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IBS WORKING PAPER 6/2016
MAY 2016

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ibs working paper 6/2016
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Jacek Suda[*] [†]

Abstract

We study the steady-state effects of working capital channel on the distribution of wages and reallocation of workers across employers. We first show that in a simple Burdett-Mortensen on-the-job search model, working capital channel matter for wage offers distributions of firms and wage distribution of workers. Both wage offer and wage distributions with working capital channel operating are stochastically dominated by distributions without this channel. We also show how working capital channel affects firm size, cross-firms flows of workers and firms' poaching behavior.

Keywords: job search, unemployment, financial market

JEL Codes: J64, E32, J31

[*]National Bank of Poland (NBP), Warsaw, Poland. E-mail: jacek.suda@nbp.pl.

[†]Any views expressed are those of the authors and do not necessarily reflect the views of Narodowy Bank Polski. This research was financed by the Polish National Science Centre grant following decision number: DEC-2011/03/B/HS4/05895.

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1 Introduction

The recent financial turmoil has been associated with a severe increase in unemployment: the salient feature of the 2008-2009 financial crisis has been high and very persistent unemployment in many advanced countries, see Elsby, Hobijn, and Sahin (2010) for the analysis of U.S. labor market developments in the Great Recession and Elsby, Hobijn, and Sahin (2013) for analysis for OECD countries. As that the financial sector has been at the center stage of the recent recession, the important question is whether financial markets could play some role in the increase of unemployment rates is important. In particular, given the increased cost of borrowing and banks' lack of willingness to borrow, it is natural to ask if such developments could influence hiring behavior of firms.

In this paper we study the steady-state effects of working capital channel on the reallocation of workers across employers (on-the-job flows). We extend the standard on-the-job-search model and introduce cost of borrowing as a production cost: firms must borrow to finance their labor input. We first show that in a such a framework the financial markets matter for the distributions of wage offers and for distribution of wage. Next, we study the effects of working capital on the firms size, across-firms flows and firms' poaching behavior.

The version of the on-the-job-search model considered in this paper is an extension of the canonical model introduced by Burdett and Mortensen (1998) and Mortensen (2003). In particular, we modify the framework adding working capital channel into the model. The working capital channel is brought by the assumption that firms must finance their variable inputs with short term loans. Such an assumption has strong empirical support. Barth and Ramey (2002) find, using US Flow of Funds data, that a substantial fraction of firmsâ€™ variable input costs are borrowed in advance. In our model, under this assumption, changes in the borrowing cost affect firmsâ€™ vacancy positing and wage offer decisions.

The paper draws on recent developments in on-the-job search models.[1]. It also connects to a recent literature that incorporates financial frictions into macroeconomic models with search frictions in the labor market. Wasmer and Weil (2004) introduce search friction into the credit market and show that it leads to a financial accelerator effect. Monacelli et al. (2011) introduce borrowing constraints into a macroeconomic model with labor search frictions and show that higher debt improves firmsâ€™ bargaining position having negative effect on wages but positive on job creation. Petrosky-Nadeau (2014) focuses on the amplification and propagation of productivity shocks in the model in which financial conditions take a direct impact on job creation of financially constrained firms.

The paper is organized as follows. Section 2 describes the set-up of a simple on-the-job search model and Section 3 describes the labor market. Section 4 introduces the working capital channel into this model. In Section 5 match-specific caputal is introduced while Section 6 concludes.

[1]See, for example, Coles and Mortensen (2011), Manning (2012), Moscarini and Poster-Vinay(2009, 2012, 2013, 2016) , Wolthoff (2014)

2 Baseline model

We consider a simple version of Burdett and Mortensen (1998) on-the-job search model in which firms make job offers to both unemployed and employed workers. The attractive feature of this framework is that in equilibrium there is substantial wage and firm size heterogeneity even though firms and workers are *ex ante* identical. Due to its simplicity, the model serves as a useful laboratory to isolate and study the effects of our extension. While not all implications of this model do no match the data, in the recent empirical analysis, Haltiwanger, Hyatt, and McEntarfer (2014) find that some core aspects of the this model do match quite well with the data.

We use the specification of Mortensen (2003) and Manning (2012). In this paper we concentrate on the analysis of *ex ante* homogenous economy in the steady state. For the model with *a priori* heterogenous firms see Antosiewicz and Suda (2016).

We later extend the standard framework and allow for some form of financial friction. In this model it takes form of working capital: firms have to finance their wage bill with the help of costly lending.

2.1 Timing

The timing in the model is as follows. First, a firm decides (i) on whether to post a vacancy and, if yes (ii) on wage to offer to a potential worker. Next, workers, both employed and unemployed, receive randomly job offers. An unemployed worker accepts the highest offer that they received as long as it exceeds their reservation wage. An employed worker accepts an offer and changes job as long as the highest offer is greater than their current wage. Finally, production takes place and some jobs are exogenously destroyed.

We now describe the behavior of firms and workers.

2.2 Firms

Firms create vacancies which are filled by workers.

A firm i uses labor provided by workers, N_i, to produce output, Y, with a constant returns to scale production technology,

$$Y_i = pN_i, \tag{1}$$

where p denotes productivity of workers employed by firm i. The productivity of a worker is the same for all workers and all firms.

After all firms post their wage offers, workers (both employed and unemployed) search among the random subset of these offers to determine the highest. We assume that there is a cost of posting a vacancy, c_V.

In our analysis in Section 4 we assume that firms must borrow to finance both the vacancy posting cost and the wage bill.

2.3 Worker

The labor input into production of firm i is provided by a group of identical workers. In each period, a worker can be either employed or unemployed. For simplicity, we assume that employed and unemployed workers are perfect substitutes and they search for jobs with equal intensity λ.

When unemployed, workers receive unemployment benefit b. An unemployed worker accepts the offer as long as it exceeds the reservation wage equal to, in this model, the unemployment benefit.

An employed worker accepts any offer that is higher than his current wage. In equilibrium, no worker will accept an offer lower than the reservation wage so the lowest wage offered is always b. Jobs are destroyed at exogenous job destruction rate δ.

3 Labor market

3.1 Unemployment

Flows to and from unemployment determine the unemployment rate, u. Employed workers flow into unemployment at rate δ while unemployed workers find jobs at the rate λ. In the steady state, inflows and outflows must be equal,

$$u \cdot \lambda = (1-u) \cdot \delta,$$

where the total number of workers is normalized to 1. Rearranging this expression we obtain a steady-state value of unemployment rate

$$u = \frac{\delta}{\delta + \lambda}. \tag{2}$$

3.2 Wage distributions

In the basic model, despite the fact that all firms and workers are identical *a priori*, in equilibrium firms offer (and workers receive) heterogeneous wages. The probability that any worker searching for a job receives a wage offer less than or equal to w is given by the wage offer distribution function $F(w)$. The fraction of employed workers that earn wage w or less is given by the wage distribution function $G(w)$.

Using these two distribution functions we can analyze the flows of workers into and out of set of any wage "bin". The flow of unemployed into the set of employed workers who earn at most w is equal to

$$in\ job(w) = u \cdot \lambda \cdot F(w),$$

that is to the probability that unemployed worker (there is a mass u of them) will be contacted by a firm (λ) offering a wage w or less ($F(w)$). The total outflow from set of workers earning the wage w or less encompasses two flows: into the unemployment, at the rate δ, and into a set of higher wages. Because an employed worker accepts only a higher paying offer, the latter occurs at rate $\lambda(1-F(w))$, and there is $G(w)(1-u)$ workers earning the wage w or less, the total outflow equals

$$out\ job(w) = [\delta + \lambda(1-F(w))]\, G(w)(1-u).$$

In the steady state, the inflow to the set of workers who earn wage w equals the outflow from that set, $in\ job(w) = out\ job(w)$, so the steady state distribution of wages earned across employed workers can be written as

$$G(w) = \frac{\lambda F(w)}{\delta + \lambda(1-F(w))} \frac{u}{1-u}. \tag{3}$$

Using equation (2) to eliminate u, we can write equation (3) as

$$G(w) = \frac{\delta F(w)}{\delta + \lambda(1 - F(w))}. \tag{4}$$

We can also compute the size (in terms of employment) of a firm that pays wage w, $N(w)$. Workers inflow from unemployment and from lower paying firms,

$$in\ firm(w) = \lambda u + \lambda(1 - u)G(w), \tag{5}$$

and outflow due to exogenous separations and due to receiving better offers

$$out\ firm(w) = \delta N(w) + \lambda(1 - F(w))N(w). \tag{6}$$

In equilibrium, a firm offering wage w has the number of inflows equal to the number of outflows,

$$\delta N(w) + \lambda(1 - F(w))N(w) = \lambda u + \lambda(1 - u)G(w),$$

and the size of a firm that pays wage w can be written as

$$N(w) = \frac{\lambda\left(u + (1 - u)G(w)\right)}{\delta + \lambda(1 - F(w))}. \tag{7}$$

3.3 Wage offer distribution

The distribution of wage offers can be derived from firms' profit condition. In the Burdett-Mortesen on-the-job-search model firms post wages to maximize their profits from posting a vacancy net of associated costs. In Section 4 we extend the standard framework by adding a working capital channel that makes it costly for firm to operate.

The value of a vacancy for a firm can be represented by a Bellman equation,

$$rV = \max_{w \geq b} \left\{\eta(v)\left[u + (1 - u)G(w)\right](J(w) - V) - c_V\right\}, \tag{8}$$

where $\eta(v) \equiv \lambda/v$ denotes the endogenous vacancy-filling rate and r is the real interest rate. The above equation uses the fact that the probability of hiring a worker equals the per vacancy rate at which workers are contacted ($\eta(v)$) times the probability that the actual offer is actually accepted by either unemployed ($u \cdot 1$) or employed worker poached from other firm ($(1 - u)G(w)$). The product of these probabilities and the resulting capital gain ($J(w) - V$) from filling a vacancy minus the cost of doing it, c_v, yield the expected net return to posting a vacancy.

The value of a filled vacancy for an employer paying wage w is given by the following Bellman equation

$$rJ(w) = p - w - c_J - \lambda\left[1 - F(w)\right](J(w) - V) - \delta J(w). \tag{9}$$

A filled vacancy brings value p to a firm at a wage cost w and additional cost c_J.[2] The match breaks due to either exogenous vacancy destruction at rate δ, or due to a worker leaving to a higher paying job. The latter happens with the probability $\lambda\left[1 - F(w)\right]$.

[2] In a standard model there is no financial cost so $c_J = 0$.

Free entry drives the expected present value of an open vacancy to zero, $V = 0$, and the value of a filled vacancy from (8) can be written as

$$J(w) = \frac{p - w - c_J}{r + \delta + \lambda\left[1 - F(w)\right]}. \tag{10}$$

Substituting equation (3) for $G(w)$ and equation (10) for $J(w)$ into equation (8), an optimal wage choice is defined by

$$0 = \max_{w \geq b} \left\{ \eta(v) \left[\frac{\delta}{\delta + \lambda(1 - F(w))}\right] \left(\frac{p - w - c_J}{r + \delta + \lambda\left[1 - F(w)\right]}\right) - c_V \right\}. \tag{11}$$

To solve this equation for the offer density function we now need to specify costs and c_J and c_V.

4 Working capital

We first consider the case of $c_J = 0$ but assume that firms cannot raise funds costlessly to pay the cost of posting a vacancy, c_V. Instead, they have to borrow this amount at the real borrowing rate, r_b. Accordingly, the actual cost that firm has to pay to post a vacancy equals

$$c_V = (1 + r_b)c. \tag{12}$$

Under this assumption equation (11) becomes

$$\frac{(1 + r_b)vc}{\lambda} = \max_{w \geq b} \left\{ \left[\frac{\delta}{\delta + \lambda(1 - F(w))}\right] \left[\frac{p - w}{r + \delta + \lambda\left[1 - F(w)\right]}\right] \right\}. \tag{13}$$

This equation can be used to solve for the steady-state value of vacancies, v. In particular, for $w = b$ we have that $F(b) = 0$ and

$$v = \frac{\lambda}{(1 + r_b)c} \cdot \frac{\delta}{\delta + \lambda} \cdot \frac{p - b}{r + \delta + \lambda}. \tag{14}$$

It is clear that v is affected by the presence of the working capital in the model. Higher borrowing cost r_b implies lower v. The requirement of financing cost of posting a vacancy with a loan does not change the distribution of offers and wages, however.

Now, assume that firms have to finance their entire wage bill with a short term loans. In this case, the wage cost becomes $c_J = r_b w$ and equation (11) becomes

$$\frac{(1 + r_b)vc}{\lambda} = \max_{w \geq b} \left\{ \left[\frac{\delta}{\delta + \lambda(1 - F(w))}\right] \left(\frac{p - (1 + r)w}{r + \delta + \lambda\left[1 - F(w)\right]}\right) \right\}. \tag{15}$$

In this case, the amount of vacancies depends on both cost

$$v = \frac{\lambda}{(1 + r_b)c} \cdot \frac{\delta}{\delta + \lambda} \cdot \frac{p - (1 + r_b)b}{r + \delta + \lambda}. \tag{16}$$

Moreover, the distribution $F(w)$ is also affected by the presence of working capital. First, note that the value of a vacancy is always equal to zero irrespectively of wage w, that is

$$\frac{(1 + r_b)vc}{\lambda} = \left[\frac{\delta}{\delta + \lambda(1 - F(w))}\right] \left(\frac{p - (1 + r_b)w}{r + \delta + \lambda\left[1 - F(w)\right]}\right)$$

for all solutions of the maximization problem in equation (11). Because it holds also for $w = b$, we can write

$$\frac{\delta}{\delta+\lambda} \cdot \frac{p-(1+r_b)b}{r+\delta+\lambda} = \frac{\delta}{\delta+\lambda(1-F(w))}\left(\frac{p-(1+r_b)w}{r+\delta+\lambda\left[1-F(w)\right]}\right). \tag{17}$$

Since the left hand side of equation (17) does not depend on w or $F(w)$, while the right hand side is decreasing in w and strictly increasing in $F(w)$, there exists a unique solution for an equilibrium offer distribution.

The lower support of the wage offer distribution equals the reservation wage b. The upper support of the offer distribution, $\overline{w_b}$, can be determined using equation (17) and noting that at largest wage paid $F(\overline{w_b}) = 1$. Therefore,

$$\overline{w}_b = b + \left(1 - \frac{(r+\delta)\delta}{(\delta+\lambda)(r+\delta+\lambda)}\right)\left(\frac{p}{1+r_b} - b\right). \tag{18}$$

Looking at equation (18) it is easy to see that the maximum wage is decreasing in r_b,

$$\frac{\partial \overline{w}_b}{\partial r_b} < 0.$$

The higher the cost of borrowing, the lower the maximal wage a firm can offer.

The offer distribution function can be computed by solving the quadratic equation in $F(w)$ given by equation (17). The positive root of this equation gives the closed form of the wage offer distribution. In particular,

$$F(w) = \frac{r+2(\delta+\lambda)}{2\lambda}\left[1 - \left(\frac{r^2 + 4(\delta+\lambda)(r+\delta+\lambda)\left(\frac{p-(1+r_b)w}{p-(1+r_b)b}\right)}{\left[r+2(\delta+\lambda)\right]^2}\right)^{\frac{1}{2}}\right]. \tag{19}$$

This expression is the same as in the standard Burdett-Mortensen model for $r = r_b = 0$.

To study the effect of the presence of working capital friction on the wage offer distribution note that

$$\frac{\partial}{\partial r_b}\left(\frac{p-(1+r_b)w}{p-(1+r_b)b}\right) < 0 \qquad \text{and} \qquad \frac{\partial^2}{\partial r_b^2}\left(\frac{p-(1+r_b)w}{p-(1+r_b)b}\right) < 0.$$

Therefore,

$$\frac{\partial}{\partial r_b}F(w) > 0 \qquad \text{and} \qquad \frac{\partial^2}{\partial r_b^2}F(w) > 0, \tag{20}$$

that is, the wage offer distribution is increasing and convex in the cost of external financing, r_b. Because,

$$F(w) - F(w)_{r_b=0} > 0 \qquad \text{for all} \quad w \in (b, \overline{w}]$$

the wage offer distribution under working capital constraint is stochastically dominated by $F(w)_{r_b=0}$ and wages offered in the economy with working capital channel are lower than without this friction.

Using this result we can compute the workers wage distribution, G(w). Substituting equation (19) into equation (4), the probability that a worker earns wage w or lower equals

$$G(w) = \frac{\delta}{\lambda}\left[\frac{2(\delta+\lambda)}{\sqrt{r^2 + 4(\delta+\lambda)(r+\delta+\lambda)\left(\frac{p-(1+r_b)w}{p-(1+r_b)b}\right)} - r} - 1\right]. \tag{21}$$

Because wages offered are lower for $r_b > 0$ the distribution of wages that workers receive is shifted to the right,

$$\frac{\partial}{\partial r_b} G(w) > 0, \tag{22}$$

and the higher the cost of borrowing the lower actual wages.

To have a sense whether these effects are quantitatively important we can plot these distributions and associated density functions,

$$f(w) = F'(w) \qquad \text{and} \qquad g(w) = G'(w),$$

for the two cases: (1) with working capital channel, (2) without working capital channel, when the cost of borrowing equal zero, $r_b = 0$.

To plot these distributions we need to make some parametric assumptions. In doing so we use the value of parameters estimated by Christensen et al. (2005), who using the Danish data for private estimated structural parameters of an on-the-job search model. Their estimate for the job destruction rate, δ, is 0.287 per year, and the estimate of λ is 0.583 per year. They calibrate the value of r to 5% but report that their estimates are almost the same for $r = 10\%$. The densities are computed for $b = 0$ and $p = 1$.

We need to assign the cost of borrowing. We consider two scenarios. First, we assume that the cost of borrowing equal the real interest rate in the economy, that is $r_b = r$. In the second scenario, we allow for the cost of borrowing to be twice as high as the real interest rate, $r_b = 2 * r$. In our calibration, the latter scenario corresponds to the real cost of borrowing equal 10% which does not seem to be excessive.

Figure 1 presents plots of wage offers cumulative distribution and density functions , $F(w)$ and $f(w)$. In Figure 2 there are plots of wage distribution and density, $G(w)$ and $g(w)$, respectively.

The increasing convex shapes of both offer and wage distributions and densities is consistent with the theoretical model does not match the data.[3] Nonetheless, this simple model can be used to shed some light how the presence of working capital and financial market frictions affect these distributions. Both Figure 1 and Figure 2 show that the these effects can be large for reasonably parameterized model. The higher the cost of borrowing the larger fraction of wage offers and wages are associated with lower wage values.

We can also determine how the size of the firm paying wage w is affected by the financial friction. Taking equations (7), (19), and (21), one can express the $N(w)$ as

$$N(w) = \frac{\lambda\delta}{(\delta + \lambda[1 - F(w)])^2}. \tag{23}$$

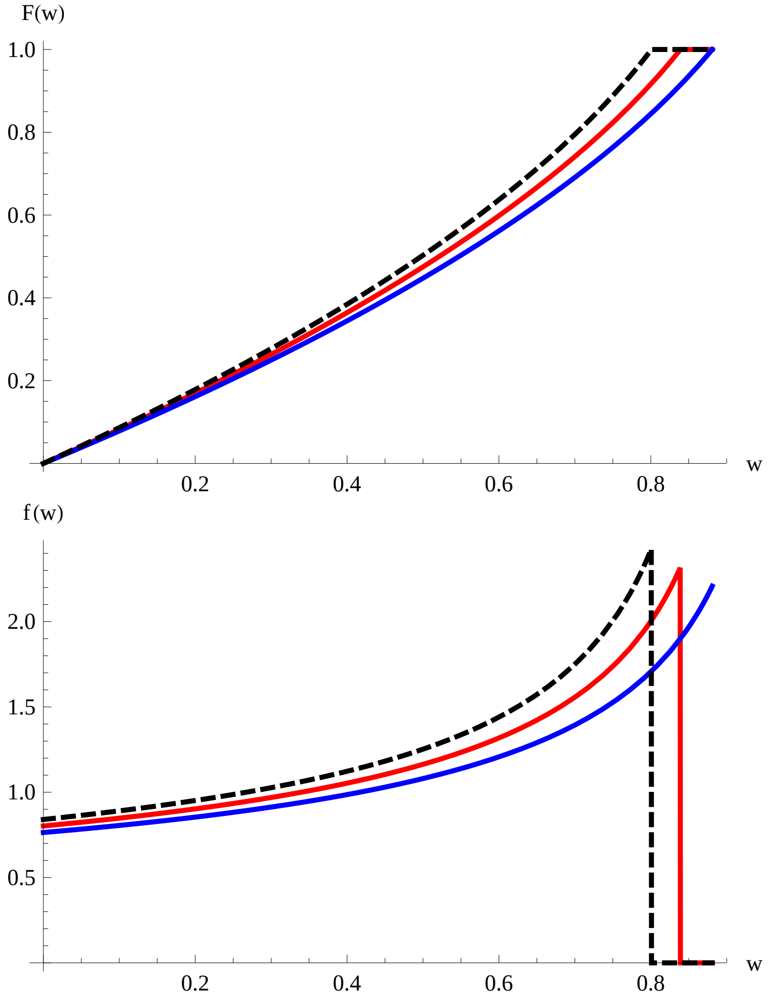
The partial derivative of $N(w)$ with respect to r_b can be determined as

$$\frac{\partial}{\partial r_b} N(w) = \frac{\lambda^2\delta}{(\delta + \lambda[1 - F(w)])^3} \frac{\partial F(w)}{\partial r_b} > 0. \tag{24}$$

Note that this result could seem counter-intuitive as it implies that in the presence of working capital requirement, firms that offer wage w are larger than in the absence of this friction. However, there are actually fewer firms that offer this wage and the entire size distribution of firms is also shifted to the right.

[3]This result is well known in the literature. The introduction of firm heterogeneity in productivity helps produce more realistic shapes of these densities.

Figure 1: Wage offers cumulative distribution, $F(w)$, and density function, $f(w)$. No friction (Blue Line), $r_b = r = 5\%$ (Red Line) and $r_b = 10\%$ (Dashed Line)

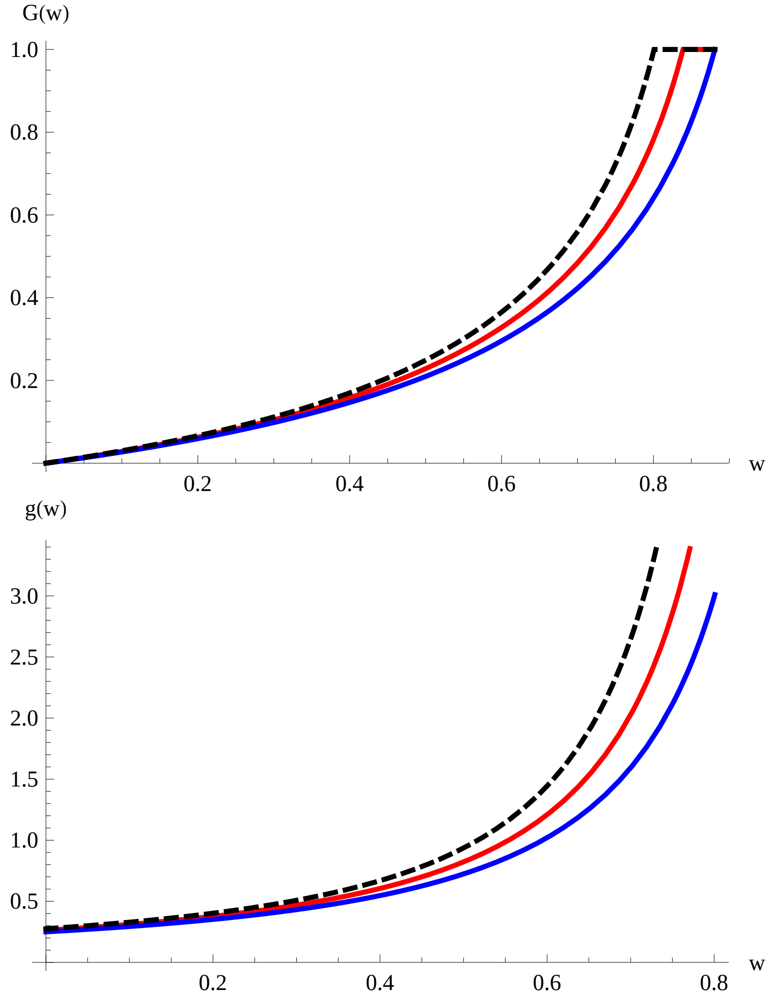


We can also ask how the poaching behavior is affected by the presence of this type of financial friction. Using the total number of hires by a firm in equation (5) and equation (4), the share of poaching hires in a firm paying wage w can written as

$$\frac{\lambda F(w)}{\delta + \lambda}, \tag{25}$$

and since $F(w)$ is increasing in r_b so is the share of hires from poaching at a wage w.

Figure 2: Wage cumulative distribution, $G(w)$, and density function, $g(w)$.
No friction (Blue Line), $r_b = r = 5\%$ (Red Line) and $r_b = 10\%$ (Dashed Line)



5 Model with capital

In this section we consider an extension of the model presented in section 2 to study to what extent the effects of the introduction of working capital into the model are affected by the simple structure of the baseline model. In the section we introduce a match-specific capital as in Acemoglu and Shimer (2000). In that setting, a worker's productivity depends on the amount of capital that firm invests in his position once they are hired, for example as a firm-specific training. Mortensen (2000) shows that in equilibrium firms that offer higher wages invest more capital and, as consequence, they are more productive. In this section we show how costly borrowing affects these results.

5.1 Firms and workers

A firm i will now use combine labor and match-specific capital, k_i, to produce output. In particular, we assume that a newly hired worker will undergo a firm-specific training that will increase his output, now equal $pf(k_i)$, where $f(k_i)$ captures the effect of firm-specific training worker's productivity. We assume that $f(\cdot)$ is an increasing concave function. The behavior of workers is assumed to be unaffected by this change. After all firms post their wage offers, workers (both employed and unemployed) search among the random subset of these offers to determine the highest. We assume that there is a cost of posting a vacancy, c_V. Once the (new) match is made a firm employs capital k_i.

5.2 Labor market

As before, a steady-state value of unemployment rate equals

$$u = \frac{\delta}{\delta + \lambda}.$$

While just like in section 3, the distribution of wages earned across employed workers is given by

$$G(w) = \frac{\delta F(w)}{\delta + \lambda(1 - F(w))},$$

and the size of a firm that pays wage w can be written as

$$N(w) = \frac{\lambda\left(u + (1-u)G(w)\right)}{\delta + \lambda(1 - F(w))} \tag{26}$$

as the distribution of offers, $F(w)$, is different in this case both $G(w)$ and $N(w)$ are affected by the introduction of the capital.

5.3 Wage offer distribution

Since the revenue, cost and, hence profit condition of firms are affected by introduction of capital, the distribution of wage offers is also altered. In particular, the values of vacancy and filled vacancies are affected.

The value of a vacancy for a firm can now be written as

$$rV = \max_{(w,k)\geq 0} \left\{ \frac{\lambda}{v} \left[u + (1-u)G(w)\right] \left(J(w) - k - V\right) - c_V \right\}, \tag{27}$$

where k denotes capital used if the vacancy has been successfully filled. This formulation highlights the fact that a one-time match-specific investment, k, is costly for the firm and while making their offers firms optimize both over the wage and the extent of their investment.

The value of a filled vacancy for an employer paying wage w and investing k in this match can be expressed as

$$rJ(w) = pf(k) - w - c_J - \lambda\left[1 - F(w)\right]\left(J(w) - V\right) - \delta J(w). \tag{28}$$

A filled vacancy brings value $pf(k)$ to a firm at a wage cost w and additional cost c_J.

Under the free entry $V = 0$ the value of a filled vacancy from (27) can be now written as

$$J(w) = \frac{pf(k) - w - c_J}{r + \delta + \lambda\left[1 - F(w)\right]}. \tag{29}$$

Using equations (3), (27), and (29), an optimal wage choice is given by the solution of

$$0 = \max_{w \geq b} \left\{ \frac{\lambda}{v} \left(\frac{\delta}{\delta + \lambda\left[1 - F(w)\right]} \right) \left(\frac{pf(k(w)) - w - c_J - k(w)(r + \delta + \lambda\left[1 - F(w)\right])}{r + \delta + \lambda\left[1 - F(w)\right]} \right) - c_V \right\} \tag{30}$$

where the optimal investment given the wage w equals

$$k(w) = \arg\max\{pf(k) - w - c_J - k(r + \delta + \lambda [1 - F(w)])\} \tag{31}$$

As before to solve this equation for the offer density function we now need to specify costs and c_J and c_V.

5.4 Working capital

In section 4 we considered two cases: (i) $c_V > 0$ but $c_J = 0$, (ii) $c_V > 0$ but $c_J > 0$. In this section we proceed in the similar fashion. Assume that firms have to borrow to pay the vacancy cost, c_V, paying the real borrowing rate, r_b^V, so that the the actual cost of posting a vacancy equals

$$c_V = (1 + r_b^V)c. \tag{32}$$

In this case, equation (30) can be written as

$$\frac{(1 + r_b^V)vc}{\lambda} = \max_{w \geq b} \left\{ \left[\frac{\delta}{\delta + \lambda(1 - F(w))} \right] \left[\frac{pf(k(w)) - w - k(w)(r + \delta + \lambda [1 - F(w)])}{r + \delta + \lambda [1 - F(w)]} \right] \right\} \tag{33}$$

with $k(w)$ given by equation (31).

Using $F(b) = 0$ we can show that the steady-state value of vacancies, v, are affected by both the presence of capital and the costly borrowing

$$v = \frac{\lambda}{(1 + r_b^V)c} \cdot \frac{\delta}{\delta + \lambda} \cdot \frac{pf(k_b) - b - k_b(r + \delta + \lambda)}{r + \delta + \lambda}, \tag{34}$$

where k_b is given by the equation

$$f'(k_b) = \frac{r + \delta + \lambda}{p}. \tag{35}$$

Like before, however, the distribution of offers and wages does not depend on r_b^V .

Now, assume that firm have to borrow not only to finance wage bill, as in Section 4, but also to cover worker's firm-specific training. In particular, assume that

$$c_J(w) = r_b^w w + r_b^k k(w), \tag{36}$$

so that equation (30) becomes

$$\frac{(1 + r_b^V)vc}{\lambda} = \max_{w \geq b} \left\{ \left[\frac{\delta}{\delta + \lambda(1 - F(w))} \right] \left(\frac{pf(k(w)) - (1 + r_b^w)w - k(w)(r_b^k + r + \delta + \lambda [1 - F(w)])}{r + \delta + \lambda [1 - F(w)]} \right) \right\}, \tag{37}$$

and

$$f'(k(w)) = \frac{r_b^k + r + \delta + \lambda [1 - F(w)]}{p}. \tag{38}$$

The steady-state level of vacancies equals

$$v = \frac{\lambda}{(1 + r_b^V)c} \cdot \frac{\delta}{\delta + \lambda} \cdot \frac{p - (1 + r_b^w)b - k_b(r_b^k + r + \delta + \lambda)}{r + \delta + \lambda}, \tag{39}$$

where

$$f'(k_b) = \frac{r_b^k + r + \delta + \lambda}{p}. \tag{40}$$

Equation (40) indicates that if a firm needs to borrow to finance the match-specific capital, the amount of firm-specific training and, as a result, the productivity of lowest paid workers is reduced. We will show numerically below that this is the case along the entire distribution of wages.

To determine the equilibrium distribution $F(w)$ note that left hand side of equation (30) holds for any value of w. Since it holds also for $w = b$, we can compute $F(w)$ as the solution to the equation

$$\frac{pf(k_b) - (1+r_b)b - k_b(r_b^k + r + \delta + \lambda)}{(\delta+\lambda)(r+\delta+\lambda)} =$$
$$= \frac{pf(k(w)) - (1+r_b)w - k(w)(r_b^k + r + \delta + \lambda\,[1-F(w)])}{(\delta+\lambda(1-F(w)))(r+\delta+\lambda\,[1-F(w)])}. \quad (41)$$

Since the left hand side of equation (17) does not depend on w or $F(w)$, while the right hand side is decreasing in w and strictly increasing in $F(w)$, there exists a unique solution for an equilibrium offer distribution.

The offer distribution function can be computed by solving equation (41) for $F(w)$. To obtain the density function $F'(w)$ we need to differentiate both sides of this equation with respect to w to obtain

$$F'(w) = \frac{1 + r_b^w}{\lambda k(w) - \frac{cv(1+r_b^V)}{\delta}\left(r + 2(\delta + \lambda\,[1-F(w)])\right)}. \quad (42)$$

Above equation shows that the working capital and costly borrowing affect the density function three-folds. First, it affects the density probability function directly through r_b^V and r_b^w. Second, it affects the amount capital invested, $k(w)$. Lastly, it affects $F'(w)$ through its influence on $F(w)$. Since the overall effect is not straightforward, we resort to numerical simulation to illustrate the effect of costly borrowing on the density and cumulative distribution functions, $F'(w)$ and $F(w)$.

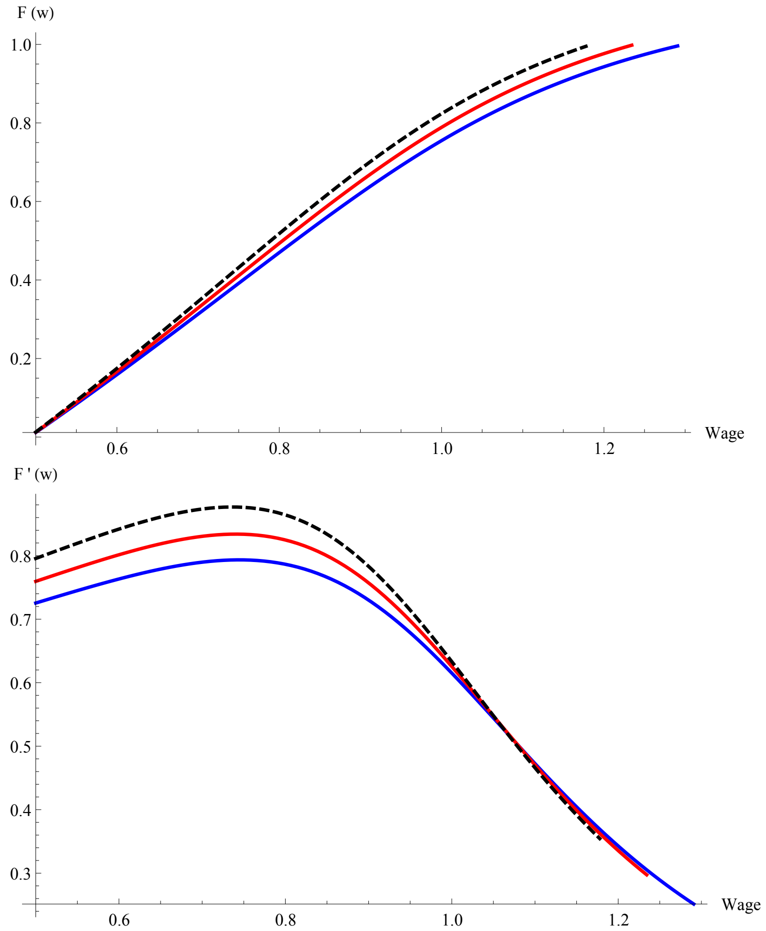
For the calibration we use the parametrization of Mortensen (2000) and Christensen et al. (2005) and set $\delta = 0.287/4$, $\lambda = 0.6$, $r = 1.25\%$ for quarterly frequency. We assume that productivity function is a Cobb-Douglas, i.e. $f(k) = k^\alpha$, and normalize p so that the productivity of least productive match was always equal 1, that is $pk_b^\alpha = 1$. We also set $\alpha = 0.2$ and $b = 0.5$, which given the normalization imply that unemployment benefits equal 50% of output per worker in the least productive job.

As in the previous section we consider two scenarios. First, we assume that the cost of borrowing equal the real interest rate in the economy, that is $r_b^k = r_b^w = r$ whereas in the second, we set the cost of borrowing to be twice as high as the real interest rate, $r_b^k = r_b^w = 2 * r$ which corresponds to the real cost of borrowing equal 10%.

Figure 3 presents plots of wage offers cumulative distribution and density functions , $F(w)$ and $f(w)$.

First, notice that the introduction of match-specific capital creates a hump-shaped density of wage offer density. Under employed parametrization, until some point the number of offers increases with wage before it starts to fall. Second, costly borrowing does not simply shift the entire distribution to the left (or up) as in the section 4. Instead, the increasing borrowing cost have two effects: (i) it lowers maximum wage offered, and (ii) it causes more firms to offer lower wages. This could imply that the presence of financial markets and possible inefficiencies associated with their functioning would have a significant on the equilibrium distribution of wages. Using equation (3) and (7) we could characterize the effect of these markets on wage and firm size distributions.

Figure 3: Wage offers cumulative distribution, $F(w)$, and density function, $F'(w)$. No friction (Blue Line), $r_b = r = 5\%$ (Red Line) and $r_b = 10\%$ (Dashed Line)



6 Conclusions

In this note, we presented the effects of introduction of working capital channel on the distribution on wage offers and on the distribution of wages in the canonical on the job search framework of Burdett and Mortensen (1998). We showed that the wage offer distribution and wage distribution shift to the left with the raise of the cost of borrowing. We also found that the size of the firm and firm's poaching behavior is affected by the working capital channel. These results are reinforced if we introduce the match-specific capital that affects the productivity of newly hired worker. The partial equilibrium analysis in this paper concentrates only on the steady-state effects. Extending the model to the general equilibrium setting is left for future work.

A Wage offer density function

A.1 Effects of r_b on maximum wage

Using $F(\overline{w}_b = 1)$ equation (17) can be written as

$$\frac{p-(1+r_b)b}{(\delta+\lambda)(r+\delta+\lambda)} = \frac{p-(1+r_b)\overline{w}_b}{(r+\delta)\delta}.$$

Using this expression we can write the largest wage paid as

$$\overline{w}_b = b + \left(1 - \frac{(r+\delta)\delta}{(\delta+\lambda)(r+\delta+\lambda)}\right)\left(\frac{p}{1+r_b} - b\right). \tag{43}$$

A.2 Effects of r_b on $F(w)$.

Consider equation (19). First, note that

$$\frac{\partial}{\partial r_b}\left(\frac{p-(1+r_b)w}{p-(1+r_b)b}\right) = -\frac{p(w-b)}{[p-(1+r_b)b]^2} < 0,$$

and

$$\frac{\partial^2}{\partial r_b^2}\left(\frac{p-(1+r_b)w}{p-(1+r_b)b}\right) = -\frac{2pb(w-b)}{[p-(1+r_b)b]^3} < 0.$$

The derivative of $F(w)$ with respect to cost of borrowing r_b equals

$$\frac{\partial F(w)}{\partial r_b} = \frac{1}{\lambda}\left(r^2 + 4(\delta+\lambda)(r+\delta+\lambda)\left(\frac{p-(1+r_b)w}{p-(1+r_b)b}\right)\right)^{-\frac{1}{2}}(\delta+\lambda)(r+\delta+\lambda)\frac{p(w-b)}{[p-(1+r_b)b]^2} > 0$$

and

$$\frac{\partial^2 F(w)}{\partial r_b^2} = \frac{1}{2\lambda}\left(r^2 + 4(\delta+\lambda)(r+\delta+\lambda)\left(\frac{p-(1+r_b)w}{p-(1+r_b)b}\right)\right)^{-\frac{3}{2}}(\delta+\lambda)(r+\delta+\lambda)\frac{2pb(w-b)}{[p-(1+r_b)b]^3} > 0$$

A.3 Distribution $G(w)$.

Taking equation (4) and substituting in equation (19) we can write $G(w)$ as

$$\begin{aligned} G(w) &= \frac{\delta F(w)}{\delta + \lambda(1-F(w))} \\ &= \frac{\delta}{\lambda}\left[\frac{2(\delta+\lambda)}{\sqrt{r^2 + 4(\delta+\lambda)(r+\delta+\lambda)\left(\frac{p-(1+r_b)w}{p-(1+r_b)b}\right)} - r} - 1\right] \end{aligned}$$

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