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BUSINESS CYCLES, WORKING CAPITAL, AND ON THE JOB SEARCH

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We study steady state and business cycle properties of a model with heterogeneous firms and on-the-job search in the spirit of Burdett and Mortensen (1998) and Moscarini and Postel-Vinay (2013). The model is solved using a novel numerical method that uses Chebyshev polynomial approximation for dealing with heterogeneity. We analyse worker flows between firms and the distribution of firm size and wages. We also introduce a simple working capital channel.

INTRODUCTION AND OVERVIEW

We build a dynamic model based on the work of Burdett and Mortensen (1998) and Moscarini and Postel-Vinay (2013). Thanks to the use of a novel numerical method we are able to embed the labour market in a full RBC framework and analyse a richer model setup. This study attempts to make a contribution in the following areas:

- Most labour markets are modelled using the Diamond-Mortensen-Pissarides framework whereas this study explores the role of on-the-job search.
- There is a growing literature on introducing heterogeneity in macroeconomic models, and in this paper we develop a robust method for handling heterogeneity which can be used in a wide array of model specifications.
- The Great Recession brought attention to financial markets and their influence on the macroeconomy and on labour markets. In this paper we explore the role of a working capital constraint in an on-the-job search framework and therefore contribute to the growing literature combining financial and labour markets.

MODEL OUTLINE

- Firms differ in productivity and are distributed according to cdf $\Gamma(p)$ on interval $[\underline{p}\ \overline{p}]$, pdf is $\gamma(p)$.
- Type-p firm produces output with labour according to: $A_t p K_t^{\alpha_K} L_t^{\alpha_L}$ and posts vacancies $v_t(p)$.
- Total vacancies are given by: $VAC_t = \int_{\underline{p}}^{\overline{p}} v_t(p)\gamma(p)dp$.
- Employed N_t and unemployed U_t send job offers with intensities: λ_e and λ_u.
- Number of potential job matches is: $M_t = \upsilon VAC_t^{1-\mu}(U_t\lambda_u + N_t\lambda_e)^{\mu}$.
- Denote probability of finding job as Φ_t and probability of filling vacancy as Ψ_t.
- Employed only move to firms of higher wage (and productivity).
- Average firm size evolves according to ($N_t(p)$ is cdf of employment):

$$L_{t+1}(p) = (1-\delta)\left(1-\Phi_t^{N}\frac{\overline{VAC_t}(p)}{VAC_t}\right)L_t(p) + U_t\Phi_t^{U}\frac{v_t(p)}{VAC_t} + (1-\delta)N_t\Phi_t^{N}\frac{v_t(p)}{VAC_t}\frac{N_t(p)}{N_t}$$

Probability of losing job or moving to a better firm — **New hires from pool of unemployed** — **New hires from lower-*p* firms**

SOLUTION METHOD

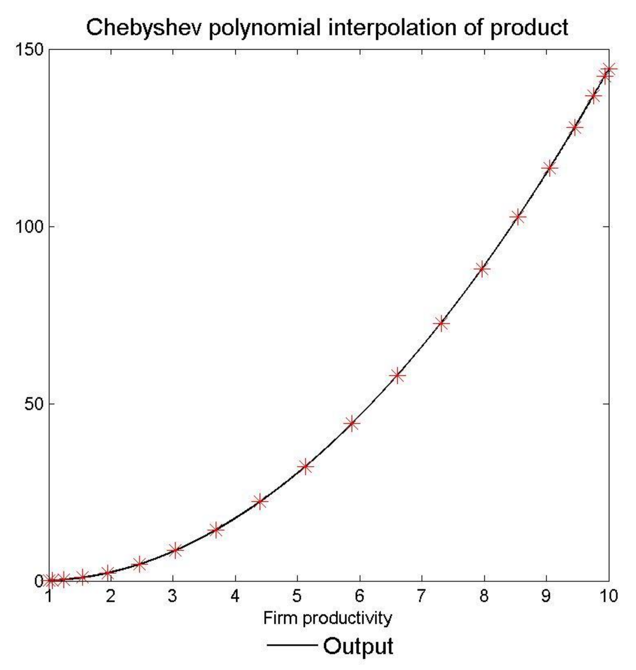
The difficulty in solving the model is due to the fact that: i) there are several functions of firm productivity that need to be tracked, and ii) there are a number of nontrivial integrals that need to be calculated. The main idea behind the numerical procedure is to use Chebyshev polynomial approximation, which means that for each function of productivity it is enough to track its value at *N* predetermined nodes.

Chebyshev polynomial approximation

For function $f(p)$ the approximation is:

$$P_N^f(p) = \sum_{n=1}^{N} b_n^f \times T_n(h(p))$$

Where: b_n^f - weights, T_n - Cheb. Polynomials, $h(p)$ - linear transformation



If we need to calculate an integral like: $\overline{V_t}(p) = \int_p^{\overline{p}} v_t(s)\gamma(s)ds$, then using the approximation:

$$\int_a^1 f(x)dx \approx \int_a^1 P_N^f(x)\,dx = \sum_{n=1}^{N} b_n^f \int_a^1 T_n(x)dx = \sum_{n=1}^{N}\frac{2}{N}\sum_{k=1}^{N} f(x_k)T_n(x_k)\int_a^1 T_n(x)dx$$

Finally we have:

$$\int_{-1}^{1} f(x)dx \approx F_{1\times N} \times W_{N\times N} \times T_{N\times 1}^{a}\ ,$$

where only *F* depends on the model's variables, and *W* and *T* are exogenous parameters. Calculating integrals boils down to a scalar product of function values and parameters, which means it is now easy to calculate the steady state and use standard methods (Judd, Uhlig, Dynare) to solve model dynamics.

RESULTS

Main result are consistent with BM and MPV models. There is improvement wrt MPV model due to addition of capital:

- Largest to smallest firm size ratio approx 20000 (4 for MPV model), which also results in employment in firm-size groups consistent with US data (Figure 1 and 2).
- Gini coefficient 0.38, close tu USA data, although highest to smallest wage ratio is low (Figure 3).

Figure 1. Distribution of workers across firms.

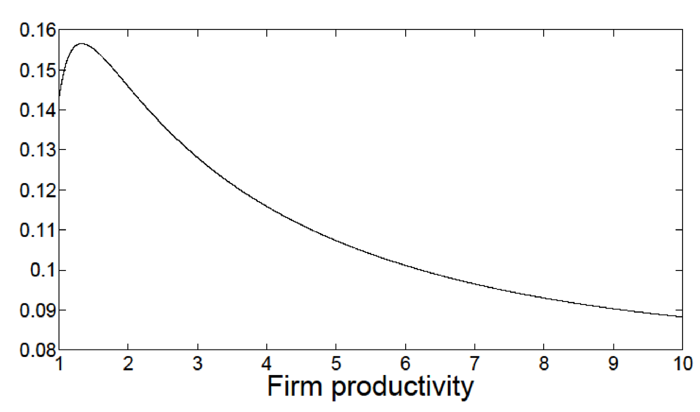


Figure 2. Average firm size.

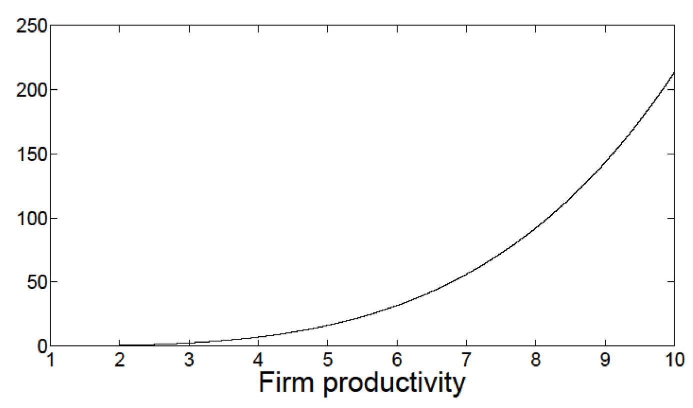


Figure 3. Distribution of wages.

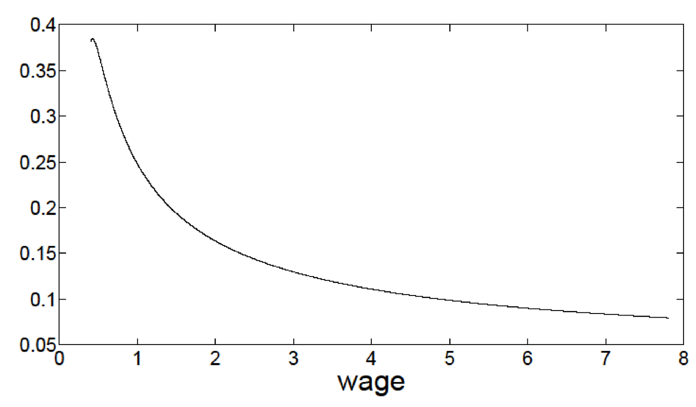


Figure 4 shows the model is consistent with empirical observations – large employers contribute more to job creation in expansions. Figure 5 shows the effect of a simple working capital constraint modelled as a markup on the wage bill. We find that the result is a shift of the distribution of workers to the right.

Figure 4. IRF of distribution function wrt ss of workers across firms to technology shock.

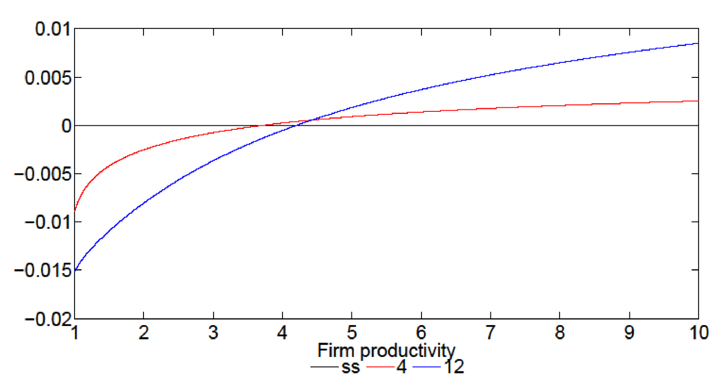
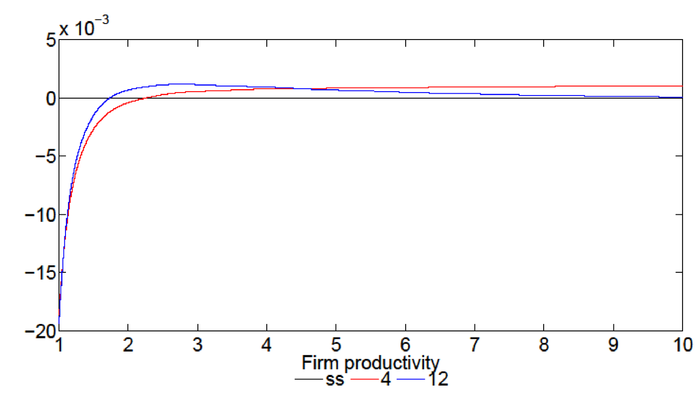


Figure 5. Effect of wage markup shock on distribution of workers.



SELECTED REFERENCES

1. Burdett, K. and Mortensen D.T. (1998). Wage differentials, employer size, and unemployment. *International Economic Review*, 257-273.
2. Moscarini, G. and Postel-Vinay, F. (2012). The contribution of large and small employers to job creation in times of high and low unemployment. *The American Economic Review*, 102(6), 2509-2539.
3. Moscarini, G. and Postel-Vinay, F. (2013). Stochastic search equilibrium. *The Review of Economic Studies*, 1545-1581.