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Taste Heterogeneity, Elasticity of Substitution and Green Growth

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Motivation

- Goal: understand the relation between elasticity of demand and the distribution of consumers' taste
- The elasticity is the primitive in a range of models
  - monopolistic competition
  - endogenous growth
  - optimal diversity
- How the results depends on the degree of consumers heterogeneity

Related Literature

- Relating elasticity of substitution and elasticity of demand to taste heterogeneity of consumers
  - Hotelling (1929), Salop (1979), Perloff and Salop (1985), Anderson, de Palma and Thisse (1988, 1989)
- love for variety of a representative consumer:
  - Dixit and Stiglitz (1975, 1977,1979, 1993), Pettingel (1979), Yang and Heijdra (1993)
- Anderson de Palma and Thisse (1987): CES as a reduced form of a discrete choice model.

Overview of the Theoretical Results

- In the presence of consumers' taste heterogeneity, price elasticity of demand can be expressed as an increasing function of
  - elasticity of substitution between goods for individual consumer and
  - dispersion of consumers' relative valuation of goods

Dixit and Stiglitz (1977)

- Representative consumer with preferences described by the CES utility function:

$$U = \left(\sum_{j=1}^{N_t} (x_j)^\rho \right)^{\frac{1}{\rho}}$$

- The share of income devoted for good j:

$$\phi_j = \frac{Q_j p_j}{Y} = \frac{p_j^{-\frac{\rho}{1-\rho}}}{\sum_k p_k^{-\frac{\rho}{1-\rho}}}$$

- and price elasticity of demand is

$$\frac{dQ_j}{dp_{jt}} \frac{p_j}{Q_j} = -\frac{1}{1-\rho}$$

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$$\frac{dQ_j}{dp_{jt}} \frac{p_j}{Q_j} = -\frac{1}{1-\rho}(1-\rho\phi_j)$$

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- and price elasticity of demand is

$$\frac{dQ_j}{dp_{jt}} \frac{p_j}{Q_j} = \left[-\frac{1}{1-\rho} (1 - \phi_j) \right] + [-\phi_j]$$

Dixit and Stiglitz with Taste Heterogeneity

- Consumer with preferences described by the CES utility function:

$$U_i = \sum_{j=1}^{N_t} (\theta_{ij} x_{ij})^{\rho}$$

- x_{ij} is the quantity of product j consumed by individual i
    - θ_{ij} is the idiosyncratic taste parameter
    - Taste heterogeneity: each consumer might have different valuation of product j

- The demand faced by the producer if $\rho < 1$ is given by:

$$Q_j = \int \int \dots \int \frac{\left(\frac{\theta_{ij}}{p_j}\right)^{\frac{\rho}{1-\rho}}}{\sum_k \left(\frac{\theta_{ik}}{p_k}\right)^{\frac{\rho}{1-\rho}}} p_j^{-1} y g\left(\underline{\theta}\right) d\underline{\theta}$$

- and price elasticity of demand is

$$\frac{dQ_j}{dp_j}\frac{p_j}{Q_j} = -\frac{1}{1-\rho}\left(1-\rho\frac{\int \int \dots \int \phi_{ij}^2 y g\left(\underline{\theta}\right) d\underline{\theta}}{\int \int \dots \int \phi_{ij} y g\left(\underline{\theta}\right) d\underline{\theta}}\right)$$

$$= -\frac{1}{1-\rho}\left(1-\frac{E\left(\phi_{ij}^2\right)}{E\left(\phi_{ij}\right)}\right) - \frac{E\left(\phi_{ij}^2\right)}{E\left(\phi_{ij}\right)}$$

Measure of taste heterogeneity

- Symetric equilibrium exists if the distribution of tastes is symmetric $\left(E\left(\phi_j\right) = E\left(\phi_k\right),\ E\left(\phi_j^2\right) = E\left(\phi_k^2\right)$ for any $j,\ k$) and if all goods have the same supply curve.
- If $\theta^{\frac{\rho}{1-\rho}} \sim Gamma\left(\frac{\mu}{D}, \frac{1}{D}\right)$, and N is the number of goods then

$$\frac{E\left(\phi^2\right)}{E\left(\phi\right)} = \frac{\mu + D}{N\mu + D}$$

If $\theta^{\frac{\rho}{1-\rho}} \sim St\left(\gamma, c\right)$, then

$$\frac{E\left(\phi^2\right)}{E\left(\phi\right)} = \left((1-\gamma) + \frac{1}{N}\gamma\right)$$

Representative Consumer

- At the point of symmetric equilibrium, the demand faced by firm is given by

$$log\left(Q_j\right) = -\frac{1}{1-\rho}\left(1-\rho D\left(\psi\right)-\frac{\rho}{N}\right) * log\left(p_j\right)$$

$$+\frac{\rho}{1-\rho}\left(\frac{-D\left(\psi\right)}{N-1}+\frac{1}{N}\right)\sum_{k\neq j}\log\left(p_k\right)+\log\left(\frac{y}{N}\right)$$

This is exactly the Walrasian demand of the *representative consumer* with $U = \left(\sum_{j=1}^{N_t}\left(x_j\right)^{\eta}\right)^{\frac{1}{\eta}}$ where $\eta = \rho\frac{1-\frac{1}{N}-D}{1-\frac{1}{N}-\rho D}$

Application to green growth theory:

- Speed of transition towards emission-free economy depends on the substitutability between dirty (emission-intensive) and clean goods
- The elasticity of substitution decreases with the dispersion of consumers' tastes
- for any given level of taste dispersion, aggregate elasticity of substitution between clean and dirty goods increases with the number of clean varieties

Application: Green Growth

- Consumer i derive utility from two types of goods: clean and dirty:

$$u_i = \left(\theta_{ci} x_c^{\rho} + x_d^{\rho}\right)^{\frac{1}{\rho}}$$

- The clean and dirty goods produced with labour and the range of machines:

$$Q_j = l_j^{1-\alpha} \int_0^1 A_{vj}^{1-\alpha} z_{vj}^{\alpha} dv$$

- Machines supplied by monopolists charging the mark-up $\mu = \frac{1}{\alpha}$ over unit cost of production, φ
- Production of dirty good is associated with CO2 emissions, $M = \vartheta Q_d$

- $$M = \vartheta Q_d$$

  In equilibrium

$$\frac{d\log(M)}{dt} = \frac{d\log(Q_d)}{d\log(A_c)} \frac{d\log(A_c)}{dt}$$

$$\frac{d\log(Q_d)}{d\log(A_c)} = \alpha - \frac{\rho}{1-\rho}(1-\alpha)\left(1 - \frac{E\left[\phi_{id}^2\right]}{E\left[\phi_{id}\right]}\right)$$

Variety of Clean Goods

- Suppose there is a variaty of clean goods and for individual consumer they are perfectly substitutable:

$$u_i = \left(\left(\sum_{k=1}^{n} \theta_{cik} x_{cik} \right)^{\rho} + x_{id}^{\rho} \right)^{\frac{1}{\rho}}$$

- Suppose also that θ_{cik} are iid with cdf given by $G(\theta)$.
- Each consumer chooses one favourite clean good.

Effect of variety on Emission Path

- $$\frac{d\log(M)}{dt} = \left(\alpha - \frac{\rho}{1-\rho}(1-\alpha)\left(1 - \frac{E\left[\phi_{id}^{2}\right]}{E\left[\phi_{id}\right]}\right)\right)\frac{d\log(A_c)}{dt}$$

$$\frac{E\left[\phi_{id}^{2}\right]}{E\left[\phi_{id}\right]} = \frac{\int \phi_d^2 G(x)^{n-1} g(x)\,dx}{\int \phi_d G(x)^{n-1} g(x)\,dx}$$

which is a decreasing function of n, number of varieties of clean good.

Conclusions

- At the aggregate level, the response of change in price to the total demand for a variety depends on the first two moments of consumers taste distribution.
- When dispersion of tastes is high, the substitution effect disappears and elasticity of aggregated demand is close to unity.
- Application to green growth theory
  - heterogeneity of consumers' tastes slows down phase-out of the dirty sector and reduction of emissions.
  - increase in the number of clean varieties can reduce the mass of consumers who are unwilling to substitute the dirty good with a clean alternative.

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THANK YOU

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