left( \left( \frac{1}{V_{t-1}^{cs}} \right)^{\lambda_J^{cs}} + \left( \frac{1}{U_{tS}^{cs}} \right)^{\lambda_J^{cs}} \right)^{\frac{1}{\lambda_J^{cs}}}$ , the inverse of matched worker-employer pairs is CES function. It has richer substitution structure comparing to standard for search models Cobb-Douglas one.

### 2.7.2 Wage and working time bargaining

In each period  $t$  negotiations of wage based on Nash bargaining mechanism take place. Let the  $\Gamma_t^{cs}$  and  $\Sigma_t^{cs}$  be respectively the surplus of household from additional employed in sector  $s$  member (measured in units of additional utility) and the surplus of the firm operating in sector  $s$  from hiring new worker:

$$\Gamma_t^{cs} = \frac{\partial E_0 \mathcal{U}_0^c}{\partial N_t^{cs}}, \quad \Sigma_t^{cs} = \frac{\partial E_0 \Pi_{0A}^{cs}}{\partial N_t^{cs}}. \quad (65)$$

Workers and firms negotiate contract regulating the hours to work (intensive margin) and wage in next period. According to Nash bargaining both sides aim to maximize total surplus from reached contract. Optimization problem is solved subject to first order conditions resulting from individual optimization problems of firm and household with respect to employment level  $N_t^{cs}$ . It also takes into consideration impact of wage changes and intensive margin on consumption and indirectly on the utility of employed. Technically, the optimization problem takes the form:

$$\begin{aligned} \max_{W_t^{cs}, h_{tN}^{cs}} \Upsilon_t^{cs}(\Gamma_t^{cs}, \Sigma_t^{cs}) \\ p.w. \\ C_{tN}^{cs} = C_{tN}^{cB} + (1 - \tau_t^{cL}) h_{tN}^{cs} \times W_t^{cs}, \\ \Gamma_t^{cs} = \frac{\partial E_0 \mathcal{U}_0^c}{\partial N_t^{cs}}, \\ \Sigma_t^{cs} = \frac{\partial E_0 \Pi_{0A}^{cs}}{\partial N_t^{cs}}. \end{aligned} \quad (66)$$

where

$$\Upsilon_t^{cs} = (\Gamma_t^{cs} \lambda_t^c)^{\xi_N^c} (\Sigma_t^{cs})^{1-\xi_N^c} \quad (67)$$

is the joint surplus from reached contract expressed in units of household's utility, where  $\xi_N^c$  reflects relative negotiation strength of workers and employers.

## 2.8 Research and development

New firms at stage II, which differentiate homogenous good are created by households as a result of new business ideas. In order to come up with new idea one has to invest in research and development. Households decision about their level is based on the impact of investment on dynamics of newly created firms. We assume the number of firms in sector  $s$  evolves according to:

$$F_{tN}^{cs} = (1 - \delta_{tF}^c) F_{t-1N}^{cs} + dF_{tN}^{cs}, \quad (68)$$

where  $\delta_{tF}^c$  denotes exogenous firm destruction rate,  $dF_{tN}^{cs}$  is the number of newly created firms in  $s$  sector. The value of new firm,  $V_{tF}^{cs}$ , is equal to discounted to current period profits stream:

$$V_{tF}^{cs} = \Pi_t^{cs} + (1 - \delta_{tF}^c) \times E_t \Lambda_{t+1}^c V_{t+1F}^{cs}, \quad (69)$$

where  $\Lambda_{t+1}^c = \beta_U \frac{\lambda_{t+1}^c}{\lambda_t^c}$  is discount rate,  $\Pi_t^{cs}$  is a profit of new firm differentiating basic product (the equality holds since the new firms set prices optimally). In order to enter to the  $s$  sector one has to pay expenditure for research and development, buying investment goods  $I_{tRD}^{cs}$  at price  $P_{tI}^{cs}$ . Hence, free entry condition implies:

$$dF_{tN}^{cs} V_{tF}^{cs} = P_{tI}^{cs} I_{tRD}^{cs}. \quad (70)$$

for innovation technology in form:

$$dF_{tN}^{cs} = \varphi_{RD}^c \times \left( I_{tRD}^{cs} \right)^{\lambda_{RD}} \left( F_{t-1N}^{cs} \right)^{\psi_{RD}}. \quad (71)$$

where parameter  $\varphi_{RD}^c$  is the product innovation technology level,  $\lambda_{RD}$  is marginal productivity of research and development expenditures and  $\psi_{RD}$  measures external effects of already implemented business ideas on the process of new ideas creation<sup>3</sup>. Proposed model of R&D module is a reformulation of Colciago and Etro (2007) model, however basic idea standing behind the proposed structure are in line with Jones and Williams (1996) results. Endogenous market structure improves empirical features of the model as it comes to the response for demand and supply shocks, comparing to the standard real business cycles models. The effect may be heuristically summed up as follows: positive technological shock increase the expected, discounted profits of the firms at stage II, that leads to further creation of the new firms. As a result mark ups decrease and the original shock has been augmented.

## 2.9 Banking sector

Perfectly competitive financial sector is represented in the model through aggregated firm collecting deposits from households, issuing bonds and giving loans to firms. It maximizes profit equal to:

$$\Pi_t^{cB} = Z_t^{cB} + R_t^{cL} L_t^{cB} - R_t^{cD} D_t^{cB} - R_t^{cB} B_t^{cB} - P_t^{cF} (\xi_L^c \times L_t^{cB} + \xi_D^c \times D_t^{cB}) \quad (72)$$

setting demand for reserves,  $Z_t^{cB}$ , the value of loans  $L_t^{cB}$  with nominal interest rate  $R_t^{cL} = 1 + r_t^{cL}$ , value of deposits for households with nominal interest rate  $R_t^{cD} = 1 + r_t^{cD}$ . Moreover bank sets the value of debt incurred on interbanking market and indirectly credit in central bank  $B_t^{cB}$  with interest rate  $R_t^{cC} = 1 + r_t^{cC}$ , regulating interbanking interest rate. Creating a new deposit or loan is costly. These costs, respectively  $\xi_D^c$  and  $\xi_L^c$  are expressed in units of final domestic financial good. In each period banking sector is obliged to keep the required rate of obligatory reserves:

$$Z_t^{cB} = \zeta^{cB} D_t^{cB}. \quad (73)$$

also the balance equation must be held:

$$Z_t^{cB} + L_t^{cB} = D_t^{cB} + B_t^{cB} + T_t^{cM}. \quad (74)$$

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<sup>3</sup>For  $\psi_{RD} > 0$  we have standing on shoulders effect, resulting is multiplication effect on the number of firms in next period. Then for  $\psi_{RD} < 0$  we have fishing-out effect, and as a result the higher the number of implemented ideas, the lower the productivity of new development investment

where  $T_t^{cM}$  is the nominal money supply issued at time  $t$  by the central bank. Equilibrium condition for loan and deposit market require:

$$D_t^{cB} = L_t^c D_t^c \quad L_t^{cB} = \sum_{s \in \mathcal{S}} L D_t^{cs}. \quad (75)$$

i.e. demand and supply for deposits and loans are equal.

## 2.10 Capital market

We assume there exist an extra level of aggregation, namely aggregated firm, owing all firms in a given country. It is owned by the household. The aggregated profits of this firm, denoted by  $\Pi_t^{cA}$ , are equal to:

$$\Pi_t^{cA} = \sum_{s \in \mathcal{S}} \left( \Pi_{tP}^{cs} + \sum_{i=1}^{F_{tN}^{cs}} \Pi_{tA}^{cs,i} - P_{tI}^{cs} I_{tRD}^{cs} \right) + \Pi_t^{cB}. \quad (76)$$

Profits are the sum of profits of firms at stage I, stage II and banking sector profits, less the costs of creating new firms. Equilibrium condition for market of shares imply:

$$d_t^c = 1, \quad (77)$$

where  $d_t^c$  is the share of household in  $c$  country in aggregated firm.

## 2.11 Central bank and monetary policy

We assume money supply is regulated by central bank and monetary policy is exogenous. Such specification is dictated by two major reasons. Firstly, in case of implementing economic growth block in the model economy it is impossible to impose endogenous monetary policy rule, as one can not to define potential output. Moreover, according to Bhattacharjee, Thoenissen (2005) models with exogenous monetary policy, comparing to the ones with endogenous, replicate better the historical data. Our specification seems to be consistent with methodological critique of different types of Taylor rule in new keynesian models provided by Chari, Kehoe, McGratten (2008).

In each period  $t$ , central bank increase money supply by transferring  $T_t^{cM}$  flow to commercial banks. The evolution of real money supply,  $M_t^c$ , and securities of central bank,  $B_t^{cB}$ , is given by:

$$\begin{aligned} M_t^c &= \frac{1}{\pi_t^c} M_{t-1}^c + T_t^{cM} \\ B_t^{cB} &= \bar{B}^{cB} + \zeta_t^B \frac{1}{\pi_t^c} B_{t-1}^{cB}. \end{aligned} \quad (78)$$

where  $\bar{B}^{cB}$  sets the ratio of central bank bonds to the total output.  $\zeta_t^B$  regulates the intensity of open market operations and  $T_t^{cM}$  is equal to:

$$T_t^{cM} = \bar{T}^{cM} + \zeta_t^M \frac{1}{\pi_t^c} M_{t-1}^c. \quad (79)$$

where  $\bar{T}^{cM}$  sets the long term inflation,  $\zeta_t^M$  is money supply shock. Exogenous monetary policy is governed by following autoregression processes:

$$\begin{aligned}\zeta_t^M &= \rho_M^c \zeta_{t-1}^M + \varepsilon_{tM}^c \\ \zeta_t^B &= \rho_B^c \zeta_{t-1}^B + \varepsilon_{tB}^c.\end{aligned}\quad (80)$$

We model also fixed exchange rate regime. Then, monetary policy is conducted subject to additional constraint, namely:

$$de_t^{fc} = 1, \quad (81)$$

where  $de_t^{fc} \equiv e_t^{fc}/e_{t-1}^{fc}$  indicates the change of nominal exchange rate between periods  $t-1$  and  $t$ . Moreover, implementing common currency between  $H$  and  $F$  country implies equalizing interest rates parity.

## 2.12 Fiscal policy

Fiscal policy in model economies is conducted by the governments operating under following budget constraint:

$$EXP_t^{cG} = REV_t^{cG} \quad (82)$$

Governmental expenditure include: spending for public consumption  $P_t^{cG}(1 + \tau_t^{cC})G_t^c$ , unemployment benefits,  $U_t^c T_{tU}^c$ , old age and disability pensions,  $NA_{tR}^c T_{tR}^c$ , lump sum transfers  $T_t^c$ . Moreover, government at time  $t$  buys out bonds issued in previous period and bought by home and foreign households,  $\frac{1}{\pi_t^c} \times B_{t-1}^{cc} + \frac{1}{\pi_t^f} \times B_{t-1}^{fc}$ . Hence, we have:

$$EXP_t^{cG} = P_t^{cG}(1 + \tau_t^{cC})G_t^c + T_t^c + U_t^c T_{tU}^c + NA_{tR}^c T_{tR}^c + \frac{1}{\pi_t^c} \times B_{t-1}^{cc} + \frac{1}{\pi_t^f} \times B_{t-1}^{fc}.$$

On the revenues side of the government balance there are: inflows from taxes levied on consumption,  $VAT_t^c$ , labor  $PIT_t^c$  and capital  $CIT_t^c$ ; income from bond issuing on domestic and foreign market  $\frac{1}{R_t^{cB}} B_t^{cc} + \frac{q_t^{fc}}{R_t^{cB} \phi_{t,RP}^f} \times B_t^{fc}$ ; profits from open market operations channeled by central bank

$$REV_t^{cG} = VAT_t^c + PIT_t^c + CIT_t^c + \frac{1}{R_t^{cB}} B_t^{cc} + \frac{q_t^{fc}}{R_t^{cB} \phi_{t,RP}^f} \times B_t^{fc} + (R_t^{cC} - 1)B_t^{cB}$$

where taxation profits are defined as follows:

$$\begin{aligned}VAT_t^c &= \tau_t^{cC} \times \sum_{r \in \mathcal{S}} P_t^{cr} Y_{tr}^{cC} \\ PIT_t^c &= \tau_t^{cL} \times \sum_{s \in \mathcal{S}} W_t^{cs} N_t^{cs} h_{tN}^{cs} \\ CIT_t^c &= \tau_t^{cK} \times \Pi_t^{cA} \times d_t^c.\end{aligned}\quad (83)$$

Government conducts exogenous fiscal policy, setting taxation burden, the level of public consumption and debt issue. The lump sum transfers equalize budget constraint. Equilibrium conditions for bond markets require:

$$B_t^c = B_t^{cc} + q_t^{fc} B_t^{fc}. \quad (84)$$

i.e. we assume the whole issued debt of government in  $c$  country will be purchased by households from both countries.

### 2.13 Equilibrium conditions

Output manufactured in sector  $s \in \mathcal{S}$  is used to produce investment, distribution, public, financial and consumption goods. It is used also as intermediate goods. Hence, the following equality must be held:

$$Y_t^{cs} = \sum_{r \in \mathcal{S}} Y_{ts}^{cr} + \sum_{k \in \{I, D, G, F\}} Y_{ts}^{ck} + Y_{ts}^{cC}. \quad (85)$$

Investment goods market equilibrium condition implies:

$$Y_t^{cI} = \sum_{s \in \mathcal{S}} (I_t^{cs} + I_{tRD}^{cs}) \quad (86)$$

Then, equilibrium on the distribution goods market means:

$$Y_t^{cD} = \sum_{s \in \mathcal{S}} \left( \kappa^{csc} \times \sum_{i=1}^{F_{tN}^{cs}} Y_{tD}^{csc,i} + \kappa^{csf} \times \sum_{i=1}^{F_{tN}^{cs}} Y_{tD}^{csf,i} + \kappa^{fsc} \times \sum_{i=1}^{F_{tN}^{fs}} Y_{tD}^{csf,i} \right), \quad (87)$$

For the public consumption market equilibrium condition implies  $Y_{tG}^{cC} = G_t^c$ . Final consumption goods production is equal to total consumption of households and financial goods production covers deposits and loans costs for banking system:

$$Y_t^{cC} = L_t^c C_t^c \quad Y_t^{cF} = \xi_L^c L_t^{cB} + \xi_D^c D_t^{cB}. \quad (88)$$

In addition, we impose  $P_t^{cC} = 1$  for  $c \in \{H, F\}$  as the consumption good is *numéraire* in both countries. The rest of the equilibrium conditions refers to open economy variables. The revenues of domestic producers from export and costs of import amount respectively to:

$$EX_t^c = \sum_{s \in \mathcal{S}} \sum_{i=1}^{F_{tN}^{cs}} \bar{P}_t^{csf,i} Y_{tD}^{fsc,i}, \quad IM_t^c = q_t^f \times \sum_{s \in \mathcal{S}} \sum_{i=1}^{F_{tN}^{fs}} \bar{P}_t^{fsc,i} Y_{tD}^{csf,i}. \quad (89)$$

Current account,  $CA_t^c$ , and capital account,  $KA_t^c$ , are respectively:

$$CA_t^c = EX_t^c - IM_t^c$$

$$KA_t^c = \frac{B_{t-1}^{cf} q_{t-1}^c q_t^f}{\pi_t^f} - \frac{B_{t-1}^{fc} q_{t-1}^f}{\pi_t^c} + \frac{B_t^{fc}}{R_t^{cB} \phi_{tRP}^f q_t^f} - \frac{B_{t-1}^{cf}}{R_t^{fB} \phi_{tRP}^c}. \quad (90)$$

Equilibrium condition for currency market requires the following equality to be held:

$$CA_t^c + KA_t^c = 0. \quad (91)$$

Nominal exchange rate  $e_t^c$ , expressing the price of foreign currency in units of domestic currency, is defined as  $e_t^c \equiv (P_t^c q_t^c)/P_t^f$ . Change of nominal exchange rate,  $de_t^c \equiv e_t^c/e_{t-1}^c$ , is hence equal to:

$$de_t^c = \frac{\pi_t^c}{\pi_t^f} \times \frac{q_t^c}{q_{t-1}^c}. \quad (92)$$

### 3 Solution of the model

Previous sections present only the optimization problems of certain agents. We use the theory of optimal control in order to find the first order conditions and solve the problems. The solution may be derived only in approximated way, using numerical analysis. In order to do so we use FORMA 2.4 numerical software, designed and developed in Institute for Structural Research.

Applied numerical algorithm is an extension of perturbation method designed by Judd (1996). It determines the optimal control, being a rule governing the decisions of agents at time  $t$ , as a function of state variables. Control must be stable, i.e. must ensure that in the long term, after temporary shock economy will converge to the steady state. As such, control satisfies the transversality conditions, which for the purpose of the content clarity were omitted in the main body of the model.

The solving procedures contains of few basic steps. In the first stage we deriv the steady state with no uncertainty, namely the vector  $y^*$  satisfying:

$$0 = E_t f(y^*, y^*, y^*, 0, 0) \quad (93)$$

where the first order conditions of the model are:

$$0 = E_t f(y_{t-1}, y_t, y_{t+1}, \sigma \epsilon_{t+1}, \sigma) \quad (94)$$

however  $y_t$  is the endogenous variables vector,  $\epsilon_t$  is a vector of i.i.d. random variables - shocks and  $\sigma$  is a small parameter. In the second step we derive the approximations of optimal controls in the following way:

$$y_t = R(u_{t-1}, \sigma \epsilon_t, \sigma) \quad u_t = P(u_{t-1}, \sigma \epsilon_t, \sigma) \quad (95)$$

where  $u_t$  is the the expansion in asymptotic series around the steady state,  $y^*$  for  $\sigma \rightarrow 0$ ,  $u_{t-1} \rightarrow 0$ . Then we compute the first (linear) factor of the expansion in two stages. At the first stage, using the symbolic computing toolbox, the conditions describing the first factor of asymptotic series expansion are derived. As result we get the matrices equation system. At the second stage, we solve this system, obtaining for  $\sigma = 1$ :

$$\begin{aligned} y_t &= y^* + Ru_{t-1} + S\epsilon_t + K \\ u_t &= Pu_{t-1} + Q\epsilon_t + L \end{aligned} \quad (96)$$

Hence we get the optimal controls, allowing for investigating the impulse response functions. As the matrices  $R$  and  $S$  are the derivatives of  $R()$  function with respect to, respectively,  $u_{t-1}$  and  $\epsilon_t$ , we can construct different type of approximation, apart from the log-linear one.

The important and innovative part of the solution procedure is its application to the problems with economic growth. We investigate the long run properties of the economy, after experiencing the permanent shock, by comparing two steady states, the one before the shock and the one after. Therefore, the nonlinear properties of the model are fully included in our analysis. We consider the semi-endogenous growth in the model assembled with exogenous stochastic trend generating systematic surge of technology. Thus, the general form of the first order conditions may be written as:

$$\begin{aligned} 0 &= E_t f(y_{t-1}, y_t, y_{t+1}, z_{t-1}, z_t, z_{t+1}, \sigma \epsilon_{t+1}) \\ \log \frac{z_t}{z^*} &= \sigma \times \left( \log \frac{z_t}{z^*} + g_z \right) \end{aligned} \quad (97)$$

that may be reduced to the form (94), where  $z_t$  is a vector of variables generating economic growth,  $g_z$  is the rate of growth of  $z_t$  variable. Using the procedure described above we find the steady state in the first step  $y^*$ ,  $z^*$  and then compute asymptotic expansion around it for  $\sigma \rightarrow 0$  we get:

$$\begin{aligned} y_t &= y^* + R^1 u_{t-1} + S \epsilon_t + K^1 \\ z_t &= z^* + R^2 u_{t-1} + K^2 \\ u_t &= P u_{t-1} + Q \epsilon_t + L \end{aligned} \quad (98)$$

where  $u_t$  is a non stationary variable. For this solution if  $g_z = 0$ , then  $K^1 = 0$ ,  $K^2 = 0$ ,  $L = 0$ . Hence,  $K^1$ ,  $K^2$ ,  $L$  matrices represent the first approximation corrections caused by implementing growth. Redefining state variable  $u_t$  and separating the stationary and non stationary parts we get from the previous equations:

$$\begin{aligned} y_t &= y^* + R^{11} \tilde{u}_{t-1} + R^{12} \tilde{v}_{t-1} + \tilde{S} \epsilon_t + \tilde{K}^1 \\ z_t &= z^* + R^{22} \tilde{v}_{t-1} + \tilde{K}^2 \\ \tilde{u}_t &= P^1 \tilde{u}_{t-1} + Q^1 \epsilon_t \\ \tilde{v}_t &= \tilde{v}_{t-1} + L^2 \end{aligned} \quad (99)$$

where all eigenvalues of  $P^1$  matrix are stationary, hence  $\tilde{u}_t$  is a stochastic process. The value of  $\tilde{v}_t$  is a priori unknown, however it is uniquely determined if we know the value of  $z_t$ . Taking  $\tilde{u}_{t-1} = 0$  and  $\epsilon_t = 0$ , i.e. if all shocks expire and assuming  $z_t = z^*$  we can derive the rate of growth of  $y_t$ . It may be interpreted as the local rate of growth around the set value of  $z_t = z^*$ . Analogically, we can define the steady state,  $\tilde{y}^*$ :

$$\tilde{y}^* = y^* + R^{12} \tilde{v}^* + \tilde{K}^1 \quad (100)$$

that does not depend on the state variables representation. To derive the local rate of growth around other value of  $z_t$  one has to determine again the steady state and the first order approximation. In order to analyze the impact of euro adoption for steady state in the growth environment we compare local steady state  $\tilde{y}^*$ , around the same value

of non stationary variables  $z^*$  in floating exchange rate regime and in common currency environment. In case of no growth the steady state  $y^*$  in both exchange rate regimes is equal (neglecting the effects of elimination of frictions on currency market). Hence, with such definition of the steady state, its shifts may be interpreted as the change of the level resulting only from growth mechanism. It is the additional first order correction with respect to  $g_z$ . Presented method of solving the models with growth requires only the properly defined steady state in case of no growth. Although, one does not have to compute reparametrization of the variables stationarizing the economy, that surely does not exist in case of our model.

## 4 Calibration

Due to large scale and short time series for Poland model is calibrated not estimated. In this section we present the calibration for Poland. The economic model has two purposes - to replicate the observed changes of macroeconomic variables measured for instance by the rate of growth or the deviation from long run trend and to fit to the average levels of economic variables. Despite the rich structure of the model both aims can not be achieved at the same time since the ability to fit to the average levels of economic variables determine almost all parameters of the model. As the basic goal of our model is the to investigate the impulse response functions for structural shocks and we do not aim to provide unconditional forecasts we have put an emphasis on replicating rather the levels of variables then on their relative changes. Hence, the calibration process is conducted in order to fit the model to expected values in the long run.

Our first step in calibration procedure was to pick the number of sectors in modeled economy. The general structure of the model allows for having an unlimited number of sectors, however each additional sector increases substantially the numerical complexity of the model. Therefore in the baseline version we restrict the number of sectors to two basic: industry *IND* and services *SERV*. Such division minimize the numerical complexity allowing at the same time for modeling, in empirically plausible way, the tradable and nontradable goods. We assume in *IND* sector tradable goods dominate and in sector *SERV* the nontradables, however both types of goods are present in each sector.

Table 1: Monetary sphere

Variable	unit	value	calibrated parameter
bonds supply	% GDP	10%	$\bar{B}^{cB}$
M0	% GDP	22%	$\mu_M^c$
obligatory reserves rate	%	3.5%	$\zeta^{cB}$
spread R-RB	% points	-0.5	$\xi_D^c$
spread R-RC	% points	-1.8	$\xi_L^c$
spread R-RD	% points	2.00	$\nu_M^c$
percentage of costs financed through loan	%	50%	$\phi_{LD}$
inflation rate	%	2%	$\zeta^{cM}$
length of price contract	quarters	3.3	$\xi_p^C$
percentage of firms setting price optimally	%	20%	$\xi_p^1$
indexation rate	%	0%	$\omega_\pi$

Table 1 presents the calibrated values of variables in steady state and parameters de-

termining given relationship. As, the value of the variable in steady state is determined by the number of parameters, relationship between the variable and parameter controlling its value is not uniquely determined. The steady state values replicate the data for Poland. Variables **spread R-RB**, **spread R-RC**, **spread R-RD** denote respectively the difference between interest rates of bonds and interbanking, households deposits and loans interest rates expressed annually. Since there is no reliable data, the percentage of operational cost financed through loan is set arbitrary at the 50% level. This parameter has however no substantial influence on the impulse response functions. We set the inflation rate at the amount similar to the target inflation of central bank. We impose the average price contract duration equal to 3.3 quarters, based on micro data provided for instance by Bils, Klenow (2004). The ratio of firms setting price optimally is equal to 20 percents. This value implies the distribution of price contracts length reaches the peak in the third quarter. We do not consider exogenous indexation in the model, taking  $\omega_\pi = 0$ . The rest of the parameters regulating the price stickiness,  $\xi_p^2$ ,  $\xi_p^3$  are described by the conditions:

$$\xi_p^2 = 2/3\xi_p^1 + 1/3\xi_p^C \qquad \xi_p^3 = 1/3\xi_p^1 + 2/3\xi_p^C$$

Table 1 presents the labor market calibration.

Table 2: Labor market

Variable	unit	value	calibrated parameter
life expectancy	years	60	$\delta_{NA}^c$
employment rate	% of population	54%	$\delta_N^{c,IND}$
inactive population	% of population	10%	$\zeta^c$
negotiatory power of employees		0.50	$\xi_N^c$
number of pensioners	% of population	15%	$\psi_R^c$
job finding probability	%	15%	$\lambda_J^c$
vacancy filling probability	%	90%	$\bar{\omega}_J^{c,s}$
job seeking cost	% of consumption	10%	$e_U^c$
unemployment rate	% of active population	6%	$\delta_D$
working time	% of time	33%	$\omega_U$
Share in total unemployed number			
unemployed for 1 quarter	% of unemployed	26.4%	$\theta_1^{cU}$
unemployed for 2 quarters	% of unemployed	12.3%	$\theta_2^{cU}$
unemployed for 3 quarters	% of unemployed	8.4%	$\theta_3^{cU}$
unemployed for 4 quarters	% of unemployed	7.6%	$\theta_4^{cU}$
unemployed for 5 quarters	% of unemployed	5.9%	$\theta_5^{cU}$
unemployed for 6 quarters	% of unemployed	5.1%	$\theta_6^{cU}$
unemployed for 7 quarters	% of unemployed	3.3%	$\theta_7^{cU}$

Source: Eurostat databases and own calculations.

We consider the 15 – 75 population in the model. Hence the unconditional life expectancy is equal to 60 years. The parameters describing the labor market structure: employment rate, the number of pensioners, unemployment rate and the unemployment time structure in steady state we calibrate according to the data for European Union. Current data for Poland may not reflect the long term state of Polish labor market, since it is experiencing intensive changes at the moment. Situation on the labor market in Poland at the moment, measured with unemployment rate or employment rate, has been explained

in the model by the supply and demand shocks resulting in deviations from long run steady state values. Working time is calibrated in line with the business cycle literature (see Cooley, Prescott (1995)) and is equal to one third of total time available. We assume the steady state probability of filling vacancy is 90%, meaning the average time required for matching worker with vacancy is 1.1 quarter. Vacancy filling probability calibration is based on the microeconomic data for Poland - see (...). The parameter determining bargaining power of the firms is set in standard way for the matching models literature at the level of 0.5 (Cheron, Langot (2004)). We set the number of endogenously inactive equal to 10 percents. Lower level precludes replicating the time structure of unemployed whereas higher level has a negative impact on the job flows matrix. As model does not contain any rigidity regarding the number of inactive population job flows between employment and inactivity are larger than reported in data (table 4). Decreasing the number of endogenously inactive adjust this shortcoming but at the cost of the improper structure of unemployed. We have set the cost of job seeking at 10% of unemployed consumption. It seems it is too much. However, the imperfect risk sharing mechanism between employed and unemployed implies low incomes of unemployed that generates high intensity of job seeking in the model. In this situation tries of lowering the job seeking costs lead to no solution of the model. Employment rate is calibrated with parameter  $\delta_N^{c,IND}$  (or  $\delta_N^{c,SERV}$ ), the second parameter  $\delta_N^{c,SERV}$  (or  $\delta_N^{c,IND}$ ) regulates the structure of employment between sectors, described below. We assume for each sector  $s, \lambda_j^s = \lambda_j^c$ , and the probability of filling vacancy is equal among sectors.

Table 3 presents the matrix of job flows between certain states on the labor market. Table 4 exhibits estimated flows for Poland (see Budnik (2007)).

Table 3: Labor market flows - model

	employment	unemployment	inactivity
employment	0.970	0.013	0.016
unemployment	0.149	0.29	0.602
inactivity	0.045	0.077	0.878

Note: data present quarterly flows between labor market states conditioned on staying in labor force. We omit the flows to pensioners.

Table 4: Labor market job flows - empirical data

	employment	unemployment	inactivity
employment	0.972	0.013	0.014
unemployment	0.091	0.851	0.053
inactivity	0.011	0.013	0.975

Source: Budnik (2007), data show average quarterly job flows in 2000-2006 period excluding migrations.

Estimated flows in period 2000-2006 from unemployment to employment are lower than calibrated. Although, in this period the situation on Polish labor market was particularly negative. In period 1996-2000 employment to employment flows were varying between 96.5 to 97 percent and unemployment to employment flows dropped from 17 to 10 percent (see Budnik (2007)). Therefore unemployment to employment flows amounting to 15

percent seems to be closer to the steady state values. Table 5 summarizes the calibration of governmental sector.

Table 5: Governmental sector calibration

Variable	unit	value
debt value	% GDP	40%
public consumption	%	15%
consumption tax	%	15%
capital tax	%	20%
income tax	%	30%
pensioners income	% wage	60%
unemployment benefit	% wage	30%

The debt level of the governmental sector is equal to 40 percent of GDP, consumption, capital and labor tax are respectively 15, 20 and 30 percents of GDP, the public consumption level amounts to 15 percents of GDP. The replacement rates for pensioners and unemployed are respectively equal to 60 and 30 percent of average wage. Table 6 delivers the calibration of sectoral structure in the model which is based on the data for Poland. Import, export, investment, private and public consumption are set according to the input-output matrices provided by the Eurostat. As the rest of the goods is classified as services, we assume, they are produced only with the use of services. The share in employment and GDP, ratio of sold to added value and the share of labor costs in added value are based on the data from Polish Statistical Office in 2006. Above calibrated data are mostly determined by technological parameters - the share of a certain good or input in overall costs. The investment and employment structure are exceptions. Having set the share of inputs and the labor in production costs, share of capital is already determined since we have the constant returns to scale technology, as assumed in the model. In order to replicate the investment and employment structure, we impose heterogenous jobs destruction rates and pace of capital depreciation among sectors. Export and import structure also may be reflected by modifying the share of domestic goods in aggregated sectoral goods production. However, we fit these shares to the data in our model by modifying distribution costs. There is a natural difference between distribution costs in services and industry - in many cases exporting services is impossible. Therefore, we assume the share of domestic goods in aggregated sectoral good production is equal in both sectors and determined by the preferences over the domestic goods - home bias. Moreover the sectoral structure of export and import is determined by the difference in distribution costs between sectors. Thus, additional calibration of distribution costs is not required.

Total share of export to GDP may be also calibrated based on distribution costs, taking into consideration the more general specification of the costs and leaving the parameter determining the domestic goods preferences as free. However, we did not proceed that way. Omitting the distribution costs and assuming the lack of preferences over the domestic goods, moreover assuming the realistic scale of foreign economy, the share of export in GDP would be extremely high, exceeding significantly 100 percents. That is the result of high ratio of export to sold production. Consequently, to replicate the observable the share of export in GDP, distribution costs should be very large. That is due to the fact we treat the foreign economy as a single one. Analyzing the foreign economy as the set of countries and at the same time differentiating the distribution costs according to the

Table 6: Sectoral structure calibration

Variable	unit	industry	services	calibrated parameter
labor costs	% of value added	49.8	47.0	$\alpha_L^{cs}$
share in export	% of export	69.8	30.2	$\kappa^{csfr}$
share in import	% of import	88.0	12.0	$\kappa^{csfr}$
share in investment	% of investment	45.3	54.7	$\delta_{tK}^{cs}$
share in employment	% of employment	46.1	53.9	$\delta_N^{cs}$
share in GDP	% of GDP	35.0	65.0	$\beta_r^{ck}$
production	% of value added	311	175.2	$\alpha_X^{cs}$
Inputs structure				
industry	% of mat.costs	74.3	39.1	$\beta_r^{cs}$
services	% of mat.costs	25.7	60.9	$\beta_r^{cs}$
consumption goods	% of mat.costs	46.0	54.0	$\beta_r^{ck}$
distribution goods	% of mat.costs	0	100.0	$\beta_r^{ck}$
financial goods	% of mat.costs	0	100.0	$\beta_r^{ck}$
governmental goods	% of mat.costs	0	100.0	$\beta_r^{ck}$
investment goods	% of mat.costs	81.2	18.8	$\beta_r^{ck}$

interval between exporting and destination country we could fit the to the observable level of export in line with realistic distribution costs. Although, in equilibrium the major part of export would be directed to the nearest countries decreasing significantly the effective scale of foreign economy.

Table 7: Calibration of the rest of technology parameters

variable	unit	value	calibrated parameter
setting up firm costs	% GDP	3.0	$FC^c$
firms destruction rate	%	3.0	$\delta_F^c$
investment expenditures	% GDP	23	$\delta_{tK}^{cs}$
share of export	% GDP	40	$HAB$
scale parameter foreign/home		10.0	$\chi^F$
share in investment project			
investment in first period	%	10	$\phi_1^{cs}$
investment in second period	%	30	$\phi_2^{cs}$
investment in third period	%	30	$\phi_3^{cs}$
investment in fourth period	%	30	$\phi_4^{cs}$
elasticity of substitution		0.20	$\theta_I^{cs}$

Table 7 provides the calibration of the rest of technology parameters. Expenditures required to set up a new firm amount to 3% of GDP and are calibrated by the level of fixed costs of monopolistic producers differentiating prices (equal in both sectors). The level of expenditures for setting up new firms is closely related to the aggregated profits of monopolists less fixed costs. As we consider free entry condition in our model this cost is equal to aggregated, expected profits of monopolists. We have not implemented fixed costs, thus, our calibration implies high monopolists profits resulting in significant costs of setting a firm. We set the rate of firms destruction amounting to 3 percent. Eurostat

provides data reporting it to be between 2 and 2.5 percent quarterly, nevertheless endogenizing the number of firms combined with low destruction rate results in very persistent impulse responses for all shocks in the model. Firms creation is an internal propagation mechanism in our model, augmenting the original shocks. Empirical impulse reaction functions determined by VAR type models focus only on short run dynamics of the economy, lasting for no more than a couple of years. Therefore the role of the propagation mechanism is difficult to assess and can not be verified empirically in a proper way. However strong internal propagation mechanism augmenting original disturbances in the long run is inconsistent with standard business cycle literature and we decide to minimize its impact by determining parameters regulating firm dynamics properly. As a result we have decreased the long run persistence of shocks. We model the time structure of investment process similarly to time-to-build time-to-plan approach proposed by Christiano and Todd (1996). The structure of this mechanism requires low elasticity of substitution between inputs in different time periods, amounting to 0.2. The share of export is calibrated with the parameter  $HAB$ , defined in following way:

$$\omega^{cs} = \frac{\chi^c \times HAB}{\chi^c \times HAB + \chi^f}$$

where  $\chi^H = 1$  and  $\chi^F$  denote relative scale foreign to home economy. For  $HAB = 1$  the share of domestic goods in aggregated good production is equal to the ratio of foreign to home economy size. As such, parameter  $HAB$  regulates the preferences towards domestic goods. We assume foreign economy is 10 times larger than the home one that is consistent with the scale of Polish economy comparing to the euro zone. Note that parameter  $\chi^F$  is not present in the core model description and is used only for calibration purposes. Table 8 describes the adjustment costs and parameters not having a clear analogs in the data.

We assume low substitution between industrial goods and services in technology of final goods production, namely equal to 0.7. Similarly, in aggregated material goods production in both sectors elasticity amounts to 0.4 and moreover the elasticity of substitution between material goods and capital is 0.4. We have conducted a number of experiments with varying elasticities of substitution, all at rather low levels, and concluded dynamic properties of the model are insensitive to this changes. There is a large disparity among estimated elasticities between domestic and foreign goods (so called Armington elasticity) reported in the literature. Saito (2004) presents highly disaggregated results on estimated elasticities of substitution between domestic and foreign goods for OECD countries. The average value of estimated Armington elasticity for industrial goods used in production process is equal to 2.6, whereas the value of elasticity for final industrial goods amounts to 0.94, both values are characterized by low standard deviation. However the differences between countries and production sectors are huge. Feentra (1994) reports the estimation varying from 2.96 to 8.38 for the USA. Based on cited estimations we set the elasticity of substitution between domestic and foreign goods at 4. We assumed lower level of elasticity, since in our model sectoral goods price includes the distribution costs resulting in lower observable substitutions. There are no data on the elasticities of substitution of services. Since exported services are mainly homogenous goods (distribution goods dominate) we assume higher in services elasticity of substitution between domestic and foreign goods than in industry. We set the elasticity of substitution between intermediate goods equal to 6.0 being a standard calibration in monetary models. This implies an average level of markups at 17 percent in model with flexible prices. The elasticity of substitution between cash and deposits is set at 10, at the level higher than standard in banking sector

Table 8: Output growth rate and inflation

variable	value	calibrated parameter
Elasticity of substitution		
final consumption good	0.70	$\varrho_r^{ck}$
final distribution good	0.70	$\varrho_r^{ck}$
final financial good	0.70	$\varrho_r^{ck}$
final governmental good	0.70	$\varrho_r^{ck}$
final investment good	0.70	$\varrho_r^{ck}$
industry, domestic and foreign goods	4.00	$\epsilon^{cs}$
services, domestic and foreign goods	6.00	$\epsilon^{cs}$
industry, fabrics	0.40	$\varrho_r^{cs}$
services, fabrics	0.40	$\varrho_r^{ck}$
between intermediate goods	6.00	$\epsilon^{csf}$
between capital and fabrics	0.40	$\rho_K^{cs}$
between cash and deposits	10.0	$\epsilon_M^c$
Number of firms at Stage II		
industry	6.00	$\varphi_{RD}^{cs}$
services	6.00	$\varphi_{RD}^{cs}$
Adjustment costs		
vacancies	1.05	$\phi_J^c$
capital utilization	1.50	$\phi_K^{cs}$
risk premium	0.01	$\phi$
imperfect risk sharing	2.0	$\theta_B$
Discount rate	0.99	$\beta$
Risk aversion	4.00	$\sigma_U$
R&D Technology		
$\lambda_{RD}$	0.10	$\lambda_{RD}$
$\psi_{RD}$	-8.0	$\psi_{RD}$

models where the Cobb-Douglas technology implies unit elasticity. However such a low substitution does not seem to be reliable. Moreover, we assume the equal number of firms in industrial and services in each sector at Stage II, namely 6. Such low number cause the real effects of Bertrand competition implemented in the model. Discount rate is stated at the standard for the real business cycle models literature level implying 4 percent real interest rate. High risk aversion augment the reaction of employment and unemployment for shocks. Moreover we assume strong external effects in firms creating process, as the technology of producing new firms in perfectly competitive R&D sector is given by:

$$dF_t = \phi_t \times RND_t$$

where  $dF_t$  denotes the number of newly created firms,  $RND_t$  indicates the level of expenditure required for setting up a new firm,  $\phi_t$  is a productivity in R&D taken by the firm as given, however equal to:

$$\phi_t = \phi \times F_{t-1}^{\psi_{RD}} (RND_t)^{\lambda_{RD}-1}$$

where  $F_t$  indicates the number of firms in sector. Imposing such technology generates *fishing-out* effects - the origin of each firm decreases the number of possible operating

opportunities on the market. As we mentioned the fundamental reason for imposing such structure is declining the magnitude and persistency of shocks in the long run by adding the mechanism generating relatively stationary number of firms. Alternative solution could be a endogenizing outs of the sector - in our model destruction rate of firms is fairly low and exogenous. Parameter regulating risk premium determines mainly a magnitude of current account impulse response. Capital utilization is set in line with standard monetary models. Vacancy posting costs are set at the minimal possible level (equal to 1). Higher costs dampen the impulse response functions of vacancies and consequently of employment and unemployment. Search models, especially these calibrated to the Polish and EU labor market, generate too little magnitude of employment and unemployment. Very low job destruction rate discourage firms to creating new jobs, as the undergone shocks in the standard business cycle models are temporary. The parameter regulating risk sharing mechanism was set at 2. The higher the values of this parameter the better the fit of labor market to the data, nevertheless much larger values leads to problems in calculating steady state. Thus, 2 is the value determined by the numerical stability of the solving algorithm.

## 5 Main results

### 5.1 The channel of lower transaction costs in exchange

At first glance, we investigate the role of lowering transaction costs for Polish economy. Now, the spread between "buy" and "sell" price on euro currency market is about 2-3 percent. Based on the data from currency market, we estimated the transaction costs of exporters and importers amount to about 1-1.5 percent of GDP in Poland, rising the operational costs of firms. In our model transaction costs in both economies, Poland and euro zone, are implemented similarly to distribution costs, in firms differentiating the homogeneous product Stage II. However instead of transportation goods, firms buy final financial goods required to exchange currency. As not all transactions of importers and exporters require exchanging currency, the costs are calibrated in two scenarios: minimal - at 1.0 percent of GDP, maximal - at 1.5 of GDP in Poland. We assume about 80 percent, the share of foreign exchange with euro area for Polish economy, of these costs will disappear after euro adoption. The rest of the costs, connected with exchange with countries outside of euro area, is unaltered, and contributes to the operational costs of export and import firms. In Tables 9 and 5.1 we present the estimated reaction for transaction costs cut of basic macroeconomics variables and labor market aggregates in Poland and euro zone in both scenarios. Permanent cut of transaction costs in foreign exchange affects mostly on exporters and importers. As the prices are sticky in short term, output can not be adjusted immediately and as such is lagged compared to foreign exchange. At first glance, export falls, as a result of inflation jump related to switching to fixed exchange regime. Due to higher discounted, expected profits of exporters and importers investment import surge. Expecting higher incomes in future households increase current consumption. Gradual capital accumulation and higher level of inputs on the basic production goods stage results in GDP level surge, amounting in long term on 0.66-1.65 percent (depending on scenario) higher comparing to no entrance scenario. Two thirds of the permanent effect is revealed five years after accession to euro zone.

**Table 9: Lowering transaction costs channel impact on Polish economy**

	$\Delta$	Minimum			Maximum		
		1 year	5 years	50 years	1 year	5 year	50 year
<b>Macroeconomic aggregates</b>							
GDP		0.23	0.42	0.66	0.58	1.04	1.65
Consumption		0.33	0.27	0.35	0.83	0.68	0.88
Investment	%	0.58	0.37	0.47	1.48	0.92	1.17
Export		-0.18	0.50	0.87	-0.46	1.26	2.17
Import		0.47	0.65	0.78	1.18	1.64	1.97
Current account	pt.	-0.19	-0.05	0.04	-0.33	-0.13	0.09
CPI Inflation	pt.	0.19	-0.01	0.00	0.46	-0.02	0.00
<b>Labor market aggregates</b>							
Active	%	0.01	0.15	0.38	0.03	0.37	0.96
Employed		0.03	0.21	0.54	0.08	0.54	1.34
industry and agriculture	%	0.03	0.20	0.58	0.07	0.50	1.46
services		0.04	0.23	0.50	0.09	0.57	1.24
Unemployed		-0.28	-0.92	-2.00	-0.69	-2.29	-4.95
for less than 1 year	%	-0.23	-0.37	-0.58	-0.58	-0.92	-1.44
long term		-0.33	-1.60	-3.72	-0.83	-3.95	-9.15
Activity rate		0.01	0.08	0.22	0.02	0.21	0.55
Employment rate		0.02	0.12	0.29	0.04	0.29	0.73
Unemployment rate	%	-0.02	-0.06	-0.14	-0.04	-0.16	-0.35
short term		-0.01	-0.02	-0.03	-0.02	-0.04	-0.08
long term		-0.01	-0.05	-0.11	-0.02	-0.12	-0.27
Average wage	%	0.09	0.10	0.06	0.22	0.26	0.16

Source: Own calculations with EUROMOD

Labor market reacts in line with output, consumption and investment. We estimate decline of transaction costs by 0.8 percent of GDP reduces the number of unemployed by 1 percent during five years and 2 percents in the long term. In the maximum scenario decline is more than proportional, respectively, 2.3 and 4.9 percents. Stronger than proportional reaction is caused by internal propagation mechanism implemented in our model, namely mechanism of firm creation, augmenting the original impulse. Especially interesting is the model predicts particularly strong reaction for this supply shock of long term unemployment. However, the surge in employment is not only due to fall of unemployment but also due to increase of activity rate - as in case of extensive margin of labor supply substitution effect dominates the income one. In principle, the cut of transaction costs on currency market have an overall positive impact for basic macroeconomic and labor market aggregates.

As the exchange with Poland reflects only about 6 percents of total euro zone exchange, where the half is export and half is import from Poland, cutting off transaction costs has a minimal impact on most of the euro zone aggregates. The only exception is foreign trade, starting from low level increases in the long term about 1.0 to 2.5 percents depending on

scenario. As a result a slight increase output and employment occur, nevertheless their scale is negligible. Euro adoption is beneficial mostly for Poland.

**Table 10: Lowering transaction costs channel impact on euro zone**

	$\Delta$	Minimum			Maximum		
		1 year	5 years	50 years	1 year	5 years	50 years
<b>Macroeconomic aggregates</b>							
GDP		0.01	0.01	0.04	0.02	0.02	0.09
Consumption		0.00	0.01	0.02	0.01	0.01	0.06
Investment	%	-0.07	-0.01	0.03	-0.18	-0.03	0.08
Export		0.61	0.89	0.96	1.54	2.24	2.42
Import		-0.04	0.74	1.05	-0.11	1.86	2.63
Current account	pt.	-0.02	0.00	0.00	-0.03	-0.01	0.01
CPI Inflation	pt.	0.00	0.00	0.00	-0.01	0.00	0.00
<b>Labor market aggregates</b>							
Active	%	0.00	0.00	0.03	0.00	-0.01	0.07
Employed		0.00	0.00	0.04	0.00	-0.01	0.10
industry and agriculture	%	0.00	0.01	0.06	0.00	0.02	0.15
services		0.00	-0.01	0.02	0.00	-0.02	0.06
Unemployed		-0.01	0.00	-0.16	-0.02	-0.01	-0.40
for less than 1 year	%	0.00	-0.01	-0.06	-0.01	-0.02	-0.14
long term		-0.01	0.00	-0.29	-0.03	0.00	-0.71
Activity rate		0.00	0.00	0.02	0.00	0.00	0.04
Employment rate		0.00	0.00	0.02	0.00	0.00	0.05
Unemployment rate	pt.	0.00	0.00	-0.01	0.00	0.00	-0.03
short term		0.00	0.00	0.00	0.00	0.00	-0.01
long term		0.00	0.00	-0.01	0.00	0.00	-0.02
Average wage	%	0.01	0.01	0.00	0.03	0.02	0.00

Source: Own calculations with EUROMOD

## 5.2 The channel of lower interest rates

Stronger integration of financial markets is one of the most important consequence of joining common currency area. It should lead to the convergence of nominal interest rates. However, not modeled in explicit way, the result of financial integration may be investigated in our model. In fact, the convergence of nominal interest rate for such country as Poland may be reckoned as increasing competitiveness of banking sector, being in fact equal to lowering costs of deposits and loans. Therefore, we model the nominal convergence through implementing permanent shock lowering operational costs of banking system being equal to lowering its purchase of financial services. One may interpret it also as lowering the price of risk assurance. Operational costs are hence equal to  $P_t^{cF}(\xi_L^c \times L_t^{cB} + \xi_D^c \times D_t^{cB})$  and determine the supply of loans and deposits, at given demand from respectively firms and households. As a result nominal interest rates on loans and deposits  $R_t^{cL}$  and  $R_t^{cD}$  are determined in equilibrium. As interest rates in both countries are the result obtained in

equilibrium equalizing them through shock implementation is not possible. Therefore, we model control the operational costs and hence the supply of loans, deposits and interest rates by imposing restrictions on spreads between risk free governmental bond rate and rates on loans and interbanking rate. Spreads are:

$$\begin{aligned} SP_t^{cRC} &= 100 * \left[ (R_t^{cB})^4 - (R_t^{cRC})^4 \right] \\ SP_t^{cRB} &= 100 * \left[ (R_t^{cB})^4 - (R_t^{cRB})^4 \right] \end{aligned} \tag{101}$$

Calibration of the spreads defined above was based on the real, average interest rates in Poland and euro zone. Hence we got also differences between nominal interest rates. As we underlined, implementing common currency requires not only switching to fixed exchange rate regime but also equalizing interbanking market interest rate, being an alternative cost of money for commercial banks. However the pace of the convergence process of interest rates depends highly on the competitiveness of Polish banking sector and its ability to lower the costs. Therefore, we investigate, similarly to transaction costs analysis, two scenarios depending on the expected pace of convergence. First - minimum- scenario assumes diminishing the difference between interest rates will take about 5 years. Second - maximal - scenario assumes it will not take more than 1 year. Both scenarios effects are presented in Table 11

Model predicts the channel of interest rates is less important than the transaction costs one. However it still generates some positive impact. The scale of gain is potentially lower as the spread between interest rates in Poland and in euro zone is not substantial. Nevertheless, as the financial costs resulting from loans in firms at Stage I declines the mark ups of firms at Stage II increases, determining new entrances. Augmented by the internal propagation mechanism decline of interest rates generates long term increase of output level by 0.45 percent, compared to no entrance scenario. Consumption, investment and export also rise, the dynamics of the latter dominates the dynamics of import and hence current account improves. As the operational costs of production firm consists mainly of wages costs, lowering interest rates on loans improves labor market aggregates. Relative (to GDP) reaction of employment and unemployment is stronger than in case of trade channel. The number of employed increases in long term by 0.36 percent and the number of unemployed declines by 1.3 percent. Like in case of transaction costs, long term unemployment is more fragile. Total rate of unemployment declines by 0.1 percentage point. The pace of interest rates convergence is crucial for the timing of these effects. Once the spread disappears after 4 quarters, the most of the effects will occur after five years. However, the slower pace does not diminish the short term effects proportionally, that may be caused by the expectation channel. Expecting gradual decline of interest rates households and firms increase the consumption and investment, improving also the labor market indicators.

**Table 11: Lower interest rates channel impact on Polish economy**

	$\Delta$	Minimum			Maximum		
		1 year	5 years	50 years	1 year	5 years	50 years
<b>Macroeconomic aggregates</b>							
GDP		-0.02	0.31	0.45	0.05	0.35	0.44
Consumption		0.11	0.18	0.23	0.13	0.20	0.23
Investment	%	-0.19	0.39	0.39	-0.03	0.41	0.38
Export		-0.18	0.09	0.31	-0.14	0.14	0.31
Import		0.12	0.22	0.26	0.18	0.23	0.26
Current account	pt.	-0.10	-0.05	0.02	-0.11	-0.04	0.02
CPI Inflation	pt.	0.05	-0.01	0.00	0.03	-0.01	0.00
<b>Labor market aggregates</b>							
Active	%	0.00	0.09	0.26	0.00	0.10	0.26
Employed		0.01	0.13	0.36	0.02	0.15	0.36
industry and agriculture	%	0.02	0.17	0.46	0.02	0.20	0.46
services		0.01	0.09	0.27	0.01	0.11	0.27
Unemployed		-0.22	-0.53	-1.29	-0.23	-0.61	-1.31
for less than 1 year	%	-0.21	-0.21	-0.35	-0.21	-0.23	-0.36
long term		-0.24	-0.92	-2.44	-0.25	-1.06	-2.46
Activity rate		0.00	0.05	0.15	0.00	0.06	0.15
Employment rate		0.01	0.07	0.19	0.01	0.08	0.19
Unemployment rate	pt.	-0.01	-0.04	-0.09	-0.01	-0.04	-0.09
short term		-0.01	-0.01	-0.02	-0.01	-0.01	-0.02
long term		-0.01	-0.03	-0.07	-0.01	-0.03	-0.07
Average wage	%	-0.01	0.11	0.07	0.01	0.11	0.07

Source: Own calculations with EUROMOD

### 5.3 Price shock

Apart from transaction costs and interest rates effects, we distinguished the price channel, namely the nonstructural shock resulting from recalculating prices in new currency. Experiences of European countries that joined the euro zone provides evidences on temporary surge of price level in some markets after euro adoption. Such phenomenon may be interpreted as reoptimizing the price at the moment of currency switch. As such currency switch relaxes the nominal rigidity of prices allowing for optimizing prices. Hence we model this phenomenon by assuming, implementing euro causes uncorrelated negative shock of price rigidity probability -  $\xi_p$ . Both types of firms, being in Taylor and Calvo part of price scheme, undergo this shock at the same magnitude. Again, we model two scenarios: minimal and maximal. In the first one the number of firms setting prices optimally is larger by 25 percents comparing to no shock scenario and in the second this number amounts to 50 percents. Due to lack of reliable data we set the shock autocorrelation at 0.5, implying shock persistency of 8 quarters. Model predictions for Poland are illustrated in Table 12. As the impact for euro zone is negligible we do not present the results.

Additional price change signal causes in Poland temporary increase of inflation, about

**Table 12: Price channel impact on Polish economy**

	$\Delta$	Minimum			Maximum		
		1 year	5 years	50 years	1 year	5 years	50 years
<b>Macroeconomic aggregates</b>							
GDP		-0.20	-0.03	0.00	-0.39	-0.06	0.00
Consumption		-0.08	-0.02	-0.01	-0.16	-0.03	-0.01
Investment	%	-0.49	0.01	0.00	-0.97	0.02	0.00
Export		-0.15	-0.04	0.00	-0.30	-0.09	0.01
Import		-0.16	-0.01	-0.01	-0.32	-0.02	-0.01
Current account	pt.	0.00	-0.01	0.00	0.01	-0.02	0.01
CPI Inflation	pt.	0.09	0.00	0.00	0.17	-0.01	0.00
<b>Labor market aggregates</b>							
Active	%	-0.01	-0.03	-0.01	-0.03	-0.05	-0.01
Employed		-0.01	-0.04	-0.01	-0.02	-0.07	-0.02
industry and agriculture	%	-0.01	-0.04	-0.01	-0.02	-0.07	-0.02
services		-0.01	-0.03	-0.01	-0.03	-0.07	-0.02
Unemployed		-0.03	0.12	0.04	-0.06	0.25	0.07
for less than 1 year	%	-0.04	0.03	0.01	-0.08	0.06	0.02
long term		-0.02	0.24	0.07	-0.03	0.48	0.13
Activity rate		-0.01	-0.01	0.00	-0.01	-0.03	-0.01
Employment rate		-0.01	-0.02	0.00	-0.01	-0.04	-0.01
Unemployment rate	pt.	0.00	0.01	0.00	0.00	0.02	0.01
short term		0.00	0.00	0.00	0.00	0.00	0.00
long term		0.00	0.01	0.00	0.00	0.01	0.00
Average wage	%	-0.04	0.00	0.00	-0.09	0.01	0.00

Source: Own calculations with EUROMOD

0.1 percentage point over steady state value. As the exchange rate regime is fixed shock can not diffuse to foreign economy. At first glance, through the cash in advance constraint, the consumption and investment demand decline and hence the employment and GDP fall. Although, shock is temporary and thus firms do not change the number of opening vacancies implying unemployment stays stable. Fall of employment results only from declining activity rate. In the long term no effects of such shock may occur and economy stays unaffected.

## 5.4 Real appreciation channel

Implementing common currency requires switching to fixed exchange rate regime, however the real rates of growth of all variables are at the same level as in the floating regime case. Adopting euro has no long run growth effects but influences the steady state values of macroeconomic variables. In case of fixed exchange rate regime real appreciation is a transmission channel for shocks. The main source of appreciation in case of Poland, as it used to be, will probably expected rate of growth of export exceeding the rate of growth of output. We analyze this impact by comparing respective steady states values.

**Table 13: Long term impact of real exchange rate appreciation**

	$\Delta$	Poland	Euro Zone
<b>Macroeconomic aggregates</b>			
GDP		0.37	0.08
Consumption		-0.05	0.17
Investment	%	-0.29	0.08
Export		1.03	-0.63
Import		-1.19	1.09
Current account	pt.	0.88	-0.07
CPI Inflation	pt.	2.32	-0.12
<b>Labor market aggregates</b>			
Active	%	-0.54	0.03
Employed		-0.58	0.03
industry and agriculture	%	0.32	0.03
services		-1.32	0.06
Unemployed		0.21	0.00
for less than 1 year	%	-1.09	0.00
long term		2.06	0.00
Activity rate		-0.32	0.02
Employment rate		-0.30	0.02
Unemployment rate	pt.	0.04	0.00
short term		-0.04	0.00
long term		0.08	0.00
Average wage	%	1.11	0.08

Source: Own calculations with EUROMOD

In Table 13 we compare the impact of fixing exchange rate by confronting the levels of variables in long term. Also the strength of inflation impulse is delivered. It should amount to 2.3 percentage points. Households decrease the cash holdings trying to avoid inflationary tax which through cash in advance constraint will dampen the consumption. Higher inflation in presence of nominal price rigidities implies lower mark ups of price setters and consequently slight surge of output and decline of producers prices. Relative home to foreign goods price also falls stimulating export and reducing import, improving current account. Model predicts worsening labor market indicators due to solely appreciation channel.

## 6 Conclusions

In this paper we estimate the effects of joining euro zone for Poland. We present the large scale two country DSGE model incorporating numerous economic mechanisms generating empirically plausible properties of the model. We investigate the impact of four shocks claimed to be direct effects of joining common currency area: reduction of transaction costs, lowering interest rates, price shock and long term currency appreciation influence. We conclude transaction costs reduction, being in fact real reallocation shock, has the largest impact measured in differences of GDP and other macro variables levels comparing to no entrance scenario. In the long term transaction costs reduction results in GDP level increase between 0.66 and 1.65 percent, depending on scenario. Lower interest rates channel should surge the GDP level in the long term by 0.45 percent. Similarly to the transaction costs reduction we incorporated this shock as a real one, resulting from lower operational costs of bank purchased from financial sector. Therefore, long run effects may occur. In case of price shock, resulting from higher percentage of firms setting price optimally at the moment of currency switch, no long run effects are observed. It is quite natural, as this shock has purely short term and incidental character. In the long run prices are set optimally despite the existence of nominal rigidities in the model. Short run impact for GDP is negative, however inflation surge is not significant and varies between 0.09 and 0.17 percentage points of CPI index. Long run effects of joining euro zone are mainly caused by the currency appreciation channel. In fixed exchange rate regime faster GDP growth, comparing to the euro zone, causes inflationary pressure and as the convergence process of Polish economy will last probably for at least a decade it may have significant meaning for macroeconomic aggregates. Results of the simulation suggests inflationary impulse resulting from convergence may reach up to 2.32 percentage points yearly. Total effect of joining euro zone for Poland is hard to estimate for two reasons. At first time horizons of different shocks is different, especially difficult is to merge long run appreciation influence with short run shocks. Secondly, even if it would be possible to add certain shocks they should be properly weighted. We leave these issues for future research.

Apart from investigating the effects of joining euro zone we propose a novel price contract in our paper. It amalgamates common in the literature Calvo and Taylor approaches and is settled in Bertrand competition scheme. In our contract probability for the firm the price to be rigid is decreasing in line with time. It allows us to control average price contract length properly and calibrate price mechanism directly on the data. Incorporating search mechanism for three labor market states, where activity is treated as quasi endogenous variable, we tried to make a small step toward fully endogenous choice between all labor market states. Together with imperfect risk sharing mechanism labor market specification in our model generates dynamics consistent with empirical data (see moments in Appendix A). Especially unemployment is significantly more volatile than output and employment varies slightly more than output.

## 7 Appendix A - properties of the model

### 7.1 Short term properties of the model

In this section we present the most important features of implemented mechanisms. We investigate the role of nominal frictions, time to build mechanism, Bertrand competition and imperfect risk sharing. In each case we compare the baseline model with the one in which certain mechanism was excluded. Table 14 presents mechanisms being compare on the impulse response functions in Figures from 3 to 9.

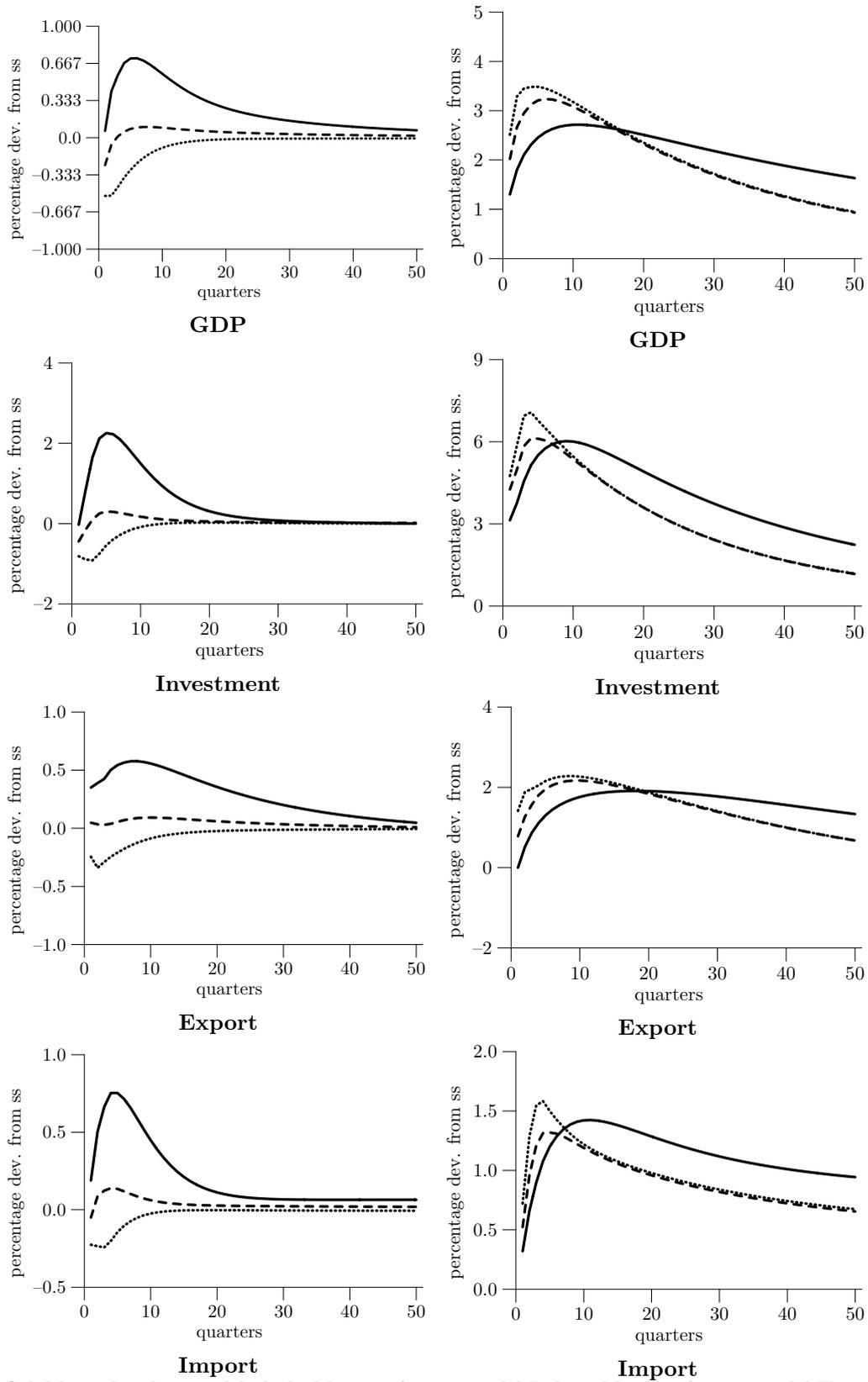
**Table 14: Mechanisms comparison**

Mechanism in baseline model	Reference models
Mixed Calvo/Taylor price contracts with price contracts length of 3.3 quarters	I - price contracts length of 2 quarters II - price contracts length of 1.5 quarters
Bertrand competition in firms at Stage II with 6 firms in each sector	I- standard monopolistic competition II - Bertrand competition with 2 firms in sector
Time to build restrictions for investment with ex post rigidities	Model with standard RBC capital accumulation
Imperfect risk sharing on the labor market	Perfect risk sharing on the labor market

Then in the Table 15 we present comparison of relative standard deviations from the data and the model. Model replicates empirical regularities better in case of Poland rather than euro zone. Nevertheless it fits the data quite well in both cases, especially considering labor market aggregates. Moments are generated through six shocks being connected with the variables in each country  $c \in \{H, F\}$ :

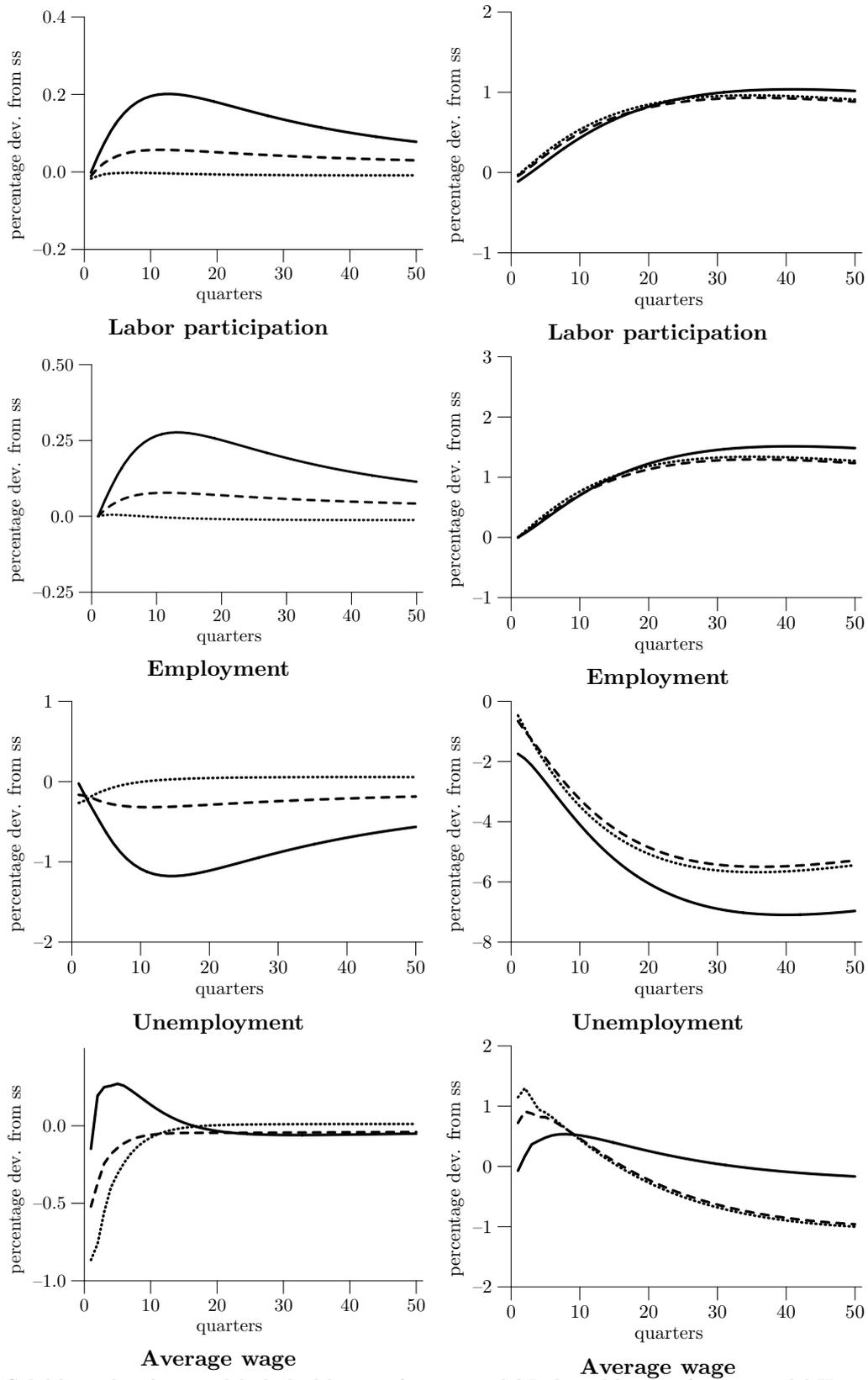
- aggregate technological shock -  $A_t^c$
- tax on labor shock -  $\tau_t^{cL}$
- standard nominal money supply shock -  $M_t^c$
- firm creation shock -  $dF_{tN}^c$
- export share shock -  $\omega^c$
- job destruction rate shock -  $\delta_N^c$

**Figure 3: Impulse responses for different scope of price rigidity - monetary shock (left panel) and technological shock (right panel).**



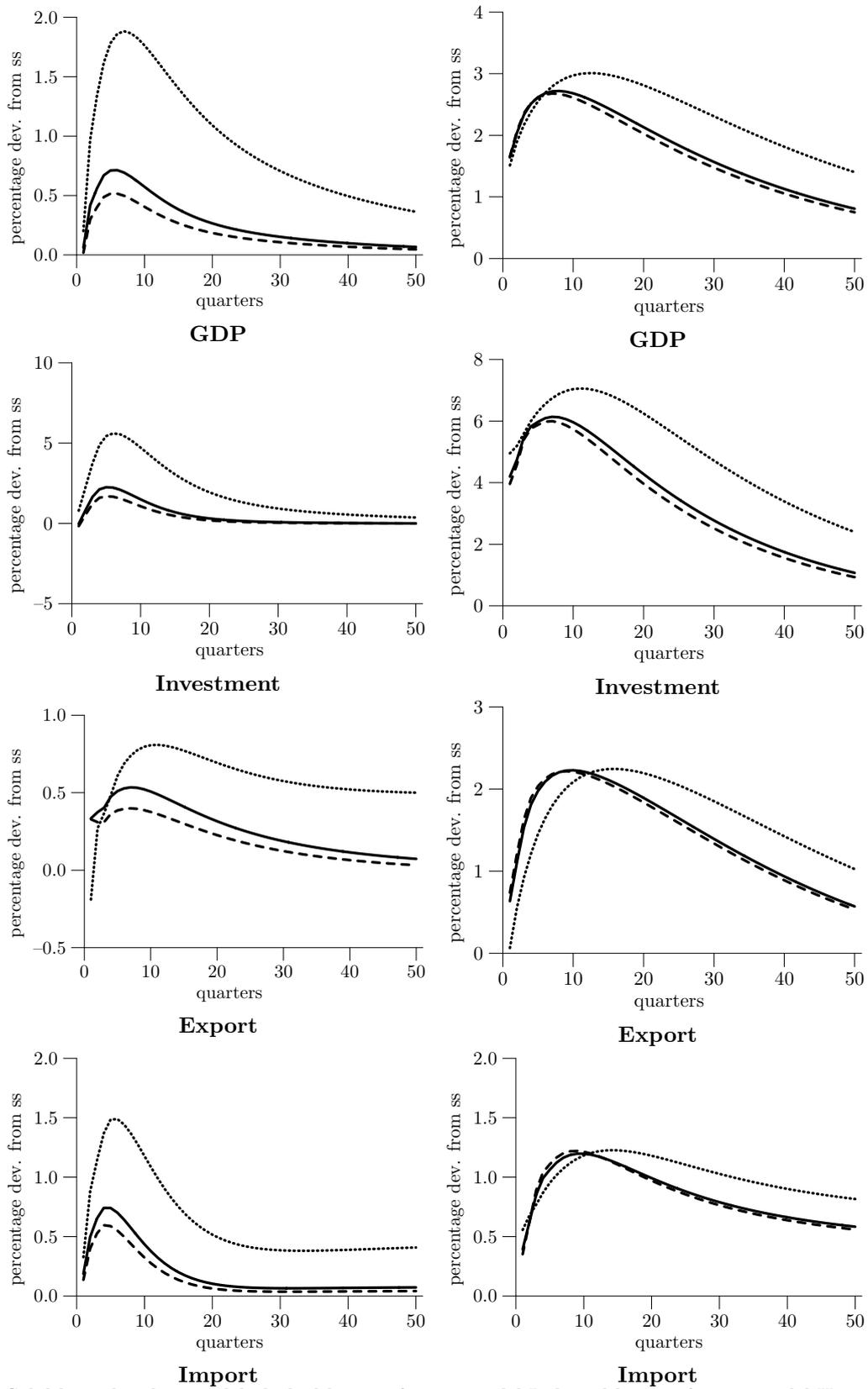
Solid line - baseline model, dashed line - reference model I, dotted line - reference model II.

**Figure 4: Impulse responses for different scope of price rigidity - monetary shock (left panel) and technological shock (right panel).**



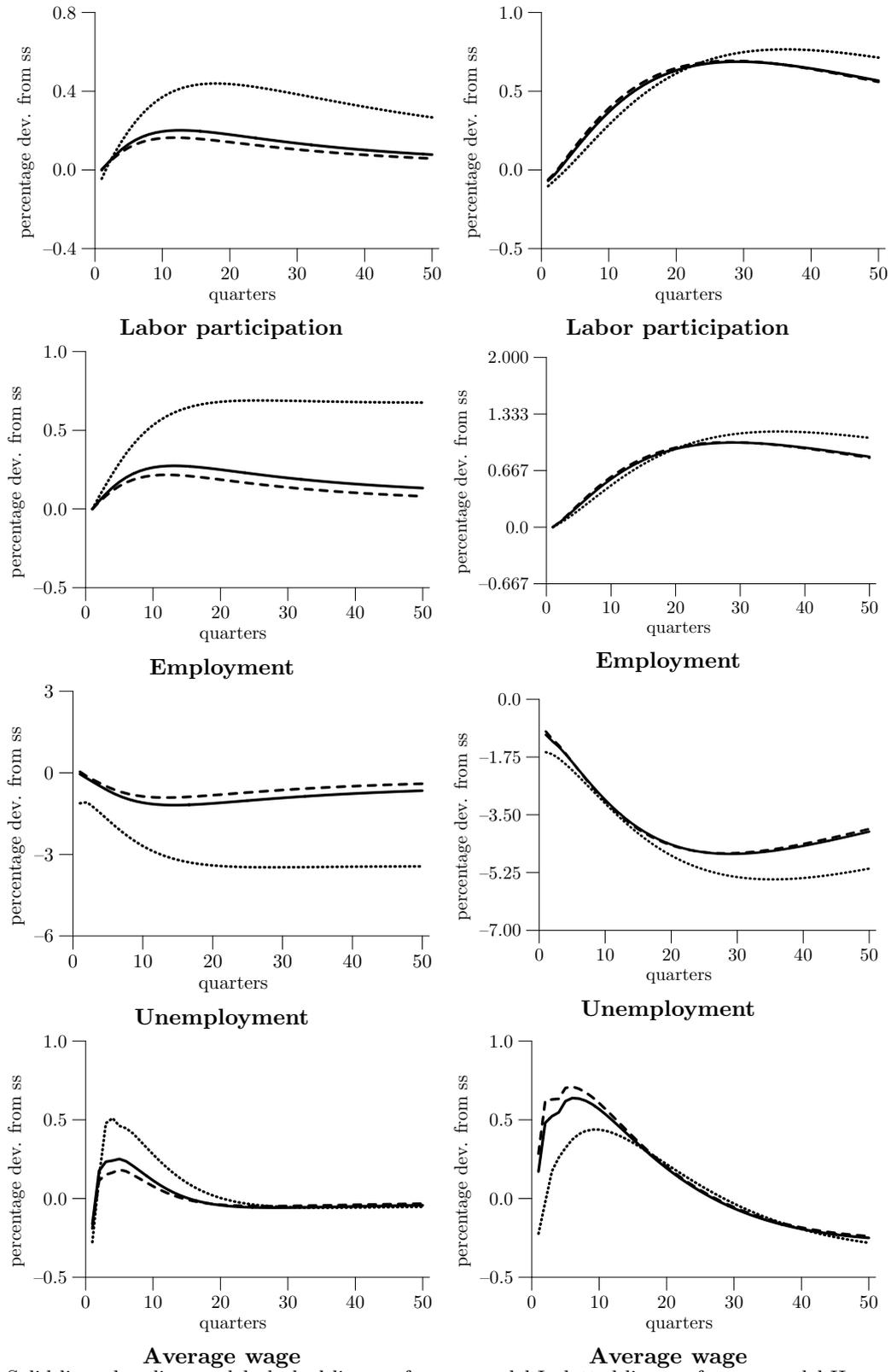
Solid line - baseline model, dashed line - reference model I, dotted line - reference model II.

**Figure 5: Impulse responses with different market structures for monetary shock (left panel) and technological shock (right panel).**

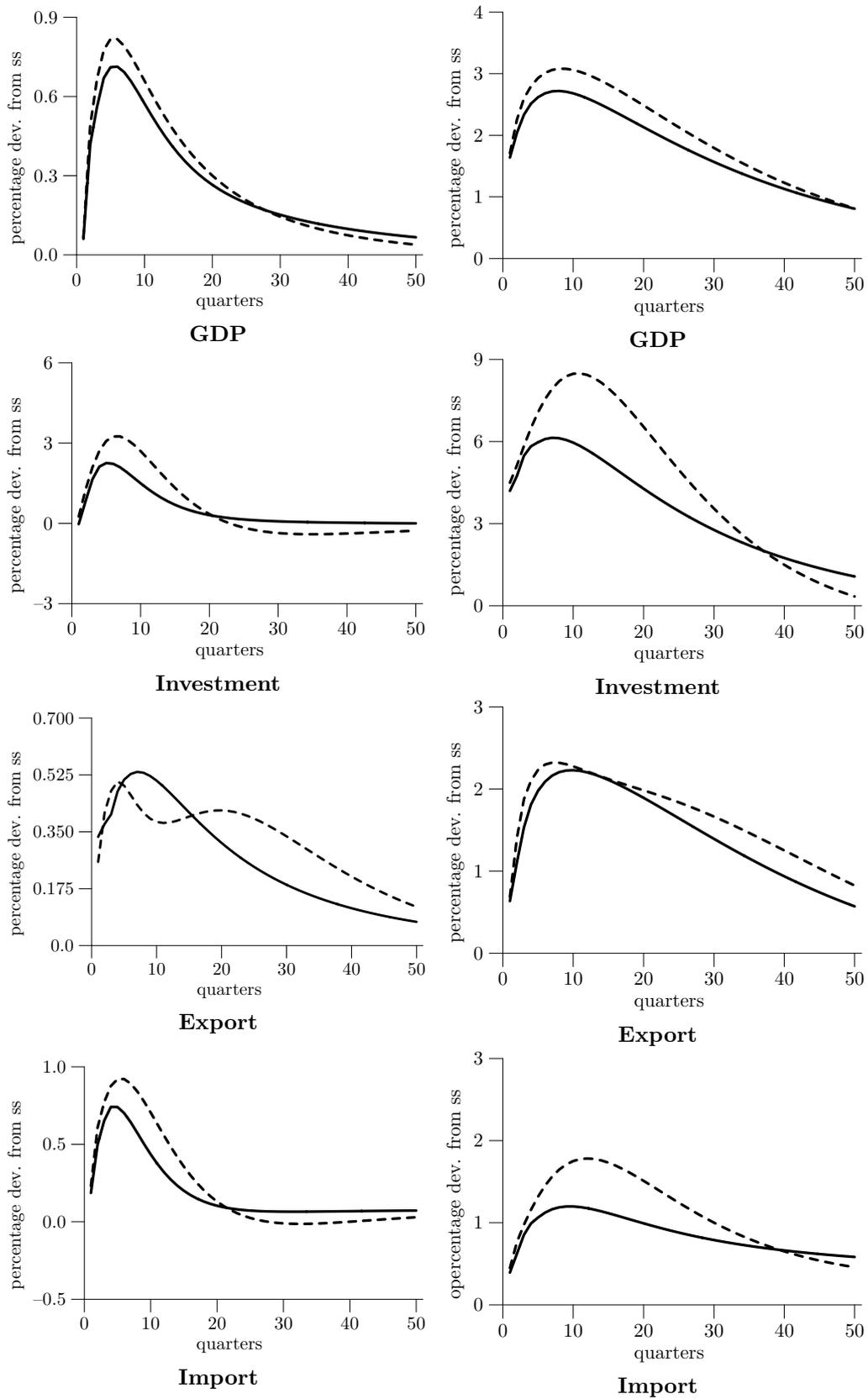


Solid line - baseline model, dashed line - reference model I, dotted line - reference model II.

**Figure 6: Impulse responses with different market structures for monetary shock (left panel) and technological shock (right panel).**

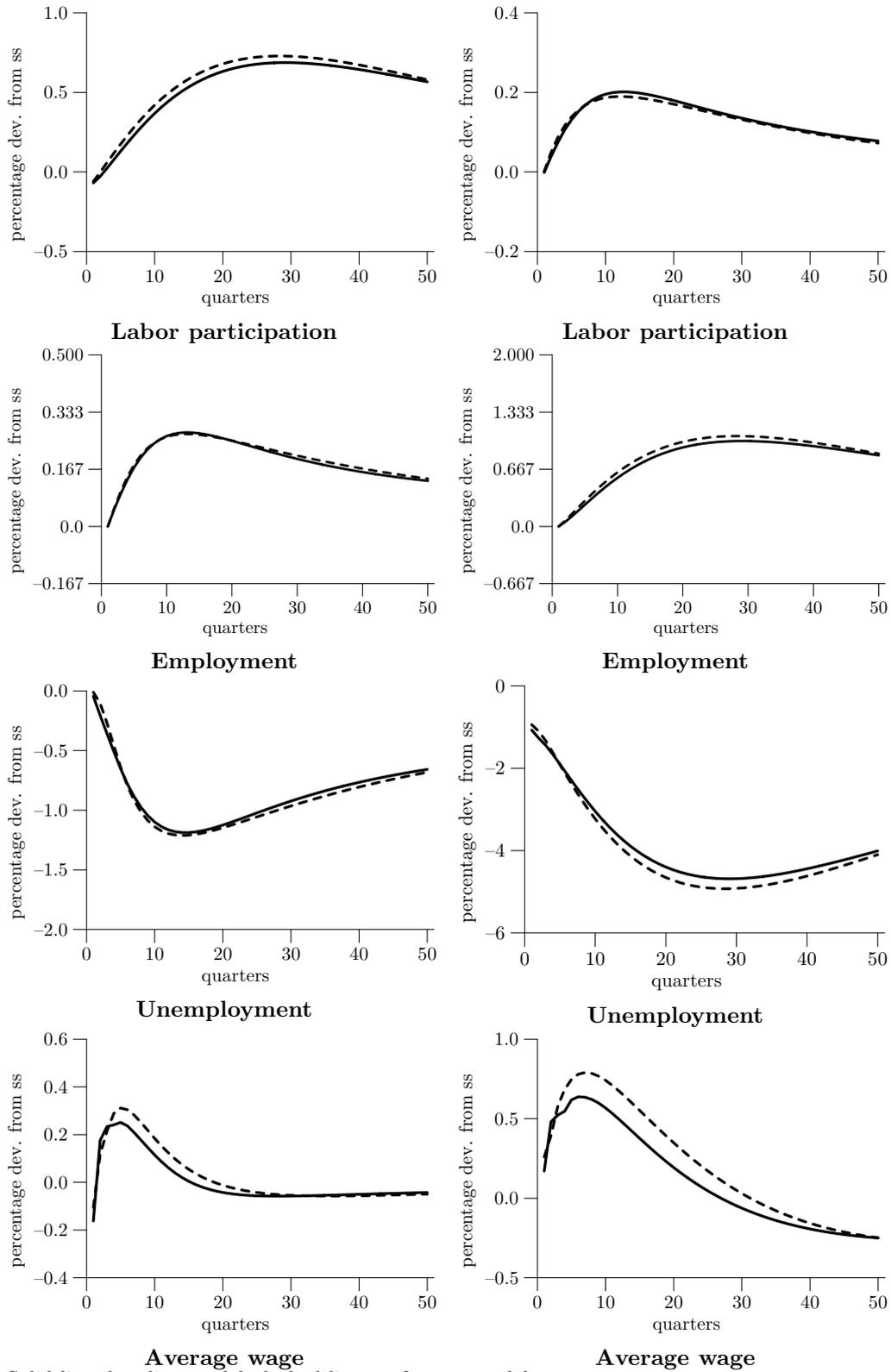


**Figure 7: Time to build investment mechanism role for impulse response functions - monetary shock (left panel) and technological shock (right panel)**



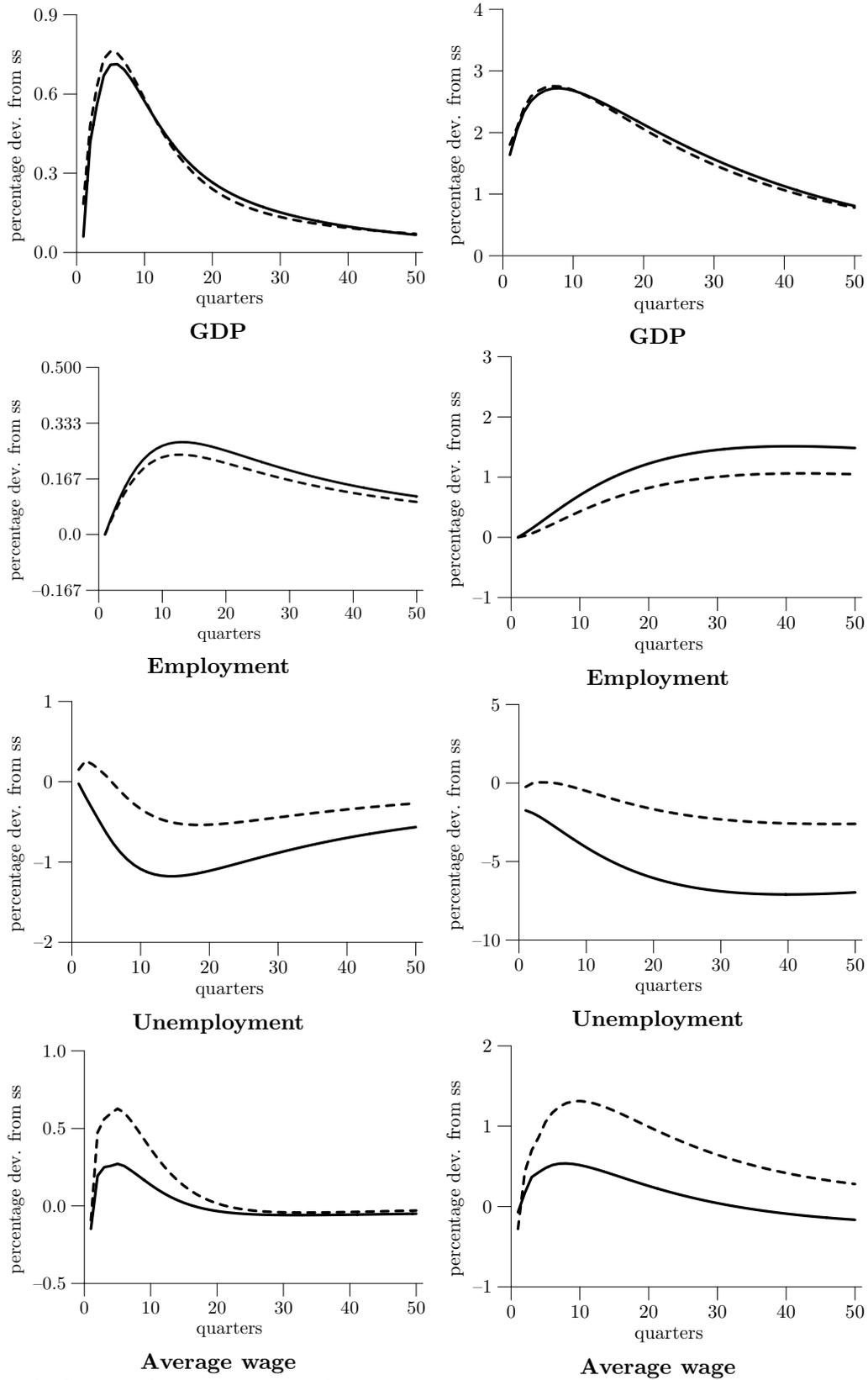
Solid line- baseline model, dashed line - reference model

**Figure 8: Time to build investment mechanism role for impulse response functions - monetary shock (left panel) and technological shock (right panel)**



Solid line- baseline model, dashed line - reference model.

**Figure 9: Impulse response functions for monetary shock (left panel) and technological shock (right panel) with different scope of risk sharing on labor market**



Solid line- baseline model, dashed line - reference model.

**Table 15: Comparison of relative standard deviations from the model and the data**

		DATA	MODEL
		$\frac{\sigma_x}{\sigma_{GDP}}$	$\frac{\sigma_x}{\sigma_{GDP}}$
GDP	Poland	1.000	1.000
	Eurozone	1.000	1.000
Private consumption	Poland	0.970	0.948
	Eurozone	1.015	1.001
Public consumption	Poland	0.796	1.012
	Eurozone	0.531	1.016
Investment	Poland	5.286	3.864
	Eurozone	3.501	2.288
Export	Poland	4.617	5.796
	Eurozone	3.244	3.927
Employment	Poland	1.339	1.266
	Eurozone	0.708	1.518
Unemployment	Poland	7.014	6.283
	Eurozone	4.616	3.311
Activity	Poland	0.642	0.912
	Eurozone	0.366	1.077
Labor productivity	Poland	1.033	0.718
	Eurozone	0.725	0.630
CPI Inflation	Poland	0.068	0.094
	Eurozone	0.255	0.163

Source: Model, own calculations and Eurostat data.

## 7.2 Long run model properties

In this section we present long run properties of the model. Table 16 presents historical data on currency appreciation in Poland and the ability of the model to replicate this regularities. In case of purely Samuelson-Balassa source of currency exchange rate movements model generates depreciation, however adding exogenous export shock improves models performance. In Table 17 the long run rates of growth of ceratin variables are presented.

**Table 16: GDP rate of growth and inflation**

	Data				Model I				Model II			
	$\pi$	$\Delta Y$	$\Delta X$	$\Delta q$	$\pi$	$\Delta Y$	$\Delta X$	$\Delta q$	$\pi$	$\Delta Y$	$\Delta X$	$\Delta q$
Poland												
industry	5.8	5.5			5.5	5.2			5.3	5.6		
services	7.5	5.2			7.9	5.2			7.7	5.0		
overall	5.9	4.2	9.4	2.3	5.9	5.2	3.7	-0.3	5.8	5.2	8.2	2.8
UE27												
industry	1.3	1.9			1.4	2.0			1.3	1.8		
services	2.5	3.0			2.5	2.6			2.4	2.7		
overall	2.0	2.3			2.0	2.3			2.0	2.4		

Note: Data in percentage points. Average values for 1996-2006,  $\pi$  - inflation,  $\Delta Y$  - value added rate of growth,  $\Delta X$  - export rate of growth,  $\Delta q$  - real appreciation rate. Model I - technological growth is the only source of appreciation, Model II - technological growth plus exogenous export growth are the sources of appreciation. Source: Eurostat and own calculations

**Table 17: Basic variables rates of growth**

Variable	Model I	Model II
Output		
industry	3.7%	4.1%
services	4.3%	3.9%
overall	4.0%	4.0%
Price levels (inflation)		
industry	1.4%	0.9%
services	2.4%	3.5%
overall	2.0%	2.0%
Appreciation of exchange rate		
real	-0.3%	2.7%
nominal	-0.3%	2.7%
Rate of growth		
output	4.0%	4.0%
private consumption	4.1%	4.2%
investment	3.9%	3.9%
export/import	3.3%	8.0%
Wages		
industry	3.8%	3.8%
services	4.0%	4.0%
overall	3.9%	4.0%

Source: Own calculations.

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