A Note on Aggregation of Heterogeneous Expectations in a New Keynesian Economy

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Abstract

This paper discusses conditions for aggregation of heterogeneous expectations in a new-keynesian economy. I show that aggregation result depends on the way agents form cross-expectations, i.e. expectations on other agents' decision variables, and I determine a maximal class of cross-expectations operators that support aggregation. Interpretation of obtained conditions in terms of equilibrium and coordination is provided. Although they are restrictive form a mathematical viewpoint, a class of operators which support aggregation and have a natural economic interpretation is discussed.

Keywords: DSGE, Heterogeneous expectations, Aggregation, Cross-expectations.

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1 Introduction

Let us consider a new-keynesian economy populated by agents who are allowed to form heterogeneous expectations in, to some extent, an arbitrary way. Expectations formation mechanism influences agents' decisions, which, when taken from individual to macro level, imply dynamics of economic aggregates. Let us assume that this bottom-up or disaggregated model is true in the sense that it defines workings of the economy. Apart from working with a true model of the economy, one can, for various reasons, employ its reformulated versions. While working with an untrue - simplified or modified - model formulation generally leads to incorrect predictions, such a model can sometimes be, e.g. for trackability or clearness of exposition, preferred to the true one. It is therefore purposeful to state questions about conditions under which reformulated models obtain representations equivalent to their true counterparts. This issue arises in the context of heterogeneity, when models are adjusted or reformulated so that their form resembles the homogeneous counterpart as closely as it gets.

[Branch and McGough (2009)] analyze conditions under which representative household framework is preserved in a heterogeneous expectations new-keynesian model. In this case reformulation consists in assuming that representative household implements an augmented expectations operator which is given by a linear combination of expectations operators of agents populating the economy. Authors give seven conditions for expectations operators which are sufficient in their model formulation for aggregation to obtain[1]. First six conditions are natural as far as their interpretation is concerned. The last one, i.e. agents' agreement on expected limiting wealth differences, is most restrictive and authors impose it without any reference to a mechanism that would render it true. Results I provide in this paper give some insight into the microeconomic mechanism underlaying a more general condition which suffices for agreement on expected limiting wealth differences to hold. More specifically, I provide a condition, which, provided that first six assumptions of [Branch and McGough (2009)] are imposed, is necessary and sufficient for aggregation in a new-keynesian economy to obtain. It turns out that aggregation result depends upon the way agents form cross-expectations, i.e. expectations on others agents' decision variables. It is therefore a structural feature of cross-expectations operators that allows for aggregation. The mechanism that stands behind obtained condition can be interpreted in terms of equilibrium or coordination. It defines a maximal class of cross-expectations operators that support aggregation. Though this class is restrictive form a mathematical viewpoint, I provide simple examples of cross-expectations operators which have a natural economic interpretation and support aggregation.

To make the paper self-contained, in sections 2 and 3 the workings of the model economy are briefly presented. Section 4 discusses the aggregation problem and section 5 concludes.

2 Averaging micro to macro-level variables

It is typical for the DSGE literature to express macro-level or economywide variables X_t as integral-CES aggregates which consist of continuum of micro-level or individual

[1] These are: fixing observables, perfect foresight on steady-state values, linearity, countable additivity, law of iterated expectations at the individual and the aggregate level and agreement on expected limiting wealth differences.

variables X_t^h, $h \in [0,1]$:

$$X_t = (\int_0^1 (X_t^h)^{\frac{1}{\lambda}} dh)^{\lambda} \tag{1}$$

where X_t^h stands for a micro-level variable[2] and $\frac{\lambda}{1-\lambda}$ equals elasticity of substitution between X_t^h and X_t^j for all $h, j \in [0,1]$, $h \neq j$ and for all t. In this paper analogous sum-aggregates will be utilized[3] for a finite number of individual variables X_t^h, $h = 1, 2, ..., m$:

$$X_t = (\sum_{i=1}^{m} \frac{1}{m} (X_t^h)^{\frac{1}{\lambda}})^{\lambda} \tag{2}$$

Derivation of macro-level variables X_t from their micro-level constituents X_t^h will be called *averaging*. The term *aggregation* will be reserved for derivation of macro-level or aggregated expectations from individual ones. If finite elasticity of substitution is not needed[4], (2) will be used with $\lambda = 1$. In this case, log-linearized version of (2) is:

$$\hat{x}_t = \sum_{i=1}^{m} \frac{1}{m} \gamma^h (X^h)$$

where $\gamma^h = \frac{X^h}{X}$ for X and X^h represent stationary, long-run values of X_t^h and X_t respectively. This means that, generally, stationary values X^h for all h are needed to obtain log linearization. It can be shown however, that model instance considered in this paper gives $\gamma^h = 1$ for all h[5].

3 Model economy

The model economy consists of $m \geq 1$ households, $n \geq 1$ firms and of a central bank. There is variety of n goods in the economy. The i-the firm produces $Y_t(i)$ of the i-th good. The h-th household consumes $C_t^h(i)$ of that good, and its total consumption equals $C_t^h = (\sum_{i=1}^n \frac{1}{n} C_t^h(i)^{\frac{1}{\lambda}})^{\lambda}$. Total consumption in the economy equals $C_t = \sum_{h=1}^m \frac{1}{m} C_t^h$.

3.1 Households

The h-th household, $1 \leq h \leq m$, solves the following stochastic dynamic optimization problem:

$$\max_{\{(C_t^h, N_t^h), t=0,1,2,...\}} \mathrm{E}_0^h \sum_{t=0}^{\infty} \beta^t U(C_t^h, N_t^h) \tag{3}$$

[2] E.g. h-th household consumption or h-th firms profit.

[3] The term $\frac{1}{m}$ in (2) ensures that for $X_t^h = \theta$ for all $h = 1, 2, ..., m$ we obtain $X_t = \theta$ as it is the case in (1) for $X_t^h = \theta$ for all $h \in [0,1]$.

[4] Finite elasticity of substitution is needed when decision functions, e.g. demand equations, are derived for X_t^h, so that agents do not choose, e.g. only the cheapest variety to consume. Infinite elasticity will apply for pure averaging only.

[5] For households do not hold bonds in the stationary state, budget constraint (5) implies that $C^h = wN^h$ for all h, where w is steady state real wage. Labour supply equation in (9) gives: $N^h = w^{\frac{1}{\phi}} (C^h)^{-\frac{\sigma}{\phi}}$, form which it follows that consumption and labour supply of all households is equal in the steady state.

where β is a discount factor, U is a regular utility function and N_t^h denotes labour services of the h-th household. E_t^h is an expectations operator of the h-th household. Decision problem (3) is solved subject to the budget constraint:

$$\sum_{i=1}^{n} \frac{1}{n} P_t(i) C_t^h(i) + Q_t B_t^h = B_{t-1}^h + W_t N_t^h \tag{4}$$

where $P_t(i)$ denotes price of the i-th good, B_t^h denotes bond holdings of the h-th households, $Q_t = \frac{1}{1+i_t}$ stands for bond price for i_t being a nominal interest rate in the economy[6], and W_t is nominal wage. *Ex post* a transversality condition $\lim_{t\to\infty} B_t^h = 0$ is imposed. Provided that $P_t = (\sum_{i=1}^{n} \frac{1}{n} P_t(i)^{1-\frac{\lambda}{\lambda-1}})^{1-\lambda}$, (4) can be rewritten as[7]:

$$P_t C_t^h + Q_t B_t^h = B_{t-1}^h + W_t N_t^h \tag{5}$$

and the corresponding Lagrangian is:

$$\mathrm{E}_0^h L = \mathrm{E}_0^h [\sum_{t=0}^{\infty} \beta^t \{U(C_t^h, N_t^h) - \lambda_t^h (P_t C_t^h + Q_t B_t^h - B_{t-1}^h - W_t N_t^h)\}] \tag{6}$$

If E_t^h is conditional expected value operator[8], or the problem is nonstochastic[9], FOCs of (3, 5) for $t = 0, 1, 2, ...$ with respect to C_t, N_t and B_t are:

$$\lambda_t^h = \frac{\partial U(C_t^h, N_t^h)}{\partial C_t^h} \frac{1}{P_t}, \quad \lambda_t^h = -\frac{\partial U(C_t^h, N_t^h)}{\partial N_t^h} \frac{1}{W_t} \quad \text{and} \quad \lambda_t^h = \frac{\beta}{Q_t} \mathrm{E}_t^h \lambda_{t+1}^h \tag{7}$$

respectively[10]. The first FOC combined with the Euler equation and with the second FOC yield a law of motion for consumption and labour respectively:

$$\frac{Q_t}{\beta} \frac{P_{t+1}}{P_t} = \frac{\frac{\partial U(C_{t+1}^h, N_{t+1}^h)}{\partial C_{t+1}^h}}{\frac{\partial U(C_t^h, N_t^h)}{\partial C_t^h}}, \quad \frac{W_t}{P_t} = -\frac{\frac{\partial U(C_t^h, N_t^h)}{\partial N_t^h}}{\frac{\partial U(C_t^h, N_t^h)}{\partial C_t^h}} \tag{8}$$

and employing the CRRA utility function, one obtains:

$$C_t^h = [\frac{Q_t}{\beta} (C_{t+1}^h)^{\sigma} \Pi_{t+1}]^{\frac{1}{\sigma}}, \quad N_t^h = [\frac{W_t}{P_t} (C_t^h)^{-\sigma}]^{\frac{1}{\varphi}} \tag{9}$$

which constitutes a recursive scheme. The first equation sets consumption given expectations on consumption and inflation, whereas the second one sets labour given consumption and real wage. In what follows, we work with first order Taylor approximations of equations (9) around logs of steady-state variables values, which,

[6] A bond acquired in period t at the price Q_t pays a unit of currency in period $t + 1$.

[7] Therefore we assume that individual prices average to economywide or macro-level prices according to this formula. It is consistent with the fact, that h-th households' demand for consumption of the i-th good is given by: $C_t^h(i) = (\frac{P_t}{P_t(i)})^{\frac{\lambda}{\lambda-1}} C_t^h$.

[8] Conditional on the model, its parameters and appropriate filtration of shocks.

[9] I.e. C_t^h and N_t^h are policy functions to be determined and all the other variables in (6) are non-stochastic exogenous processes.

[10] These FOCs are interpreted as equations for marginal utility from consumption and labour, and an Euler equation respectively.

acknowledging the fact, that from time t perspective $(t+1)$-time variables can only be predicted, are[11]:

$$\hat{c}_t^h = \mathrm{E}_t^h\{\hat{c}_{t+1}^h\} + \frac{1}{\sigma}(\hat{\pi}_{t+1} + \hat{q}_t), \quad \hat{n}_t^h = \frac{1}{\varphi}(\hat{w}_t - \hat{p}_t - \sigma\hat{c}_t^h) \tag{10}$$

What if E_t^h is not conditional expectations operator and the problem is stochastic? In what follows let us interpret h-th households decision rules as consisting of two components. The first one constitutes a policy function derived for the deterministic case, and the second one specifies *ad hoc* how agents form expectations from period to period.

3.2 Firms and the central bank

The i-th out of $n \geq 1$ firms produces output $Y_t(i)$ according to production function $Y_t(i) = A_t N_t(i)$ where A_t is stationary technology process and $N_t(i)$ is labour hired[12]. Firms operate on a monopolistic competitive market and are subject to Calvo-type pricing frictions. Profits are transferred to households uniformly[13]. I assume firms are rational, markets are complete and firms engage in a perfect risk sharing mechanism, hence a standard new-keynesian inflation dynamics is generated: $\pi_t = \mathrm{E}_t\pi_{t+1} + \chi\hat{mc}_t$ where $\hat{mc}_t$ stands for log deviation of marginal cost from its stationary state value. The central bank implements a Taylor rule of the form $i_t = \rho + \phi_y \mathrm{E}_t\tilde{y}_{t+1} + \phi_\pi \mathrm{E}_t\pi_{t+1}$ where ρ is consistent with a zero steady state inflation and $\tilde{y}_{t+1}$ denotes output gap.

4 Aggregation of expectations

Let $E_t^h(\,)$ be an expectations operator of agent $1 \leq h \leq m$ for all t, let X_t^h be her decision variable in period t and let Y_t be a macro-level variable that is not subject to disaggregation into individual components[14]. Let X_t^h be governed by a law of motion of the form[15]:

$$X_t^h = \mathrm{E}_t^h X_{t+1}^h + \mathrm{E}_t^h Y_{t+1} \tag{11}$$

Equation (11), given definition of $\mathrm{E}_t^h(\,)$, defines how X_t^h evolves over time at the individual or micro level. Evolution of X_t at the macro level is than given by the averaged formula, see section (2):

$$X_t = \sum_{h=1}^{m} \frac{1}{m} X_t^h \tag{12}$$

This is assumed to be the true model for X_t. Let $\mathbb{E}_t(\,)$ denote an aggregated expectations operator. In what follows let us assume that:

$$\mathbb{E}_t(\,) = \sum_{h=1}^{m} \frac{1}{m} \mathrm{E}_t^h(\,) \tag{13}$$

[11] For a generic variable X_t, its log-deviation from the steady state is denoted by $\hat{x}_t = (\log X_t - \log X)$, assuming that $X_t > 0$ for all t and that $X > 0$.

[12] Agents labour services form a homogeneous labour which is hired by firms according to their demands.

[13] One can think of households as being given a stock index at time zero.

[14] E.g. inflation.

[15] Such a parsimonious form is presented for simplicity of exposition, in fact it is not even solvable. What follows holds however for more general and solvable equation forms, in particular the argument is outlined in a way, that it can easily be applied to the law of motion of consumption: $\hat{c}_t^h = \mathrm{E}_t^h \hat{c}_{t+1}^h + \frac{1}{\sigma}(\hat{E}_t^h \pi_{t+1} + \hat{q}_t)$, see eq. (7).

Aggregated law of motion of X_t is assumed to be of the form[16]:

$$X_t = \mathbb{E}_t X_{t+1} + \mathbb{E}_t Y_{t+1} \tag{14}$$

Definition (aggregation) If the true, i.e. disaggregated, law of motion of variable X_t obtains representation equivalent to the aggregated one (or the other way round), we say that aggregation obtains for X_t. If this holds for all t, we say that aggregation obtains for X_t for all t.

Now conditions for aggregation will be provided[17] Let $E(t) = [e_{hj}(t)]$ for $1 \leq h, j, \leq m$ where $e_{hj}(t) = \mathrm{E}_t^h(X_{t+1}^j)$. We can call $E(t)$ a one-period expectations matrix for X_t, since, for fixed t, it shows what values for X_{t+1}^j are expected by all agents in the economy for all $j = 1, 2, ..., m$. Diagonal elements of $E(t)$ are self-expectations, and off-diagonal elements are cross-expectations.

Proposition 1 (necessary and sufficient conditions for aggregation)

Aggregation obtains for X_t for all t if and only if $\sum_{h,j=1}^m e_{hj}(t) = m \times trE(t)$ for all t.

Proof: From (11) and (12) it follows that the true law of motion for X_t is given by:

$$X_t = \sum_{h=1}^m \frac{1}{m} \mathrm{E}_t^h X_{t+1}^h + \sum_{h=1}^m \frac{1}{m} \mathrm{E}_t^h Y_{t+1} = \sum_{h=1}^m \frac{1}{m} \mathrm{E}_t^h X_{t+1}^h + \mathbb{E}_t Y_{t+1} \tag{15}$$

whereas (14) and (12) yield an aggregated dynamics of X_t, which is:

$$\begin{aligned} X_t &= \mathbb{E}_t X_{t+1} + \mathbb{E}_t Y_{t+1} = \mathbb{E}_t \sum_{j=1}^m \frac{1}{m} X_{t+1}^j + \mathbb{E}_t Y_{t+1} = \\ &= \sum_{h=1}^m \frac{1}{m} \mathrm{E}_t^h \{\sum_{j=1}^m \frac{1}{m} X_{t+1}^j\} + \mathbb{E}_t Y_{t+1} = \frac{1}{m^2} \sum_{h,j=1}^m \mathrm{E}_t^h X_{t+1}^j + \mathbb{E}_t Y_{t+1} \end{aligned} \tag{16}$$

these two formulations are equivalent if and only if:

$$\sum_{h=1}^m \frac{1}{m} \mathrm{E}_t^h X_{t+1}^h = \frac{1}{m^2} \sum_{h,j=1}^m \mathrm{E}_t^h X_{t+1}^j \tag{17}$$

which constitutes the thesis.

Proposition 1 states, that aggregation obtains for X_t for all t if and only if average of self-expectations, i.e. of all terms of the form $E_t^h X_{t+1}^h$, in the economy equals the joint average of self- and of cross-expectations, i.e. of all terms of the form $E_t^h X_{t+1}^j$, in that economy. This can be interpreted in terms of equilibrium between over- and under-expectations in the economy:

Definition (over- and underexpectations)

Let $\Delta_{hj}(t) = e_{hj}(t) - e_{jj}(t)$ for $1 \leq h, j, \leq m$, where $e_{hj}(t) = \mathrm{E}_t^h(X_{t+1}^j)$. Let

$$\Delta^+(t) = \sum_{\Delta_{hj}(t) \geq 0} \Delta_{hj}(t) \quad \text{and} \quad \Delta^-(t) = \sum_{\Delta_{hj}(t) \leq 0} \Delta_{hj}(t)$$

be called over- and underexpectations in the economy in period t respectively.

Proposition 2 (aggregation as clearing of over- and underexpectations)

Aggregation obtains for X_t for all t if and only if $\Delta^+(t) + \Delta^-(t) = 0$ for every t.

[16]The same remark as in footnote 15 applies here.

[17]I assume that first six conditions of [Branch and McGough (2009)] hold.

Proof:

It is clear that $\Delta^+(t) + \Delta^-(t) = \sum_{h,j=1,h\neq j}^{m} \Delta_{hj}(t)$. Aggregation condition from Proposition 1 can be written as (ignore time index t):

$$m\sum_{j=1}^{m} e_{jj} = \sum_{h,j=1}^{m} e_{hj} = \sum_{j=1}^{m} e_{jj} + \sum_{h,j=1,h\neq j}^{m} e_{hj} \tag{18}$$

or:

$$0 = \sum_{h,j=1,h\neq j}^{m} e_{hj} - (m-1)\sum_{j=1}^{m} e_{jj} = \sum_{h,j=1,h\neq j}^{m} (e_{hj} - e_{jj}) \tag{19}$$

which constitutes the thesis.

Notice that neither disaggregated dynamics of X_t, nor the aggregated one, involves cross-expectations explicitly, yet they matter for aggregation to obtain. It is definition of the aggregated expectations operator (13) that introduces cross-expectations when applied to macro level variable X_t. Its form, while typical to the literature, is nonetheless *ad hoc*[18]. As long as agents form only self-expectations and expectations on variables which are not subject to disaggregation (Y_t), as is the case in the disaggregated economy with heterogeneous expectations, one does not need to know how cross-expectations are formed to investigate dynamic behavior of the economy. Cross-expectations emerge, however, even in the disaggregated model, if one allows agents to form expectations on macro-level variables which are subject to disaggregation:

$$\mathrm{E}_t^h X_{t+1} = \mathrm{E}_t^h \sum_{j=1}^{m} \frac{1}{m} X_{t+1}^j = \sum_{j=1}^{m} \frac{1}{m} \mathrm{E}_t^h X_{t+1}^j \tag{20}$$

which is the case in a real world. Having assumed that aggregated expectations operator is given by (13), aggregation obtains or not, depending on how agents *actually* form these cross-expectations.

Definition (cross-expectations operators supporting aggregation) We say that cross-expectation operators $E_t^h X_{t+1}^j, 1 \leq h, j \leq m$ support aggregation for X_t if aggregation obtains for X_t for all t under these operators.

A natural question is what are the cross-expectations that support aggregation. Conditions involved in Propositions 1 or 2 turn out to be very restrictive. Let us fix the analysis to a single period t. Let us define an expectations vector of the h-th agent as $e_h = (e_{h1}, e_{h1}, ..., e_{hm}) \in \mathrm{R}^m$. Assume that $m-1$ out of m agents in the economy have already fixed their expectations according to their expectations formation mechanisms, i.e. they have fixed values of $e_{h,j}$ for $1 \leq j \leq m$ and, say, $1 \leq h \leq m-1$. It turns out, that aggregation obtains for X_t if and only if expectations vector of the m-th agent, e_m, lays on a hyperplane L of the form:

$$L = \{e_h \in \mathrm{R}^m : \sum_{j=1}^{m-1} e_{mj} - (m-1)e_{mm} + p = 0\}$$

where p is a function of e_{hj} for $1 \leq h \leq m-1$ and $1 \leq j \leq m$:

[18] If it is not a real concept, but an auxiliary one, which role is to obtain an elegant model formulation in macro-level variables, one can easily define it in such a way, that no aggregation problems occur: $\mathbb{E}_t X_{t+1} = \sum_{j=1}^{m} \frac{1}{m} \mathrm{E}_t^h X_{t+1}^h$. Such a formulation, nevertheless, does not simplify the matter, since it does not provide the shortcut along which expectations on individual variables are not needed to present model formulation.

Proof: Recall condition (17) and rewrite it using e_{hj} symbols:

$$\frac{1}{m^2}\sum_{h,j=1}^{m} e_{hj} - \sum_{h=1}^{m}\frac{1}{m}e_{hh} = 0$$

Now assume e_{hj} for $1 \leq h \leq m-1$ and $1 \leq j \leq m$ are fixed and note that it is equivalent to:

$$\sum_{h=1}^{m-1}(\sum_{j=1}^{m} e_{hj} - m e_{hh}) - (m-1)e_{mm} + \sum_{j=1}^{m} e_{mj} = 0$$

which constitutes the thesis.

For arbitrary[19] expectations operators of $m-1$ agents in the economy, aggregation obtains, if the last one, or generally any agent, coordinates expectations formation by setting his own expectations anywhere on the hyperplane L[20]. The coordinating agent does not form his expectations according to any *independent* rule in the sense, that his expectations vectors are not formed entirely according to his type of expectations formation but its values (at least one) depend on what other agents have expected. There are, however, examples of natural expectations operators which support aggregation, which are independent in the above sense and which have a meaningful economic interpretation. Consider the following two boundary cases:

$$\mathrm{E}_t^h X_{t+1}^j = \mathrm{E}_t^h X_{t+1}^h \quad \text{and} \quad \mathrm{E}_t^h X_{t+1}^j = \mathrm{E}_t^j X_{t+1}^j \tag{21}$$

These two operators support aggregation, yet their interpretation is completely different from the micro-level perspective. In the first case agents do not try to get to know anything about other agents variables but simply expect that they will evolve as their own are expected to. On the contrary, the second mechanism assumes that agents perfectly know what other agents expect to happen. Also cross-expectations given by linear combinations of $\mathrm{E}_t^h X_{t+1}^h$ and $\mathrm{E}_t^j X_{t+1}^j$ support aggregation:

Proposition:

Cross-expectations operator $E_t^h X_{t+1}^j = \alpha \mathrm{E}_t^h X_{t+1}^h + \beta \mathrm{E}_t^j X_{t+1}^j$ supports aggregation if and only if $\alpha + \beta = 1$.

Proof: Substituting $E_t^h X_{t+1}^j = \alpha \mathrm{E}_t^h X_{t+1}^h + \beta \mathrm{E}_t^j X_{t+1}^j$ to (17) gives:

$$\alpha \sum_{h=1}^{m}\frac{1}{m}\mathrm{E}_t^h X_{t+1}^h + \beta \sum_{j=1}^{m}\frac{1}{m}\mathrm{E}_t^j X_{t+1}^j - \sum_{h=1}^{m}\frac{1}{m}\mathrm{E}_t^h X_{t+1}^h = 0$$

which is equivalent to:

$$(\alpha + \beta - 1)\sum_{h=1}^{m}\frac{1}{m}\mathrm{E}_t^h X_{t+1}^h = 0$$

which constitutes the thesis.

This operator has a natural interpretation for $\alpha, \beta \geq 0$. Agents form cross-expectations taking into account both their own situation and situation of the other

[19] As long as first six conditions hold.

[20] It can be for example, that she forms $m-1$ out of m expectations in a way which is typical to her type, e.g. adaptive or rational, but forms the last one so that aggregation is coordinated. It follows from the fact, that values on $m-1$ axes can be chosen arbitrarily and still it is possible to choose value for the remaining axis so that the expectations vector lays on L, as long as projection of L on all the axes is nondegenerate, i.e. of positive measure.

agent, according to preassigned weights. If we allow weights to vary with h, i.e. with agent type, then, even if $\alpha_h + \beta_h = 1$ for all h, aggregation-supporting weights cannot be generally time-invariant, as we would like them to be:

Proposition:

Cross-expectations operator $E_t^h X_{t+1}^j = \alpha_h \mathrm{E}_t^h X_{t+1}^h + (1-\alpha_h)\mathrm{E}_t^j X_{t+1}^j$ supports aggregation if and only if:

$$\sum_{h=1}^{m} (\alpha_h - \mu) \mathrm{E}_t^h X_{t+1}^h = 0 \tag{22}$$

for all t, where $\mu = \frac{1}{m}\sum_{h=1}^{m} \alpha_h$.

Proof: Substituting $E_t^h X_{t+1}^j = \alpha_h \mathrm{E}_t^h X_{t+1}^h + (1-\alpha_h)\mathrm{E}_t^j X_{t+1}^j$ to (17) gives:

$$\frac{1}{m}(m \sum_{h=1}^{m} \alpha_h \mathrm{E}_t^h X_{t+1}^h + m \sum_{j=1}^{m} \mathrm{E}_t^j X_{t+1}^j - \sum_{h=1}^{m} \alpha_h \sum_{j=1}^{m} \mathrm{E}_t^j X_{t+1}^j) - \sum_{h=1}^{m} \alpha_h \mathrm{E}_t^h X_{t+1}^h = 0$$

which reduces to:

$$\sum_{h=1}^{m} \alpha_h \mathrm{E}_t^h X_{t+1}^h - \frac{1}{m} \sum_{h=1}^{m} \alpha_h \sum_{j=1}^{m} \mathrm{E}_t^j X_{t+1}^j = 0$$

and reindexing j for h one obtains the thesis.

5 Concluding remarks

In the paper I provide a maximal class of cross-expectations operators which support aggregation of expectations in a new keynesian economy and give some insight into the microeconomic mechanism that goes in line with operators in this class. As far as interpretation is concerned, this class consists of operators which, on one hand, entail a balance between over- and underexpectations in the economy, and, on the other, that drive expectations formation in an coordinated way. The need for coordination explains why it is restrictive from a mathematical viewpoint for an arbitrary operator to be a member of this class. Nonetheless, I give examples of simple operators which support aggregation, are economically meaningful, yet do not involve coordination in an explicit way.

References

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